SIMULATION VERSUS ANALYTICAL MODELLING FOR SUPPLY CHAIN DYNAMICS ANALYSIS

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ABSTRACT

Simulation has become an important information technology tool for analysis and improvement of entire supply chain operation. One of the most important supply chain operation stability measures is a bullwhip effect value. The bullwhip effect can lead to holding an excessive inventory, insufficient capacities and high transportation costs. It is important to investigate the magnitude of this effect by quantifying it. There are a variety of methods, which address bullwhip effect modelling. Nevertheless, there is a lack of methods for its numerical evaluation in a supply chain. This paper proposes statistics and probability theory based analytical methods which allow quantification of the bullwhip effect value in a supply chain that operates under uncertain market demand. Simulation technique is used to validate results obtained from the analytical model. Based on validation results, the logic of the analytical model is examined, and some specifications of the analytical model are analysed and described.

Keywords: simulation, analytical modelling, the bullwhip effect, supply chain under uncertain demand

1. INTRODUCTION

A precise representation of dynamic, time-dependent changes in a system operation is important for analysis and evaluation of the bullwhip effect in a supply chain. The most common distinction between dynamic models is drawn between the so-called continuous and discrete models (Crosbie 2000). A continuous model is usually based on ordinary or partial differential equations with time as an independent variable. The state of the system is assumed to change in a continuous fashion so that at any particular instant in time the state of the system will be uniquely defined. A discrete model assumes that the state of the system changes only at specific times, often referred to as events, and the state of the system is unchanged between these times.

Both differential equation (more typical for technical system models), and discrete-event simulation (more typical for business system models) could be used to evaluate the bullwhip effect value in a supply chain.

The bullwhip effect evaluation methods are classified as follows:

1. Continuous system modelling (analysis of an order flow):
   - control theory based (Laplace transformation, z-transformation);
   - system dynamics.
2. Discrete-event system/Analytical modelling (analysis of an individual order):
   - discrete-event system simulation;
   - analytical simulation.

Simon (1952) was one of the first to use classical Laplace transform techniques to analyse simple production-inventory systems (Riddalls et al. 2000). This move was quickly translated into the newly favoured discrete z-domain by the Operations Research community (Dejonckheere et al. 2003). Contributions that utilise the Laplace transform are more numerous than those utilising the z-transform. This is probably due to more tractable algebraic manipulation required when using the Laplace transform. However, as the z-transform is a special case of the Laplace transform, many tools, techniques and best practices developed for the Laplace transform are readily exploited in the z-domain – usually after a small change in notation.

Disney and Towil (2002) developed a z-transform model from which an analytical expression for the bullwhip effect is derived that is directly equivalent to the common statistical measure, the Coefficient of Variation (COV) often used in simulation, statistical and empirical studies to quantify the bullwhip effect (Disney and Towill 2002). Dejonckheere et al. (2003) developed a control engineering insights based methodology (the transfer function, the frequency response plot) for the analysis of the bullwhip effect value using different demand forecasting methods (Dejonckheere et al. 2003). Forrester (1961) started to analyse a supply chain phenomena – demand amplification using a continuous time model (Forrester 1961). This work facilitates the development of the systems dynamics in the production and inventory control fields. Sterman (1989) characterised causes of unsuccessful decisions in supply chain management and realised a number of experiments using system dynamics principles (Sterman 1989). Barlas and Aksogan (1997) developed apparel industry supply chain model, using system dynamics simulation and analyse effectiveness of inventory and production policies in supply chain management (Barlas and Aksogan 1996). Hennet (2005) proposed a system dynamics based method to analyse multi-stage supply chain taking into consideration random variation of demand for final product under classical distributed production and ordering policies (Hennet 2005).

Zhang and Zhang (2007) used discrete-event simulation to evaluate the bullwhip effect value in three-stage supply chain with different information sharing strategies (Zhang and Zhang 2007). They also studied possibilities of decreasing the lead time uncertainty and the effect of this on the bullwhip effect using simulation (Zhang et al. 2006).

Ingals et al. (2005) analysed the impact of a control-based forecasting method on the bullwhip effect value by performing simulation-based experimental study (Ingals et al. 2005). Chandra et al. (2001) investigated information sharing impact on the demand forecast accuracy and the bullwhip effect value in a supply chain through discrete-event simulation (Chandra et al. 2001).

