Information Fusion in Decision Making under Uncertainty

Alexander Valishevsky, Arkady Borisov

1. Introduction

In this paper a decision aid methodology based on f-granules is presented. F-granules are at the kernel of the approach, as they are used to describe alternatives. An important feature of f-granules is that it is possible to use both fuzzy and random information to describe problem domain. F-granules used can be considered to be probability distribution of fuzzy values.

Distinctive feature of the approach is that alternatives are evaluated based not on their performance on criteria, as in most methodologies, but on the likelihood that performance of alternatives on criteria will be high. The likelihood is presented in the form of interval probabilities, which make ranking of alternatives a non-trivial task and allows for alternatives to be non-comparable. Thus, in the general case only a partial ranking of alternatives is obtainable. Which is in full consent with intuition – one cannot expect a methodology to give a precise answer to an extremely imprecise question – and it replicates the situation in such decision aid methodologies as PROMETHEE or ELECTRE [1, 2]. Usually it is not possible to obtain a complete ranking without loosing some information, as the decision problem can be too hard for any methodology to be able to make a smart trade-off between conflicting alternatives. Thus, it can be advantageous to build a partial ranking and to delegate trading-off to the decision maker.

As with any decision aid methodology that is used for decision-making under uncertainty, in our setting the notion of optimality is vague. Thus, the main task of the methodology is not to extract an optimal alternative, in whatever way it is defined, but rather to use such tools as ranking and sensitivity analysis to help decision-maker to make a better and more informative decision.

This methodology differs from similar decision aids in several aspects. The main difference is that the proposed decision aid can be used when the available information is very uncertain. It may be so uncertain that it will not be possible to construct a performance table or to describe the problem domain using any crisp methodology or framework, including interval probabilities. Regardless of the fact that this methodology uses interval probabilities, they are all derived with the methodology described in [3] from fuzzy and probabilistic information, provided by the decision maker or an expert.

In the next chapter we give a brief introduction to conditional f-granules and then we describe the methodology.

2. Conditional fuzzy granules

As was mentioned above, conditional fuzzy granules can be viewed as probability distribution of fuzzy values, or to be more exact, they introduce probabilistic constraints on possibility distribution of criteria. Detailed description of fuzzy granules can be found in [3].

Fuzzy granules can be represented in the form of IF-THEN rules, where the premise part corresponds to different possible non-deterministic outcomes. Outcomes that correspond to the same event or phenomenon are grouped into evidence. The consequence part contains fuzzy value of a criterion, which is a possibilistic constraint on its value.

To clarify the notion of fuzzy evidence, let us consider a simple example. Assume that we assess risk of a construction project based on type of soil, which we are uncertain about. In this setting risk is the criterion and the type of soil is a non-deterministic phenomenon. After consulting experts and examining samples of soil we may have to come to the following evidence that contains three granules:

1) IF (*soil* is rocky) with probability *0.8*
   THEN *risk* is VERY LOW
2) IF (*soil* is sandy) with probability *0.15*
   THEN *risk* is ABOVE AVERAGE
3) IF (*soil* is swampy) with probability *0.05*
   THEN *risk* is VERY HIGH

Each evidence is a probability distribution of fuzzy values. For example, we can interpret the aforementioned evidence as the following probability distribution:

P(*risk* is VERY LOW) = *0.8*
P(*risk* is ABOVE AVERAGE) = *0.15*
P(*risk* is VERY HIGH) = *0.05*

Given a set of such evidences, which describe criterion "*risk*" and obviously also other criteria, we can determine what is the probability that the risk will be LOW for a particular construction project. Value of *Prob(risk=LOW)* is an interval value. Details on the calculation of the probability bounds can be found in [3].

This is the main tool for describing alternatives and for assessing their performance, which is actually the probability that a particular alternative will perform well.

