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MARKOV APPROACH TO THE CONSTRUCTION OF NONLINEAR AUTOREGRESSIVE MODELS

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Abstract. Time series forecasting used by the theory of Markov chains are considered. For this purpose the main task is to find the transition probabilities of Markov chain on the basis of observed values of the time series. It is proved, that to find the transition probabilities of Markov chain with all necessary requirements one can use the quadratic programming on simplex. The consistent and unbiased estimations of the transition probabilities are building due to solving the above mentioned quadratic programming problem in MATLAB. Then these estimations are checked up by experimental way with the method of Monte-Karlo.

Key words. Time series, Markov chain, transition probability, regression model, statistical estimation

Mathematics Subject Classification: 60J10, 62F12, 11K45

1 Introduction

One of main problems of modern econometrics is development of time series $\{x_t, t \in Z\}$ methods of analysis through regression models without a priori information about the form of dependence of the conditional expected value from its past values (see, for example, [1,2]). Principal reason of waiver of traditional linear models is no Gauss type of random values, describing the dynamics of the real models. We will remind that assumption about normal distribution of time series allows to calculate the conditional expected value of phase variable as linear functional of its past values $\{x_t, s < t\}$. Null information about the distribution law does not allow to calculate the conditional expected value mentioned above analytically as some function with unknown parameters and to take part a problem to the estimation on a least-squares method, as it is accepted in a linear gauss theory. Therefore it is necessary to deal with the estimation of unknown function in nonlinear difference equation of the first order with usual kind of information about the distribution law in many applied problems of regression analysis for time series $\{x_t, t \in N\}$ already in simplest case [1,3,4,5]

$$x_t = f(x_{t-1}) + h_t, \tag{1}$$

where h_t are the uncorrelated tailings, on the average equal to the zero. The problem of analysis of time series described higher got the name of nonparametric estimation of autoregression [6]. If we designate $\mathcal{F}^t$ minimum sigma-algebra, in relation to which random values $\{h_s, s \le t\}$ are measured, the needed function can be defined through the conditional expected value $f(x_{t-1}) := \mathbb{E}\{x_t / \mathcal{F}^{t-1}\}$. To use the sequence of sigma-algebra $\{\mathcal{F}^t, t \in \mathbb{Z}\}$ and conditional dispersion $\sigma_t^2 := \mathbb{E}\{h_t^2 / \mathcal{F}^{t-1}\}$, tailings h_t can be present [7] in form work of "white noise" $\{\xi_t, t \in \mathbb{Z}\}$ in equation (1), (i.e. sequences of the independent identically distributed (i.i.d.) random values with zero mean and by single dispersion) and with conditional standard deviation: $h_t := \sigma_t \xi_t$.

In respect of conditional dispersion, it can be random $\mathcal{F}^{t-1}$ measurable value which depends on $\{\xi_t, s \le t-1\}$, and also be a subject of estimation. This property of tailing's dispersion is called [7] as conditional heteroskedasticity and can be modulate through linear difference equations with coefficients, linearly depending on white noise (GARCH (p,q) processes). For searching for the function $f(x)$ the set of values we can break up enough small length δ on intervals and then on every interval we can use either least-squares method or minimize specially built functional as an integral with the kernels of different form (nuclear estimation [2,6]).

2 Description of model

We will suppose that is observed random process of type

$$x_{n+1} = f(x_n) + \sigma_n \xi_{n+1} \quad , \tag{2}$$

ξ_n is a random error of observations, (i.i.d.) . $\mathrm{E}\{\xi_n\} = 0$,

$f(x_n)$ is a nonlinear function of the elements of chain .

Equation (2) can be interpreted so, that a random sequence depends on the «history».

