THE GOLDEN SECTION TASK AS A TOOL OF ONTOLOGY LOGICAL INFERENCE

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Introduction

Ontology is a new method of knowledge and query representation and processing. It is able to accurately and effectively describe the semantics of data for a certain domain; it has its own logical inference as well. Due to that, ontologies are widely used nowadays, the logical inference of ontologies being of particular interest.

In the papers of Khakhalin [3, 4], the ontology inference procedures have been described, but the solution to the task has not been shown on an example in full. The authors want to emphasize that it is the inference rules that are a distinctive feature of the ontology, which in turn, make the ontology a tool for solving various problems.

The purpose of this paper is to study the ontological logical inference used in mathematic tasks solving. We have focused our research on three main areas. First, we have developed the taxonomy-based ontology. Second, we have explored the inference procedures. Finally, we have applied ontology as a tool for solving mathematical tasks. This paper gives the answers to the following questions: 1) what is the difference between taxonomy and ontology, 2) how do the ontology inference procedures find the solution to the task.

Difference between taxonomy and ontology, logical inference

<u>Taxonomy</u> – A set of categories or terms organized into a hierarchy with parent child relationships and implied inheritance. Meaning, a child term (i.e.; Dog) has all of the characteristics of its parent term (i.e.; Mammal). A taxonomy only contains broader and narrower relationships.

<u>Ontology</u> - A set of terms and relationships that represents the complete set of knowledge about a domain. Ontologies allow for more sophisticated relationship types than a taxonomy. For example, in the taxonomy of currency, Penny would be a narrower term to U.S. Coins. An ontology would add additional information to Penny such as Made of: Copper, Value: 1 cent, President Featured: Lincoln. [1]

In other words, ontology is a precise specification of a certain domain area, which includes a glossary of terms in the domain and a set of relationships (such as "element-class", "part-whole"), which describes how these terms relate to each other. In fact, this is a hierarchical conceptual skeleton of the domain area.

Ontologies can be categorized as follows:

1) Task ontology – an ontology that formally specifies the terminology associated with the type of task, e.g., scheduling, planning etc.
2) Method ontology – an ontology that formally specifies the definitions of the relevant concepts and relations used for specifying the reasoning process to accomplish a task.
3) Domain ontology – an ontology defined for conceptualizing the particular domain, e.g., job-shop scheduling, nurse assignment, air-gate assignment etc.

4) Application ontology – it contains the essential knowledge in order to model a particular application under consideration.

There is no general rule how to develop an ontology, so ontology is mostly developed based on a taxonomy.

The development of the taxonomy contains the following steps (see [2] for a detailed description of each step):

1) The taxonomy team is selected;
2) Role in corporate strategy is determined;
3) Business purposes and requirements for taxonomy are determined;
4) Pre-existing information is gathered and reviewed;
5) Survey and interviews are conducted;
6) Inventories are created;
7) Classification is rationalized;
8) Taxonomy is finalized.

The development of ontology has been done by including to the developed taxonomy the following things: the relationships among the taxonomy terms, the properties of those terms, constraints on and rules involving those terms.

The use of ontology as a domain area model is defined by some set of inference procedures, such as: inheritance, valuation, additional procedures and associated procedures.

Inheritance procedure – is an inference engine, in which each subclass inherits the values of the properties of its superclasses. Valuation procedure – is a procedure, where some or all of the input variables take specific values. Fragments of ontology in solving specific tasks are the material to describe the situation, which is determined by the input data. In addition to these fragments, concepts and relationships defined by the input conditions are to be inserted. This is accomplished with additional procedures. The associated procedure is a method by which special procedures for handling certain concepts and relations are executed.

Problem solving with ontology logical inference

The methods of inference have been considered on the following tasks:

Task 1:

The triangle ABC is inscribed in a circle (Fig.1). Find the radius (R) of the circle if it is known that the side AB is equal to 6 cm, the side AC is equal to 4 cm, and the angle CAB is equal to 60 degrees.

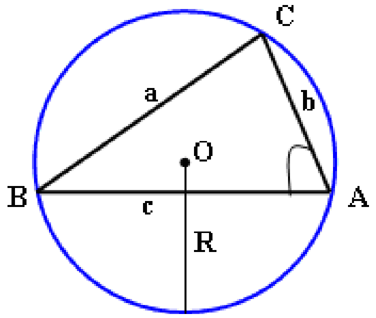

Figure 1. Task 1.