3. Overview of the methodology

In this chapter we give an overview of the methodology that uses f-granules to describe available alternatives. The use of f-granules gives several advantages. It allows one to use both fuzzy and probabilistic information to describe alternatives. This can be seen as a trade-off between crisp decision analysis methodologies such as described in [1, 2, 4, 5], which do not allow to use fuzzy descriptions of alternatives, and rigorous approaches that make extensive usage of fuzzy information, such as [6], but which require extensive computational power. Recently new methods started to appear that deal with multiple sources of uncertainty in decision-making tasks. For example, in [12] an aggregation procedure is presented, that enables one to process simultaneously probabilistic and fuzzy information. However, in this method is based on converting all sorts of uncertainty into the probability density functions simply by normalization. This is against intuition, since fuzziness and randomness are two different types of uncertainty, which are to be processed distinctively.

Thus, it is evident that there is a need for a decision-making methodology that can effectively process different types of uncertainty. The methodology described in this paper can be divided into a number of steps, which can be represented as a workflow, which is shown in Figure 1. Let us consider the steps involved in greater details.

3.1. Step 1: Determine set of alternatives and criteria

First it is necessary to determine what alternatives are there and which criteria will be used to evaluate these alternatives. Nature of the proposed decision aid methodology requires the set of alternatives to be finite and each alternative to be discrete and definite.

Like in other approaches, different alternatives and criteria can be determined during brainstorming sessions or by careful analysis of decision problem.

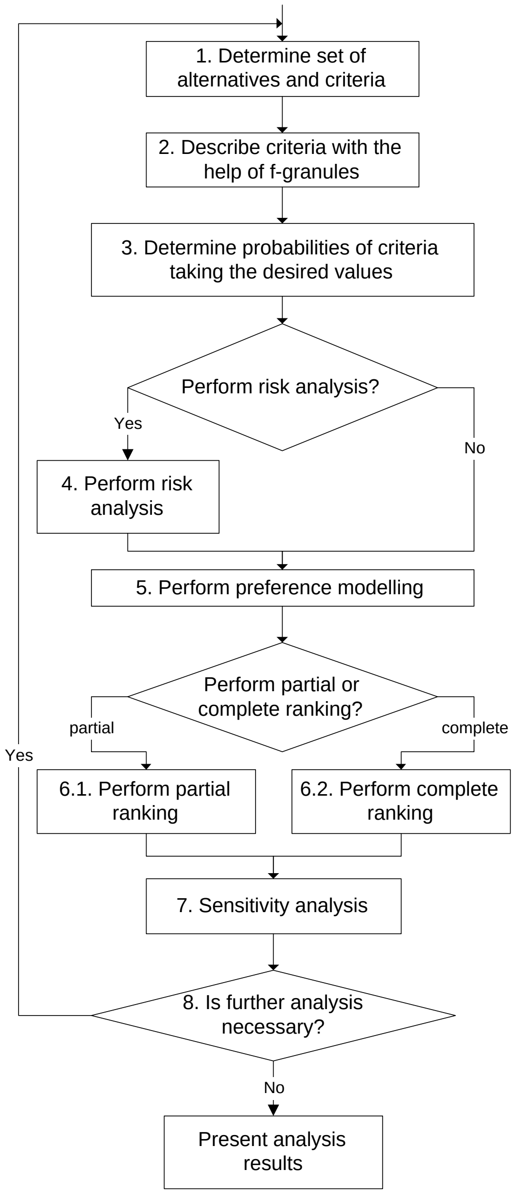

Figure 1. Workflow of the f-granule based decision aid methodology

3.2. Step 2: Describe criteria with the help of f-granules

After having determined a set of alternatives and criteria it is necessary to construct evidences that consist of f-granules, as described earlier in the paper. Each evidence contains granules that describe possible outcomes of some non-deterministic event and consequence of these outcomes on the value of a criterion. Usually description of an alternative contains a set of evidences, which describe performance of the alternative on criteria, defined in the first step. However, as will be seen later, alternatives are compared based on the probabilities that their performance on criteria will be high.

Information about one criterion can be distributed among several evidences. However, each evidence should describe only one criterion. Further information about the possibilities of using fuzzy granules in decision support models can be found in [7]. When a set of evidences is defined and agreed upon, if there is a group of decision makers, we can extract information from these evidences by determining the likelihood that the criterion will be equal to some value.