Also we can write that the conditional expected value of random variable looks like

$$\mathbf{E}\{x_{n+1} \mid F^n\} = \mathbf{E}\{x_{n+1} \mid x_n\} = \sum_y p(x, y) \cdot y = f(x_n) \tag{3}$$

that determines non-linearity of functional dependence x_{n+1} from x_n. The purpose of our researches is to describe the dynamics of chain $\{x_n\}$. This means to find the functional dependence $f(x_n)$, due to equation (3). For searching for of function $f(x_n)$ we need to create separate discrete intervals of values and then on every interval we can use either least-squares or minimize specially built functional as an integral with the kernels of different form. We will consider the model of phase space discretization and presentation of him in form eventual number of no splitting areas $\{S_k, k = 1,...,r\}$ which can be examined as the states of some Markov chain. Since the probabilistic behaviour of a Markov chain is determined by the transition probability matrix ***P*** and a probability distribution over the initial state X_0 , if we are given X_0 and ***P***, we may want to determine the probability distribution for each random variable X_n or possibly we may be interested in the limiting distribution of X_n as $n \to \infty$,, if such a distribution exists. Within this context, if a chain is irreducible and aperiodic and thus ergodic, then there exists a unique row vector $\boldsymbol{\pi} = (\pi_1, \pi_2,..., \pi_r)$, such as

$$\lim_{m\to\infty} p_{ij}^{(m)} = \pi_j, \quad i, j = 1,2,...,r,$$

where $p_{ij}^{(m)}$ is the *(i,j)*th element of $\boldsymbol{P}^t$,

$$p_{ij}^{(m)} = P(X_m = j \mid X_0 = i)$$

and

$$0 \le \pi_j \le 1$$
$$\sum_j \pi_j = 1, \quad j = 1,2,...,r$$

and

$$\boldsymbol{\pi} = \boldsymbol{\pi} P.$$

3 The micro maximum likelihood estimator

When a sample of repeated observations of the Markov chain exists, and time ordered data that reflect the intertemporal movements of the micro units over the states are available, then, as shown by [1], equation

$$P(x_0, x_0,..., x_T) = P(x_0) \cdot \prod_{t=1}^{T} P(x_t \mid x_{t-1}), \tag{4}$$

yields a likelihood function and serves as a basis for obtaining estimates of the transition probabilities and making certain tests of hypotheses about these parameters. Samples of time-ordered micro data and the availability of methods for estimation and inference formed the basis for the applications of the Markov model that were previously noted. Unfortunately, data involving time-ordered detailed changes are frequently not available, are often too expensive to obtain or are incomplete, and what we must work with are their aggregated sample counterparts. For example, census data usually give only the frequency distribution of the number of individual units in each size class for each census year and report no information on the time path behaviour of each individual. Assume further that we are given $n_i(0)$ individuals in stale i at time $t=0$, and that the elements of an observation indicate the sequence of states the individuals are in at $t = 0, 1, \ldots, T$. Let us specify the probability of an ordered sequence for a stationary Markov process as equation (4). If we let $n_{ij}(t)$ denote the number of individuals for which $x_{t-1} = i$, $x_t = j$, and

$$n_{ij} = \sum_t n_{ij}(t)$$

the probability of a given ordered set of sequences for the n individuals $\mathbf{n}(t)' = (n_1(t), n_2(t), \ldots, n_r(t))$, within the context of eq. (4), may be specified with $\propto$ denoting proportionality as

$$P(x_0, x_1, \ldots, x_T \mid \mathbf{n}) \propto P(x_0) \cdot \prod_{i,j} p_{ij}^{n_{ij}}, \tag{5}$$

and as shown in [1], the n_{ij} form a set of sufficient statistics.

The distribution of the $n_{ij}(t)$ may be obtained by considering the $n_i(t-1) = \sum_j n_{ij}(t)$ observations on a multinomial distribution with probabilities p_{ij}. One can shows that maximum likelihood (ML) estimator is

$$\hat{p}_{ij} = \frac{n_{ij}}{\sum_j n_{ij}}. \tag{6}$$

It is well known that ML estimator is consistent but it is not generally unbiased. But one can shows when the sample size increases, the bias tends to zero and the estimates are asymptotically normally distributed.

4 The estimation of transition probabilities from macro data

The transition probability estimators developed in the following parts of the article proceed under the assumption that the individual time traces, $n_{ij}(t)$, relative to the sequence of states are unavailable and only the sample aggregate proportions relating to the number of individuals in each state for each time period t are known. If the $n_{ij}(t)$ sample observations are not available and only the aggregate outcome data, $n_j(t)$, which are equal to $n_j(t) = \sum_i n_{ij}(t)$, are available, then one way to make use of the observed proportion data in estimating the transition probabilities is to make use of the argument of conditional probability in the following way:

$$P(x_{t-1} = i, x_t = j) = P(x_{t-1} = i) P(x_t = j \mid x_{t-1} = i) \tag{7}$$

Then using the generalized addition law of probability, then

$$P(x_t = j) = \sum_i P(x_{t-1} = i) P(x_t = j \mid x_{t-1} = i)$$

or

$$q_j(t) = \sum_i q_i(t-1) \, p_{ij} \tag{8}$$

where $q_j(t)$ and $q_j\ (t — 1)$ represent the unconditional probabilities $P(x_t = j)$ and $P(x_{t-1} = i)$ respectively. If the unconditional probabilities $q_j(t)$ and $q_j\ (t — 1)$ in eq. (8) are replaced by the actual observed proportions $y_j(t)$ and $y_j(t-1)$, then there will be no set of transition probabilities that will satisfy this relation with probability one. Thus, if errors are admitted in eq. (8) to account for the difference between the actual and estimated occurrence of $y_j(t)$ based on the right-hand side of eq. (8), then the sample observations may be assumed to be generated by the following stochastic relation:

$$y_j(t) = \sum_i y_i(t-1) \, p_{ij} + u_j(t) \tag{9}$$

We are proposed using this stochastic relation as a basis for specifying a linear statistical model for estimating the transition probabilities. It is to this proposition that we now turn.

5 The unrestricted least squares transition probability estimator

In developing approach for estimating the transition probabilities in (9) from sample proportion data, let us rewrite the stochastic relation given in (9) in matrix form as

$$\mathbf{y}_j = X_j \, \mathbf{p}_j + \mathbf{u}_j, \tag{10}$$

where $\mathbf{y}_j = \{ y_j(t)\}$ is a (T × 1) vector of sample proportions, $\mathbf{p}'_j = (p_{1j}, p_{2j}, ..., p_{rj})$ is a (r × 1) vector of unknown transition parameters to be estimated, $\mathbf{u}_j$ is a (T × 1) vector of random disturbances and X_j is the following (T × r) matrix:

$$X_j = \begin{bmatrix} y_1(0) & y_2(0) & \cdots & y_r(0) \\ \vdots & \vdots & & \vdots \\ y_1(t-1) & y_2(t-1) & \cdots & y_r(t-1) \\ \vdots & \vdots & & \vdots \\ y_1(T-1) & y_2(T-1) & \cdots & y_r(T-1) \end{bmatrix}. \tag{11}$$

It is assumed that the matrix X_j has rank $\boldsymbol{r}$. We make the following assumptions about the random disturbance vector $\mathbf{u}_j$ in (10):

$$\mathrm{E}(\mathbf{u}_j) = 0$$

$$\mathrm{E}(\mathbf{u}_j \mathbf{u}_i') = \sigma_j \omega_{jj} ,$$

where ω_{jj} is a *(TxT)* positive definite diagonal matrix.

The set of equations of which (9) and (10) is a part, may then be written as

$$\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_r \end{bmatrix} = \begin{bmatrix} X_1 & 0 & \cdots & 0 \\ 0 & X_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & X_r \end{bmatrix} \begin{bmatrix} \mathbf{p}_1 \\ \mathbf{p}_2 \\ \vdots \\ \mathbf{p}_r \end{bmatrix} + \begin{bmatrix} \mathbf{u}_1 \\ \mathbf{u}_2 \\ \vdots \\ \mathbf{u}_r \end{bmatrix} \tag{12}$$

or

$$\mathbf{y} = X\ \mathbf{p} + \mathbf{u}, \tag{13}$$

with X is the block diagonal matrix on the right-hand side of (12) with $X_1 = X_2 = ... = X_r$

$$\mathrm{E}(\mathbf{u}) = 0$$

$$\mathrm{E}(\mathbf{u}\ \mathbf{u}') = \Sigma ,$$

where Σ is a $(T\,r \times T\,r)$ non-diagonal, singular matrix.

Given the multivariate linear statistical model (12) or (13),and assuming T strictly greater than r, in [1] suggested the use of the conventional least squares estimator as a basis for obtaining estimates of the transitional probabilities. That is, he viewed the problem as one of finding the estimate p which minimizes the positive definite the quadratic form

$$\varphi = \mathbf{u}'\mathbf{u} = (\mathbf{y} - X\mathbf{p})'(\mathbf{y} - X\mathbf{p}) \tag{14}$$

Solving this extremum problem in the conventional way yields the minimizing solution

$$\hat{\mathbf{p}} = (X'X)^{-1} X'\mathbf{y} , \tag{15}$$

provided that $X'X$ is non-singular. Since Xj, $j = 1,2, \ldots, r$, has been assumed to be of rank $\boldsymbol{r}$, $X'X$, a block diagonal matrix with matrices $X'jXj$ on the main diagonal, will be non-singular. Since the matrix $X'X$ is positive definite (and also symmetric), both the necessary and sufficient conditions for $\hat{\mathbf{p}}$ to minimize (14) are fulfilled.

