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ON AVERAGING OF NONSTATIONARY MARKOV DYNAMICAL SYSTEMS WITH ANTICIPATING SWITCHING

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Abstract. This paper deals with Markov impulse dynamical system with anticipating switching and coefficients, which are dependent on time. It proposes the aveaging and the conditions of exponential stability in mean square of the trivial solution of this system. The aveaging principle of the mentioned system with non-anticipating jumps without direct dependence on time was proved by Ye.Tsarkov in [4]. A.Pola in [3] have proved the aveaging principle for the system with non-anticipating switching with dependence on time. And the limit theorems for the system with anticipating switching, but without direct dependence on time were proved in [2] by O.Pavlenko. Both results were combined in this article.

Key words and phrases. Markov dynamical system, averaging, stability, second Lyapunov method.

Mathematics Subject Classification. 60H10, 60H30.

1 Introduction

The problem of asymptotical analysis of dynamical systems with a small positive parameter under random perturbations has aroused in many mathematical and engineering papers. But mainly this problem is discussing for perturbations of the vector field and the phase trajectories have remained continuous functions of time. In the present paper the perturbations are connected not only with the vector field, but in moments of switching of the tor field the phase trajectories also have jumps. These processes appear, for example, if dynamical system should be considered jointly with some flow of refusals and limit theorems should be proven both for the dynamical system and for the queuing system.

To prove the limit theorems for the above dynamical systems the methods and results of paper [1] can be successfully applied. But in this paper the author uses some specially constructed recurrent equation in the Markov moments of switching and does not use any infinitesimal characteristics of the Markov processes $\{x(t), \xi(t)\}$. This approach is poorly consented with the Second Lyapunov method which is mainly used for stability analysis of stochastic dynamical systems. Therefore in our paper we apply the martingale approach and well known Dynkin's formula. It allows us not only to prove limit theorems but also to prove that the limit equations can be used for stability analysis of the initial impulse dynamical system. It should be mentioned that to prove the convergence on probability of the solutions of the system to the solutions of the averaged equation $u(t)$ we apply some modification of the Second Lyapunov method applying the weak infinitesimal operator of the Markov process $\{x(t), \xi(t), u(t)\}$ the specially constructed functionals of the phase coordinates.

To formulate the problems which are discussed in this paper we need to describe the Markov process $\{\xi(t), t \geq 0\}$ in detail.

Let's $\{\xi_\varepsilon(t), t \geq 0\}$ be the right-continuous homogeneous Feller-Markov process on a probability space$(\Omega, \Im, P)$ with the infinitesimal operator

$$(Qv)(y) = \frac{a(y)}{\varepsilon} \int\limits_{\mathbf{Y}} [v(z) - v(y)] \, p(y, dz), y \in Y, \tag{1}$$

where ε is a small positive parameter, Y is a compact, $p(y, A)$ is a transition probability of Markov chain with Feller's property.

We will assume that spectrum of operator Q has the form

$$\sigma(Q) = \sigma_{-\rho} \cup \{0\}, \sigma_{-\rho} \subset \{z \in C : \mathbf{Re} z \leq -\rho < 0\}$$

and zero has multiplicity one. Then the process $\xi_\varepsilon(t)$ is ergodic and has unique invariant probability measure $\mu \in C^*(Y)$. Process $\xi_\varepsilon(t)$ has jumps at time moments $\{\tau_j, j \in N\}$ and time periods between jumps are exponential distributed with intensity $\frac{a(y)}{\varepsilon}$, where $a(y)$ is some continuous positive function.

$$\mathbf{P}\{\tau_j - \tau_{j-1} < t / \xi_\varepsilon(\tau_{j-1}) = y\} = 1 - e^{-\frac{a(y)}{\varepsilon} t}.$$

Let $\{x^\varepsilon(t)\dot{;} \quad t \geq 0\}$ be the right continuous n-dimension vector function which satisfies the following system:

-the differential equation :

$$\frac{dx^\varepsilon(t)}{dt} = f\left(\frac{t}{\varepsilon}, x^\varepsilon(t), \xi_\varepsilon(t)\right), \qquad \tau_{j-1} < t < \tau_j; \tag{2}$$

-the conditions of jumps:

$$x^\varepsilon(t) = x^\varepsilon(t-) + \varepsilon g\left(\frac{t}{\varepsilon}, x^\varepsilon(t-), \xi_\varepsilon(t-), \xi_\varepsilon(t)\right), \qquad t \in \{\tau_j, j \in N\}; \tag{3}$$