3.3. Step 3: Determine probabilities that the criteria will be equal to the desired values

Given a set of evidences that describe some criteria, we can determine the likelihood that the criterion will be equal to a particular fuzzy value. E.g., considering the example presented above, we can determine the probability that risk will be LOW. The obtained probability is interval. It is in consent with our intuition, as the description of alternatives is uncertain – with randomness and fuzziness present – so we cannot expect the method to give a precise answer.

Framework for analysing fuzzy granules and calculating upper and lower bounds of the probability is presented in [3]. One can use it as follows: first it is necessary to determine which values of criteria are desirable and then it is necessary to calculate the probabilities that alternatives will perform that well on these criteria, i.e. the probabilities that evaluations of criteria for these alternatives will be equal to the desirable values. These are maximization decision attributes, i.e. we want to maximize the probability that criteria will take desirable values. If there is a need to avoid some values, then it is necessary to calculate the probabilities of criteria taking undesirable values, which are the minimization decision attributes.

3.4. Step 4: Calculate information value of each alternative

Throughout the paper it was mentioned that conditional f-granules allow one to perform decision analysis under uncertainty, when the information available is very limited. But how much information is lacking? How much information is out there? Which alternative is more informative? To answer these and other questions an information value calculation methodology has been developed. The method is described in [8]. The idea of the method is to measure the amount of lacking information by calculating entropy value for each alternative. Entropy value shows how much information is lacking, so it can be considered as a measure of uncertainty for an alternative.

In order to calculate entropy it is necessary to present decision problem in information theoretic terms. Interval evaluations are used throughout the decision aid methodology, so it is necessary to generalize Shannon’s entropy to the case of interval probabilities. Such generalization is presented in Chapter 4.

Information obtained in this step can be used to check whether information availability for the alternatives is more or less equal. In case there is much less information available about one alternative, than about another, it would be advisable to spend some time collecting additional information, if it is reasonable.

3.5. Step 5: Perform preference modelling

We propose to use weighting scheme in order to model preferences. There are a number of weighting schemes readily available for use. For example, it is possible to adapt a value-tree-based approach used in AHP [5]. Moreover, we can use such methods for weight evaluation as SMART, SWING or their generalization to interval case [9]. The basic idea of the latter approach is to fix one so-called reference attribute, to assign it some value and then to determine relative importance of other attributes by assigning them evaluations relative to that of the reference attribute. Weights are then determined by normalizing the sum of these evaluations to one.

In its turn, AHP consists of constructing a hierarchy of criteria, sub-criteria. Weights are assigned based on pair-wise comparisons of criteria that have a common parent.

3.6. Step 6: Perform ranking

The considered decision aid methodology is intended for solving multi-criteria decision analysis tasks under uncertainty. Thus, it is extremely difficult, if not impossible to define an optimality criterion in this setting. The main tools that help a decision maker to make a better decision are ranking and sensitivity analysis. Comparison of alternatives is based on interval probabilities, and in a general case it is possible to construct a partial ranking only.

Let us introduce partial ranking that is similar to that used in the PROMETHEE method [1], and is accomplished by evaluating outgoing and incoming flows. First let us introduce several definitions. Let us assume that we have a set of alternatives $A = \{a_1, a_2, \dots, a_n\}$, where n is the number of alternatives. Set of criteria that is used to evaluate alternatives is $C = \{c_1, c_2, \dots, c_m\}$, where m is the number of criteria. Let us assume that using a weighting scheme the following weights have been assigned to criteria: $w_1, w_2, \dots, w_m$. Each alternative is evaluated on m criteria and the evaluations are interval-valued probabilities. For alternative A_i we have the following evaluations: $E_1^i, E_2^i, \dots, E_m^i$. Evaluations are interval, so: $E_j^i = \left({}^{\min}e_j^i, {}^{\max}e_j^i\right)$.