Although the set of relations (12) are 'disturbance related', since $X_1 = X_2 = ... = X_r$, the j equations may be estimated separately or jointly by generalized least squares with the same results. Thus, the unrestricted 'conventional' least squares estimator for $\hat{\mathbf{p}}_j$, (a subvector of $\hat{\mathbf{p}}$, is, from (15),

$$\hat{\mathbf{p}}_j = (X_j' X_j)^{-1} X_j' \mathbf{y}_j , \quad j = 1,2,...,r , \tag{16}$$

The question that now arises is whether or not these conventional least squares estimates of the transition probabilities, p_{ij} , satisfy the following non-negativity and row sum conditions :

$$0 \le p_{ij} \le 1 \tag{17}$$

$$\sum_j p_{ij} = 1, \quad i = 1,2,...,r . \tag{18}$$

It is not difficult to prove [8] that the estimation of transitional probabilities on a least-squares method automatically meets the condition of the rate fixing. It, unfortunately, it is impossible to say about the condition of nonnegative. It means that the application of ordinary least-squares method (without the account of limitations) can give a result, that the above mentioned probabilities can be negative (or to exceed unit).

In [1] was recognized that the conventional least squares estimator may violate the condition $0 < p_{ij} < 1$. In this case was suggested that the appropriate estimate will lie on the boundary of the restricted parameter subset and concluded that the estimate on the boundary of the subset which minimizes the quadratic form (14) should be used. Based on [1] the authors of [3] have proposed an inequality restricted estimator which satisfied Goodman's conditions. They chose estimates which minimize the quadratic form (14) subject to constraints

$$G\mathbf{p} = \boldsymbol{\eta}_r , \tag{21}$$

$$\mathbf{p} \ge 0 . \tag{22}$$

Since (14) appears as a quadratic form in $\boldsymbol{p}$ and the restrictions are linear, this problem is a typical quadratic programming problem. Following [7], by making use of the Kuhn-Tucker equivalence theorem for non-linear programming and the duality theorem for quadratic programming, we may reduce the problem to the following linear programming specification: Find $\tilde{\mathbf{p}}^c$ that maximizes

$$(X'\mathbf{y} - X'X\,\tilde{\mathbf{p}}^c)'\,\mathbf{p}, \tag{23}$$

subject to

$$G\mathbf{p} \le \boldsymbol{\eta}_r , \tag{24}$$

$$-G\mathbf{p} \leq -\boldsymbol{\eta}_r, \tag{25}$$

$$\mathbf{p} \geq 0, \tag{26}$$

where $\tilde{\mathbf{p}}^c$ is the optimal restricted least squares estimator.

By virtue of the duality theorem of linear programming the corresponding dual linear programming is to minimize

$$\begin{bmatrix} \boldsymbol{\lambda}'_1 & \boldsymbol{\lambda}'_2 \end{bmatrix} \begin{bmatrix} \boldsymbol{\eta}_r \\ -\boldsymbol{\eta}_r \end{bmatrix}$$

subject to

$$\begin{bmatrix} G' & -G' \end{bmatrix} \begin{bmatrix} \boldsymbol{\lambda}'_1 \\ \boldsymbol{\lambda}'_2 \end{bmatrix} \geq X'\mathbf{y} - X'X\,\tilde{\mathbf{p}}^c$$

and

$$\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2 \qquad\qquad \geq \; 0,$$

where $\boldsymbol{\lambda}_1$ and $\boldsymbol{\lambda}_2$ are *(r* x 1) vectors of dual variables.