The developed analytical models usually support an analysis of different factors impact on the bullwhip effect value, but not evaluation of the value itself. For example, Simchi-Levi et al. (2002) explained that the increase in demand variability with the necessity for each supply chain stage makes orders based on the forecasted demand of the previous stage (Simchi-Levi 2002). Since variability in placed orders is significantly higher than that in customer demand, the supply chain stage is forced to carry more safety stock in order to meet the same service level. The proposed quantifying of the magnitude of increase in variability between two neighbour supply chain stages is expressed as a function of a lead time between the orders receiving and the number of demand observation on which forecast is made:

$$\frac{Var(Q)}{Var(D)} \geq 1 + \frac{2L}{p} + \frac{2L^2}{p^2}, \quad (1)$$

where

Var(Q) – the variance of the orders placed by the supply chain stage;

Var(D) – the variance of the demand seen by this supply chain stage;

L – lead time between the orders receiving;

p – number of observation on which further demand forecast is based.

As a result, the bullwhip effect is magnified with increasing the lead time and decreasing the observations number.

Luong and Phien (2007) evaluated the bullwhip effect value in two echelon supply chain using AR(2) autoregression model. However, it is necessary to determine autoregression coefficients and for providing a precise evaluation of the bullwhip effect value it is recommended to use a higher degree autoregression model that leads to complication of the bullwhip effect value calculation (Luong Huynh Trung et al. 2007). Kelle and Milne (1999) suggested to evaluate a variance of placed orders (bullwhip effect) in inventory systems that implement the *S-s* inventory control policy using approximations of the quantitative model, developed in accordance with asymptotic renewal theory (Kelle and Milne 1999).

The performed analysis of the bullwhip effect evaluation methods elicits that the developed analytical models allow analysis of the impact of different factors on the bullwhip effect value, but not evaluation of the value itself. Researches in this area are still developing and are considered perspective in supply chain management.

This paper develops statistics and probability theory based methods for analytical evaluation of the bullwhip effect value based on the numerical measures of the customer demand distribution.

The rest of the paper is organised as follows. In the next section, the importance and impact of the stochastic factors on the management of a supply chain is discussed. This is followed by the description of the supply chain analysed. An explanation of the elaborated statistics and probability theory based methods for the evaluation of the bullwhip effect value is given. A numerical example and a simulation-based validation of the results of the analytical solution are discussed. Finally, conclusions are provided.

2. STOCHASTIC FACTORS IN A SUPPLY CHAIN MANAGEMENT

Stochastic factors have a major influence on the behaviour of a supply chain and its management. Stochastic nature of the customer demand is established as one of the most critical factors in decision-making, since many of the uncertainty sources can be handled adequately only at the tactical level (Van Landeghem and Vanmaele 2002), and planning cycle of supply chain processes is coordinated with material flow, which is characterised by demand volume. That's why, planning and controlling problems of a supply chain at the tactical level under the stochastic customer demand are analysed in the paper. Operation of any supply chain depends on the customer demand and its fluctuation. However internal demand between supply chain stages also plays an important role due to it considerably changes the information about the required product amount. An important phenomenon in the supply chain management is the increase in variability of the demand as it moves through the supply chain in the direction from customer to supplier. This phenomenon's name is the bullwhip effect (Simchi-Levi et al. 2002) because even small disturbances in demand at the customer level cause the demand amplification for the next supply chain member (see Fig. 1.).

A measure of the demand variability is the standard deviation of demand, σ. An increase in this value at each supply chain stage directly indicates the existence of the bullwhip effect. The bullwhip effect is considered to be an important characteristic of supply chain operation stability.

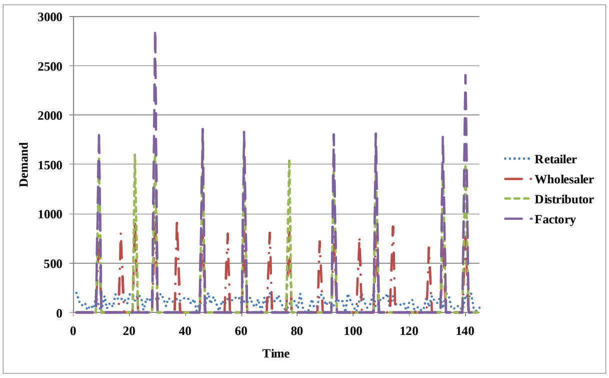

Figure 1: The Bullwhip Effect in a Supply Chain

The main consequence of the bullwhip effect appearance is holding an excessive inventory or/and inventory allocation in inappropriate supply chain stages.

3. SUPPLY CHAIN ANALYSED

The main objective of the research is to develop analytical methods for evaluation of the bullwhip effect value based on the numerical measures (mean and standard deviation) of the customer demand distribution in a supply chain.