We define outgoing and incoming flows as follows:

$$\phi^-(A_i) = \sum_{k=1}^{m} w_k \cdot {}^{\min}e_k^i$$

$$\phi^+(A_i) = \sum_{k=1}^{m} w_k \cdot {}^{\max}e_k^i$$

Partial ranking is performed by the following preorder, where a and b are two alternatives from set A, while P is outranking relation, I is indifference relation and R is incomparability relation:

$$\begin{cases} aPb & if\ \phi^-(a) > \phi^+(b), \\ aIb & if\ \phi^-(a) = \phi^-(b) and\ \phi^+(a) = \phi^+(b), \\ aRb & otherwise. \end{cases}$$

If we present some folding of interval evaluations, then a complete ranking would be possible. But, as noted elsewhere in the literature, complete ranking is possible only if some information is hidden or ignored.

A natural way to fold interval probabilities is the following:

$$e_i^j = \frac{{}^{\min}e_i^j + {}^{\max}e_i^j}{2}$$

Now let us define performance p_i of an alternative a_i as follows:

$$p_i = \sum_{k=1}^{m} w_k e_k^i$$

Then the complete ranking is defined according to the following preorder, where a_i and a_j are two alternatives from set A:

$$\begin{cases} a_i P a_j & (a_i \text{ outranks } a_j) & if\ p_i > p_j \\ a_i I a_j & (a_i \text{ is indifferent to } a_j) & if\ p_i = p_j \end{cases}$$

Once again we would like to point out that the complete ranking might be deceptive, as it hides away features of alternatives that may make them incomparable.

This concludes the part about ranking, which is one of the most important tools for multi-criteria decision-making under uncertainty. Next chapter considers how it is possible to perform sensitivity analysis.

3.7. Step 7: Perform sensitivity analysis

Sensitivity analysis is an important and powerful tool of a decision maker, as it helps to determine how sensitive a decision model is to changes in its parameters.

Like in other approaches, in the proposed decision aid methodology it is possible to change values of weights and to follow alternations in ranking of alternatives.

Should it be necessary, definition of alternatives or desired criteria values can be changed as well. These changes require that probabilities be re-calculated (step 3).

There is another approach that can aid in performing sensitivity analysis. It is based on determining which features of alternatives are desirable. It can be considered as a sort of reverse engineering: first we define which values of criteria are desirable and then we determine a 'dream alternative', which is close to satisfying these desires. Results of such an analysis can be used for pinpointing critical features of alternatives, which we should pay our attention to. This kind of analysis is based on the adaptive network ANGIE (Adaptive Network for Granular Information and Evidence processing) that is described in [10]. Description of the ANGIE-based sensitivity analysis can be found in [11].

3.8. Step 8: Analysing results

At this point it should be decided whether further analysis is required. Decision analysis is necessarily an iterative task, so one should not neglect data adjustment and revision. Several reasons for revising data are: risk analysis shows that there is too little information available about an alternative; there are critical features that have not been properly examined; further sensitivity analysis is required; new data is available and so on.

4. Generalization of Shannon's Entropy to the Case of Interval Probabilities

In this chapter the generalization problem is presented as an optimization task with inequality constraints and the attention is focused on solving this task. We give a formal statement of the problem and present its analytic. The problem is solved using method of Lagrange multipliers [13], as it has both equality and inequality constraints. The traditional single-valued entropy is described in [14].

4.1. General Problem Statement

Let us assume that a system can be in *n* states, and the probability that the system is in the *i*-th state is interval and is equal to $[p_i^{min}, p_i^{max}]$. In case when probabilities are single-valued rather than interval-valued, it is required that probabilities sum to 1, i.e.

$$\sum_i p_i = 1. \qquad (1)$$

If probabilities are interval-valued, then (1) can be rewritten as (2):

$$\sum_i p_i^{\min} \le 1 \le \sum_i p_i^{\max}. \qquad (2)$$

It is easy to show that (1) is a special case of (2) when $\forall i : p_i^{\min} = p_i^{\max}$.

The entropy is interval as well: $H = [H^{\min}, H^{\max}]$. General problem statement for the calculation of the interval-valued entropy can be stated in the following way:

Let $[p_1^{min}, p_1^{max}]$, $[p_2^{min}, p_2^{max}]$, … , $[p_n^{min}, p_n^{max}]$ be interval-valued probabilities, then lower and upper boundary of entropy can be calculated as follows:

$$H^{\min} : -\sum_{i=1}^{n} p_i \ln p_i \to \min \text{ and } H^{\max} : -\sum_{i=1}^{n} p_i \ln p_i \to \max,$$

subject to

$$p_i^{\min} \le p_i \le p_i^{\max}, \qquad i = \overline{1,n}.$$

$$\sum_{i=1}^{n} p_i = 1 \tag{3}$$

Natural logarithms are used, as it will simplify calculations that will follow, but it does not change the essence of the problem.