In order to develop a solution algorithm, let us remove the $\tilde{\mathbf{p}}^c$ restrictions on $\boldsymbol{p}$ in the primal and dual specifications and define the following primal-dual programming formulation: To maximize

$$(X'\mathbf{y} - X'X\,\mathbf{p})'\mathbf{p} - \boldsymbol{\lambda}'_1\boldsymbol{\eta}_r + \boldsymbol{\lambda}'_2\boldsymbol{\eta}_r = -\boldsymbol{\lambda}'_1\boldsymbol{\alpha}_1 - \boldsymbol{\lambda}'_2\boldsymbol{\alpha}_2 - \boldsymbol{\beta}'\mathbf{p} \leq 0, \tag{27}$$

subject to

$$G\mathbf{p} = \boldsymbol{\eta}_r, \tag{28}$$

$$G'\boldsymbol{\lambda}_1 - G'\boldsymbol{\lambda}_2 + (X'X)\mathbf{p} - \boldsymbol{\beta} = X'\mathbf{y}, \tag{29}$$

$$\mathbf{p}, \boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2, \boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \boldsymbol{\beta} \geq 0, \tag{30}$$

where $\boldsymbol{\alpha}_1$, $\boldsymbol{\alpha}_2$ and $\boldsymbol{\beta}$ are the vectors of slack variables to primal and dual respectively.

The above problem can be readily solved by the use of the standard simplex version of the quadratic programming algorithm. The characteristics of the formulation and algorithm are reflected in the tableau given in table 1.

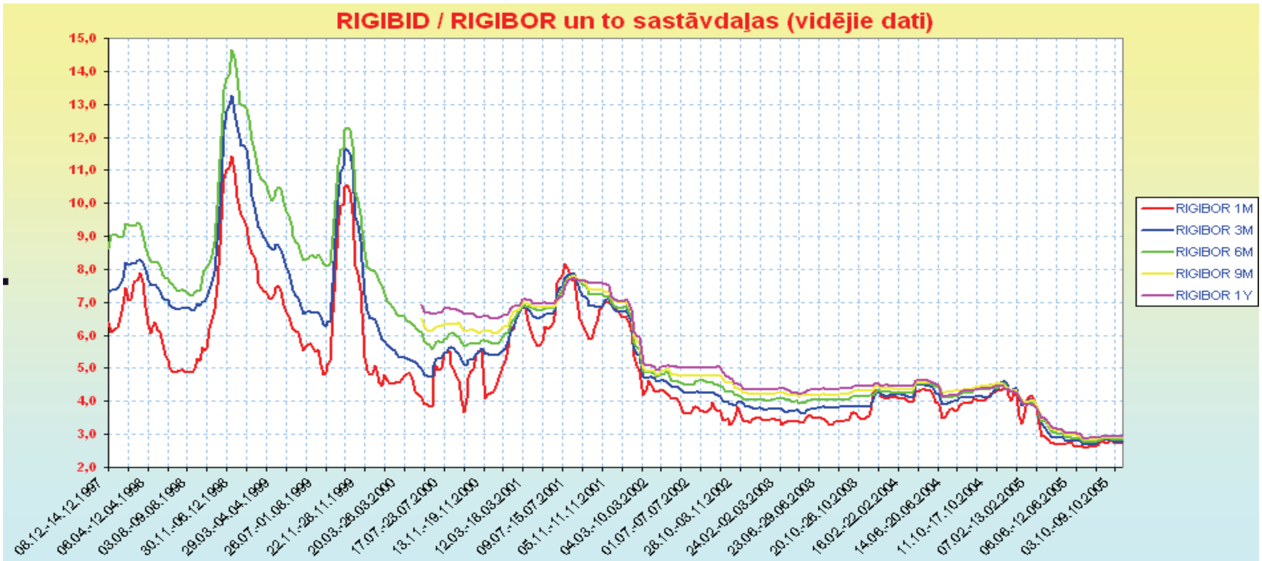
TABLE 1

Quadratic programming simplex table for the classical restricted least squares estimator

B_0	λ_1	λ_2	$\mathbf{p}$	α_1	α_2	β
η_r			G	I		
$-\eta_r$			-G		I	
$X'\mathbf{y}$	G'	- G'	$X'X$			-I

Given the difficulty of determining the sampling properties of the inequality restricted least squares transition probability estimators and the fact that for many of the other estimators for transition probabilities, only asymptotic results are available, we are concerned with specifying a probability model that may be used to generate data that will form a basis for evaluating the finite sample performance of alternative estimators via Monte Carlo experiments. A basis for gauging the performance of alternative estimators is discussed and the sampling results for the micro and macro estimators are presented.