-the initial conditions:

$$x^\varepsilon(0) = x, \quad \xi_\varepsilon(0) = y. \tag{4}$$

2 Averaging of impulse systems

Side be side with the system of equations (2)-(4) we consider the averaged equation

$$\frac{du}{dt} = \bar{f}(u) \tag{5}$$

with initial conditions

$$u(0) = u, \tag{6}$$

where

$$\bar{f}(u) = \lim_{t\to\infty} \frac{1}{t} \int_0^t \int_{\mathbf{Y}} \left[f(s,u,\xi) + a(\xi) \int_{\mathbf{Y}} g(s,u,\xi,z)\, p(\xi,dz) \right] \mu(d\xi)\, ds. \tag{7}$$

By the definition (7) the function $\bar{f}(u)$ has continuous bounded derivative.

Lemma Let there exists such constant $c > 0$, that

$$\sup_{t\in R^+, x\in R^n} \left| \int_0^t \left[\hat{f}(s,x) - \bar{f}(x) \right] ds \right| \le c \left\| f \right\|_x \tag{8}$$

where

$$\hat{f}(s,x) = \int_{\mathbf{Y}} \left[f(s,x,y) + a(y) \int_{\mathbf{Y}} g(s,x,y,z)\, p(y,dz) \right] \mu(dy) \tag{9}$$

$$\bar{f}(x) = \lim_{t\to\infty} \frac{1}{t} \int_0^t \hat{f}(s,x)\, ds \tag{10}$$

$$\left\| f \right\|_x = \sup_{t\in R^+, y\in Y} \left| f(t,x,y) \right|. \tag{11}$$

Under above conditions the equation

$$\frac{\partial h(t,x,y)}{\partial t} + Qh(t,x,y) = \bar{f}(x) - f(t,x,y) - a(y) \int_{\mathbf{Y}} g(t,x,y,z)\, p(y,dz) \tag{12}$$

has solution in the form

$$\begin{aligned} h(t,x,y) = -\int_t^\infty e^{-Q(t-\tau)} \left[\hat{f}(\tau,x) - f(\tau,x,y) - a(y) \int_{\mathbf{Y}} g(t,x,y,z)\, p(y,dz) \right] d\tau \\ + \int_0^t \left[\bar{f}(x) - \hat{f}(\tau,x) \right] d\tau \end{aligned} \tag{13}$$

and there exists such positive constant $c_1 > 0$ that for any $r > 0$

$$\sup_{t\in R^+, y\in Y} \left| h(t,x,y) \right| \le c_1 \left\| f \right\|_x \tag{14}$$

uniformly for all $x \in U_r = \{x \in R^n : |x| < r\}$.

Theorem (Averaging principle). Under the above mentioned hypothesis the solution of (2)-(4) converges as $\varepsilon \to 0$ to the solution $u(t,x)$ of the averaging equation (5) uniformly on $x \in U_r = \{|x| < r\}$ and $t \in [0,T]$, for any $r > 0$ and $T > 0$, that is

$$\lim_{\varepsilon \to 0} \sup_{x \in U_r \quad ,y \in Y} \mathbf{P}_{x,y} \left\{ \sup_{0 \le t \le T} |x^\varepsilon(t) - u(t,x)| > \delta \right\} = 0 \tag{15}$$

for any $\delta > 0$.
Now we consider the impulse system (2)-(4) under assumptions, that

$$f(t,0,y) \equiv 0, g(t,0,y,z) \equiv 0. \tag{16}$$

Let us call the trivial solution of (5) by exponentially stable, if there exist such positive constants M and γ , that for any $x \in R^n$ and $t \ge 0$

$$|u(t,x)| \le M e^{-\gamma t} |x|. \tag{17}$$

Then the following theotem can be proven.
Theorem (Stability condition) Let the above mentioned hypotheses be fulfilled and the trivial solution of (5) be exponentially stable. There exists such $\varepsilon_0 > 0$ that for any $\varepsilon \in (0, \varepsilon_0)$ the trivial solution of (2)-(4) is exponentially stable in mean square, i.e. there are such positive constants M_1 and γ_1, that for any $x \in R^n$, $y \in$, $\varepsilon \in (0, \varepsilon_0)$ and $t \ge 0$

$$\mathbf{E}_{x,y} |x^\varepsilon(t)|^2 \le M_1 e^{-\gamma_1 t} |x|^2. \tag{18}$$

3 Conclusion

The averaging procedures can be successfully used for the analysis of the dynamical system of impulse type with anticipating switching and coefficients, which are dependent on time. If ε is sufficiently small, the averaged equation can be used for stability analysis of the initial impulse system.

References

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