4.2. Analytical Solution of the Problem

We use method of Lagrange multipliers [13] in order to solve problem (3). It is necessary to convert all inequality constraints $\ge$ into $\le$ by multiplying by -1. Moreover, it is necessary to convert a minimization problem into maximization problem.

First let us consider solution of the maximization problem (4).

$$H^{\max} : -\sum_{i=1}^{n} p_i \ln p_i \to \max$$

subject to

$$-p_i \le -p_i^{\min}, \quad p_i \le p_i^{\max}, \qquad i = \overline{1,n}.$$

$$\sum_{i=1}^{n} p_i = 1 \tag{4}$$

The Lagrangian for program (4) is the following:

$$L(p_1,\dots,p_n,\lambda,\mu_{11},\mu_{12},\mu_{21},\mu_{22},\dots,\mu_{n1},\mu_{n2},) = -\sum_{i=1}^{n} p_i \ln p_i + \lambda\left(\sum_{i=1}^{n} p_i - 1\right) + \sum_{i=1}^{n}\left(\mu_{i1}\left(p_i - p_i^{\min}\right) + \mu_{i2}\left(p_i^{\max} - p_i\right)\right) \tag{5}$$

Partial derivatives of the Lagrangian (5) are:

$$\frac{\partial L}{\partial p_i} = -\ln p_i - 1 + \lambda + \mu_{i1} - \mu_{i2}, \qquad i = \overline{1,n}.$$

Thus, it is necessary to solve the following problem:

$$-\ln p_i - 1 + \lambda + \mu_{i1} - \mu_{i2} = 0, \qquad i = \overline{1,n}, \tag{6}$$

$$\sum_{i=1}^{n} p_i = 1, \tag{7}$$

$$\mu_{i1}\left(p_i - p_i^{\min}\right) = 0, \qquad \mu_{i2}\left(p_i^{\max} - p_i\right) = 0, \qquad i = \overline{1,n}, \tag{8}$$

$$p_i^{\min} \le p_i \le p_i^{\max}, \quad \mu_{ij} \ge 0, \qquad i = \overline{1,n}, \quad j = 1,2, \tag{9}$$

Now it is necessary to consider 2^{2n} cases depending on values of μ_{ij} in complementary slackness conditions (8), namely, depending whether a particular μ_{ij} is or is not equal to zero.

1) If $\mu_{ij} = 0,\ i = \overline{1,n},\ j = 1,2$ and each interval probability contains value $\frac{1}{n}$, then the maximum entropy is reached at this point. Otherwise proceed to the next step.

2) Check each configuration in which a number of μ_{ij} 's are not equal to zero for some *i* and *j*. It is useful to note that if μ_{i1} or μ_{i2} is not equal to zero, then $\left(p_i - p_i^{\min}\right)$ or $\left(p_i^{\max} - p_i\right)$ must be equal to zero according to (8). Thus, if μ_{i1} or μ_{i2} is not equal to zero, then, accordingly, $p_i = p_i^{\min}$ or $p_i = p_i^{\max}$.

Hence, first for all positive μ_{i1} or μ_{i2} determine values of probabilities. Values of all other probabilities must be equal, which follows from (6) and from the fact that $\mu_{j1} = 0$ and $\mu_{j2} = 0$ for all p_j that cannot be determined. Values of p_j can be determined using (7) and the fact that all unknown probabilities are equal. At this point values of all probabilities for a particular configuration have been determined.

Now it is necessary to check whether probabilities sum to *1* and whether all μ_{ij} 's are nonnegative according to (9). In order to determine the latter it is necessary to calculate a system of linear equations derived from (6).

If all conditions are satisfied, calculate value of entropy at the corresponding point. After all the configurations have been processed choose the biggest value, which is the sought upper boundary of entropy $H^{\max}$.