Solution:

The task is implemented using Protégé ontology editor. Ontology development in Protégé contains the following steps:

1) Domain and scope of the ontology are determined;
2) Existing ontologies reusing is considered;
3) Important terms in the ontology are enumerated;
4) The classes and the class hierarchy are defined;
5) The properties of classes – slots are defined;
6) The facets of the slots are defined;
7) Instances are created.

This task cannot be solved in one step (Fig.2): first, you should find the third side of a triangle, then - the area of the triangle, and only then you can find the radius of the circle.

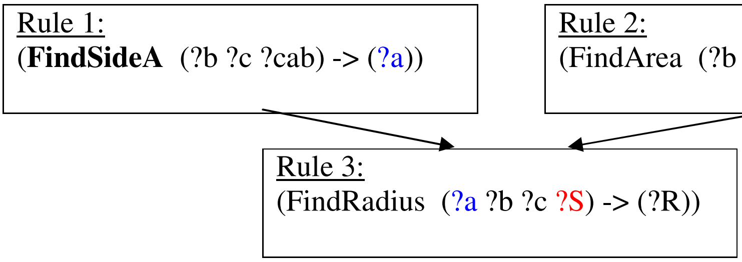

Figure 2. Task solving steps.

Domain ontology for this problem was constructed as follows: classes and their hierarchy were defined, the properties of classes (slots) and the facets of the slots were defined and instances were created (Listing 1, Figure 3).

```
(defclass TASK (is-a :THING))
(defclass Perimeter (is-a TASK)
        (slot has_value_of_perimeter (type string)) )
(defclass Radius (is-a TASK)
        (slot has_value_of_radius (type string)) )
(defclass Sides (is-a TASK)
        (slot has_length_AB (type string))
        (slot has_length_BC (type string))
        (slot has_length_AC (type string)) )
(defclass Area (is-a TASK) )
(defclass Area_of_circle (is-a Area)
        (slot has_value_of_circle_area (type string)) )
(defclass Area_of_triangle (is-a Area)
        (slot has_value_of_triangle_area (type string)) )
(defclass Angles (is-a TASK)
        (slot has_value_of_angle_ABC (type string))
        (slot has_value_of_angle_CAB (type string))
        (slot has_value_of_angle_BCA (type string))
        …
(make-instance Triangle_angles of Angles)
(slot-set Triangle_angles has_value_of_angle_CAB “60”)
(make-instance Triangle_sides of Sides)
(slot-set Triangle_sides has_length_AB “6”)
(slot-set Triangle_sides has_length_AC “4”)
```

…

Listing 1. Textual ontology example.

The solution, using a domain ontology is as follows:

1) Some or all of the input variables take specific values using the valuation procedure;

2) The procedure of inheritance would inherit the properties (formulas) of a right triangle of the triangle.

3) The associated procedures FindSideA and FindAreaOfTriangle find the unknown, but necessary for the task solution variables (Listing 2):

```
(defrule FindAreaOfTriangle
        ?x <- (object (is-a Area_of_triangle))
        (object (is-a Sides) (has_length_AB  ?AB) (has_length_AC  ?AC))
        (object (is-a Angles) (sinCAB  ?a) )
        =>  (slot-set ?x has_value_of_triangle_area (* 0.5 ?AB  ?AC ?a) ) (halt) )
```

Listing 2. Example of FindAreaOfTriangle associated procedure.

4) The associated procedure FindRadius finds the solution of the problem.

The solution to the task is not unique as shown above, but also can be solved in a different way. When there are several ways to solve the task, an ontology-based system chooses the shortest one.