For selective estimations the homogeneous Markov chain of the first order, got by the analysis of time series of RIGIBOR 6M index, is used.



As we see from the behavior pattern reflected by this time series indicates (1) a strong tendency of individuals to remain within a given state from one time period to the next, and (2) the most probable outcome, excluding remaining in the same state, is that individual either move up or down one state at a time. This type of behavior appears to be consistent with that observed for many kinds of economic choice units.

To construct the model of the transition matrix , we need an initial probability vector to define a Markov process. If we let $y(0)$ be the $(1 \times r)$ initial vector (proportion data) and $y(t)$ be the $(1 \times r)$ probability vector (or aggregate data) at time t, then from the definition of a Markov process,

$$\mathbf{y}(t) = \mathbf{y}(0)P^{t},$$

Obviously, different starting vectors *y*(0) generate different patterns of aggregate data *y*(*t*). In our model, there are four possible starting states and the patterns of changes in the aggregate data for the different starting states are shown in figure.

6 Results from the sampling experiment macro data

a) Unrestricted least squares. Making use of experimental data generated from a population of 1000 individuals, information on the proportion of individuals in each state for time period 1 through 25 was used to obtain the unrestricted least squares estimates of the transition probabilities. For each case, the non-negativity condition on the set of transition probabilities was violated. The mean and root mean square error statistics for the unrestricted estimates for each level of 50 problems are presented in table 2.

TABLE 2

Means and root mean square errors for the unrestricted least squares estimates

Sample size	Means				Root mean square error			
25	0,5165	0,5089	0,0180	-0,0434	0,1583	0,2228	0,2134	0,1407
	0,1504	0,3866	0,4466	0,0164	0,1437	0,2170	0,2541	0,1608
	-0,0104	0,1408	0,6067	0,2629	0,1127	0,1272	0,1795	0,1265
	-0,0001	0,0007	0,1586	0,8408	0,0802	0,0973	0,1503	0,1099
100	0,5480	0,4857	-0,0218	-0,0119	0,1115	0,1526	0,1845	0,1268
	0,1724	0,2553	0,4719	0,0004	0,1494	0,2017	0,2295	0,1615
	0,0350	0,1806	0,6501	0,2043	0,0824	0,1235	0,1137	0,0858
	0,0144	-0,0260	0,1114	0,9002	0,0411	0,0688	0,0655	0,0513

For each sample value, three or four of the means violate the non-negativity condition. In addition, they show up very badly when their means and root mean square errors are compared with the maximum likelihood results from micro data. As expected, the root mean square errors decrease as the sample size increases. When the root mean square errors are summed over all elements of the transition matrix without weighting the individual elements, the aggregated errors are 2.49, 2.05 and 1.95 for the estimates based on samples of sizes 25, 50 and 100 respectively.

b. Restricted least squares. The means and root mean square error statistics for the restricted least squares estimator for each set of 50 estimates are presented in table 3.
The restricted least squares estimates of the transition probabilities appear much superior to the unrestricted estimates given in table 2. Superiority is indicated in that the means, of course, are non-negative and the root mean square errors are absolutely smaller than those of the corresponding unrestricted estimates.

TABLE 3

Means and root mean square errors for the restricted least squares estimates

Sample size	Means	Root mean square error
25	0,4992 0,4272 0,0723 0,0013 0,1315 0,4593 0,3943 0,0149 0,0160 0,0992 0,6344 0 2504 0,0051 0,0262 0,1361 0',8326	0,1521 0,1513 0,1369 0,0058 0,1011 0,1603 0,1406 0,0291 0,0307 0,0649 0,1324 0,0921 0,0143 0,0407 0,1123 0,1149
100	0,5657 0,3953 0,0390 0,00001 0,1224 0,4754 0,3959 0,0063 0,0043 0,1060 0,6875 0,2022 0,0017 0,0113 0,0929 0,8941	0,0868 0,1147 0,0840 0,0001 0,0712 0,1199 0,0912 0,0145 0,0099 0,0393 0,0629 0,0404 0,0049 0,0232 0,0459 0,0432

7 Conclusion

Therefore, we can estimate the transition probabilities when we use the set selection and methods of parameters estimation of Markov chains with stationary assumption of process (1).

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