Solution of the minimization problem (10) is very similar to that of (4).

$$H^{\min} : -\sum_{i=1}^{n} p_i \ln p_i \to \min$$

subject to

$$-p_i \le -p_i^{\min}, \quad p_i \le p_i^{\max}, \qquad i = \overline{1,n}.$$

$$\sum_{i=1}^{n} p_i = 1 \tag{10}$$

In order to solve (10) it is necessary to convert it into maximization problem by multiplying the target function by -1. After this operation the program (10) looks as follows:

$$H^{\min} : \sum_{i=1}^{n} p_i \ln p_i \to \max$$

stated that

$$-p_i \le -p_i^{\min}, \quad p_i \le p_i^{\max}, \qquad i = \overline{1,n}.$$

$$\sum_{i=1}^{n} p_i = 1 \tag{11}$$

Solution of program (11) is completely analogous with that of (4), except for the last step where it is necessary to choose the smallest value, which will be the lower boundary of the interval entropy $H^{\min}$. Moreover, Lagrangian and partial derivatives do not look the same, namely:

The Lagrangian for (11) is:

$$L(p_1,\ldots,p_n,\lambda,\mu_{11},\mu_{12},\mu_{21},\mu_{22},\ldots,\mu_{n1},\mu_{n2},) = \sum_{i=1}^{n} p_i \ln p_i + \lambda\left(\sum_{i=1}^{n} p_i - 1\right) + \sum_{i=1}^{n}\left(\mu_{i1}\left(p_i - p_i^{\min}\right) + \mu_{i2}\left(p_i^{\max} - p_i\right)\right) \quad (12)$$

And the partial derivatives of (12) are:

$$\frac{\partial L}{\partial p_i} = \ln p_i + 1 + \lambda + \mu_{i1} - \mu_{i2},\ i = \overline{1,n}.$$

4.3. Additivity

In order to prove that the interval entropy calculated above is additive we can use the following reasoning. According to [14], if two systems *x* and *y* are independent, then $H(x,y) = H(x) + H(y)$.

If the entropy values are interval $H(x) = [H_x^{\min}, H_x^{\max}]$ and $H(y) = [H_y^{\min}, H_y^{\max}]$, then in order to determine entropy value of a joint system with minimum possible entropy value, it is necessary to sum the lower boundaries of corresponding intervals. If there is a system with smaller value, then it means that either systems *x* and *y* are not independent or the boundaries $H_x^{\min}$ or $H_y^{\min}$ are chosen incorrectly. Assume that there exists some $`H_x^{\min} + `H_y^{\min} \le H_x^{\min} + H_y^{\min}$, then either $`H_x^{\min} \le H_x^{\min}$ or $`H_y^{\min} \le H_y^{\min}$, but since $H_x^{\min}$ and $H_y^{\min}$ are minimum possible entropy values for individual systems, then: $`H_x^{\min} = H_x^{\min}$ and $`H_y^{\min} = H_y^{\min}$.

Proof for the upper boundary is analogous.

Hence, we have proven that if $H(x) = [H_x^{\min}, H_x^{\max}]$, $H(y) = [H_y^{\min}, H_y^{\max}]$ and systems *x* and *y* are independent, then:

$$H(x,y) = [H_x^{\min} + H_y^{\min}, H_x^{\max} + H_y^{\max}].$$

This finishes the generalization of Shannon's Entropy to the case of interval probabilities.

5. Conclusion

In this paper a decision aid methodology is proposed. F-granules are at the core of this methodology, which enables one to use both fuzzy and probabilistic information to describe alternatives. This approach has several similar and distinctive features to other methodologies. It is similar in preference modelling, as almost arbitrary weighting scheme can be applied to the described methodology. Moreover, ranking is similar to that applied in other approaches, for example in [1]. However, such decision analysis tools as risk and sensitivity analysis are not similar to those used elsewhere. References to papers containing descriptions of both types of analysis are included. Moreover, the method described does not require the decision maker to construct a performance table. Besides that, the generalization of Shannon's entropy to the case of interval probabilities is presented.

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