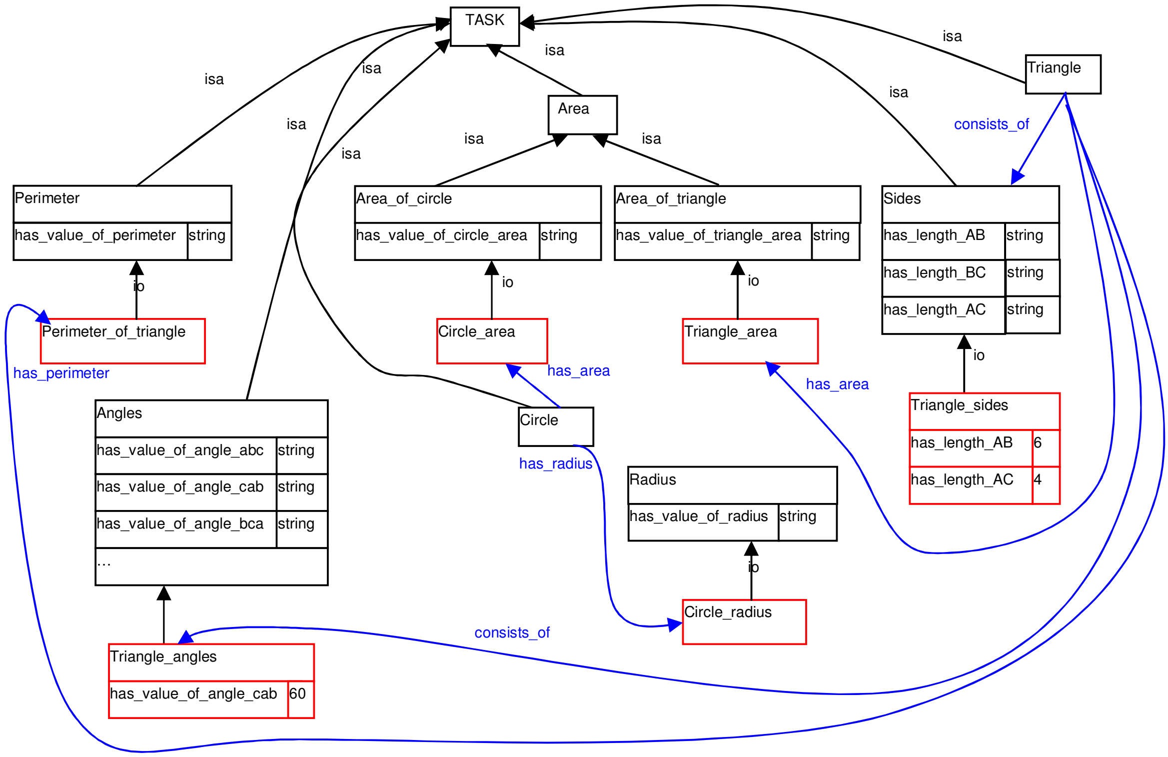

Figure 3. Ontology of task 1.

Task 2:
The inference with dependencies of the golden section: find the height, diameter and height of the dome of the church Basil (chapel), if the following is known: the height of the St. Basil’s Cathedral (is equal to 65 m) and that the cathedral was built on the principles of the golden section.

Solution:
In mathematics and the arts, two quantities are in the golden ratio if the ratio of the sum of the quantities to the larger quantity is equal to (=) the ratio of the larger quantity to the smaller one (Fig.4). The golden ratio (golden section) is an irrational mathematical constant, approximately 1.6180339887.

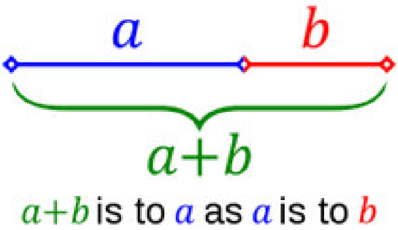

Figure 4. Golden section.

St. Basil's Cathedral is a symmetrical ensemble of eight churches surrounding the central church with the marquee. If we take the height of the cathedral as a unit, the basic proportions, defining the division of the whole into parts, make a series of the golden section (Fig.5).

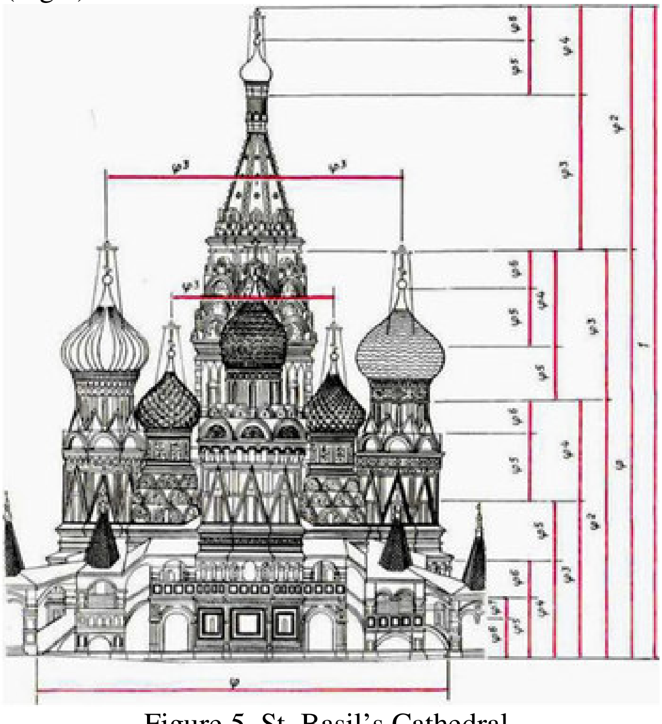
Figure 5. St. Basil’s Cathedral.

This task cannot be solved in one step, but the values of the chapel parameters can be calculated if the only one size, the height of the cathedral, is known.
Domain ontology for this problem was constructed as follows: first, classes and their hierarchy were defined, then the properties of classes (slots) and the facets of the slots were defined and instances were created (Listing 3).

```
(defclass TASK (is-a :THING))
(defclass Cathedral (is-a TASK)
        (slot cathedral_height (type string))
        (slot cathedral_weight (type string)) )
        …
(defclass Churches (is-a TASK)
        (slot church_height (type string))
        (slot dome_diameter (type string))
        (slot barrel_diameter (type string)) )
        …
(make-instance Church_central of Churches)
(make-instance Church_middle_E of Churches)
(make-instance Church_middle_N of Churches)
…
(make-instance Church_small_NE of Churches)
(make-instance Basil_Church of Churches)
(make-instance StBasilCathedral of Cathedral)
(slot-set StBasilCathedral cathedral_height "65")
```

Listing 3. Textual ontology example.

The solution, using a domain ontology is as follows:
1) Some or all of the input variables take specific values using the valuation procedure;
2) The procedure of inheritance brings together the characteristics of medium-height churches, that is, they are identical in size and decorations. By the same principle of inheritance the procedure works for small churches.
3) The associated procedures find the unknown variables needed to solve the problem. Some of the associated procedures are shown in Listing 4.

```
(deffunction GoldenSection (?x)  (return (/ ?x 1.618) ) )
(deffunction GS (?x)  (return (/ ?x 2.67) ) )
(defrule HeightOfMiddleChurch1 (object (is-a Churches)) (object (is-a
Cathedral) (cathedral_height ?x) ) => (slot-set Church_Middle_E church_height
(GoldenSection ?x)) (slot-set Church_Middle_N church_height (GoldenSection
?x)) (slot-set Church_Middle_S church_height (GoldenSection ?x)) (slot-set
Church_Middle_W church_height (GoldenSection ?x)) (halt) )
…
(defrule BasilChurchHeight3 (object (is-a Cathedral) (Church_Middle_E_height
?x) ) => (slot-set StBasilCathedral Basil_church_height (GoldenSection ?x))
(halt) )
…
```

```
(defrule DiameterMiddleBarrel (object (is-a Churches)) (object (is-a Cathedral)
(Church_Middle_E_height ?x) ) => (slot-set Church_Middle_E diametr_of_barrel
(GoldenSection (GS ?x))) (slot-set Church_Middle_N diametr_of_barrel
(GoldenSection (GS ?x))) (slot-set Church_Middle_S diametr_of_barrel
(GoldenSection (GS ?x))) (slot-set Church_Middle_W diametr_of_barrel
(GoldenSection (GS ?x))) (halt) )
…
```

Listing 4. Example of HeightOfMiddleChurch1, BasilChurchHeight3 and DiameterMiddleBarrel associated procedures.

Using the ontology and inference procedures of this domain any parameter of the cathedral can be calculated, knowing one of its variables (height, width or diameter of the barrel of the Cathedral).
The task is also implemented using Protégé ontology editor and its solution is described in detail.

Conclusions

Based on geometric relations of parameters of the triangle and ratios of calculating the parameters (sizes) of St. Basil's Cathedral, the use of the ontology and inference procedures for calculating the unknown geometric parameters was shown.
Inference rules make the ontology a tool for different task solving; without them it is just a taxonomy - a set of hierarchically constructed terms. One of the advantages of the use of the ontology is a systematic approach to the study of the domain area; besides, the development of the ontology allows one to restore the missing logical links in their entirety.

References

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