The supply chain is analysed from the inventory management point of view, when it is represented as a serially connected inventory management systems chain. The considered supply chain consists of the end customer, retailer and supplier. The retailer supplies the customer single-item products according to the demand received, and replenishes its inventory by placing orders to the supplier. Customer demand is stochastic and stationary. For managing the inventory, the *s-S* inventory control strategy (see Fig. 2.) is used.

The magnitude of increase in variability of the placed order size with regard to variability of received demand value characterises the bullwhip effect. Its value can be expressed analytically, taking into consideration the numerical measures of the customer demand distribution – an expected value *E(X)* and variance *D(X)*.

It is assumed that the demand X_1, X_2, …, X_i is a discrete random sample observed from some population. Equivalently, these data are independent and identically distributed (IID) observations on some underlying random variable *X* whose distribution governs the population. Values that numerically characterise the population/distribution, such as an expected value *E(X)* and a variance *D(X)* of the discrete random variable *X* are given.

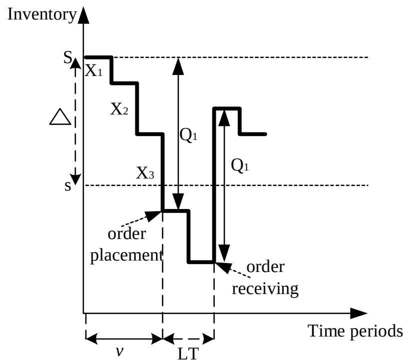

Figure 2: Inventory Control Strategy in the Retailer Stage

The inventory level to which inventory is allowed to drop before a replacement order is placed (reorder point level) is found by a formula:

$$s = E(X) * LT + STD(X) * \sqrt{LT} * z, \qquad (2)$$

where

LT – constant lead time between replenishments;

$STD(X) = \sqrt{D(X)}$ - standard deviation of the mean demand;

z - the safety stock factor, based on a defined in-stock probability during the lead time.

The total requirement for the stock amount or target stock level *S* is calculated as a sum of the reorder point level and a demand during the lead time value:

$$S = s + E(X) * LT \qquad (3)$$

The order size Q_i is demanded when the on-hand inventory drops below the reorder point; it is equal to the sum of the demand values between the order placements:

$$Q_i = X_1 + X_i + ... + X_v, \qquad (4)$$

where

v – random variable, the number of period in which order is placed.

Regular or cyclical in nature inventories with additional safety stock are considered. A strategy to control such inventories assumes that the conditions of demand level, its variability and lead time are known and involves the following main steps:

1. find the current on-hand inventories at the stocking point;
2. establish the stock availability level at the stocking point after the demand satisfaction;
3. calculate total requirements that is the amount of cycle stock plus additional quantities needed to cover the uncertainty in demand;
4. determine an order size as the difference between the total requirements and the quantity on hand in case if the on-hand inventory drops

below the allowed level when a replacement order should be placed.

4. ANALYTICAL METHODS FOR THE BULLWHIP EFFECT EVALUATION

4.1. Statistics-based method

Provided that the demand *X* is uncertain and the aforementioned inventory control strategy is employed, the placed order size *Q* is expected to be a random variable that depends on the demand values. The expected value *E(Q)* and variance *D(Q)* of the function $Q = \varphi(X)$ are estimated using the following formulas proposed by Feller (1967):

$$E(Q) = E(X) * E(v)\ , \quad (5)$$

and

$$D(Q) = E(v) * D(X) + D(v) * [E(X)]^2\ , \quad (6)$$

where

E(v) – expected value of a number of periods between placed orders;

D(v) – variance of a number of periods between placed orders.

The number of periods between placed orders *v* characterises the frequency of order placements but its probabilistic behaviour is estimated by numerical characteristics: an expected value and a variance.

The multi-experimental realisation of the following algorithm allows one to collect statistics of *v* values (v_i, $i = \overline{1,n}$) and evaluate their probabilities p_i by relative frequencies $\hat{p}_i$ of their occurrence, which in its turn allows one to estimate *v* expected value and variance:

if $X_1>\Delta$ THEN v=1 AND STOP
ELSE generate X_2
if $X_1<\Delta$ and $X_1+X_2>\Delta$ THEN v=2 AND STOP
ELSE generate X_3
…
if $X_1+X_2+\ldots+X_{n-1}<\Delta$ and $X_1+X_2+\ldots+X_n>\Delta$ THEN $v=n$
STOP

The performance of the supply chain is evaluated under various factors such as end customer mean demand *E(X)* and its standard deviation *STD(X)*, safety stock factor *z* and a lead time *LT*. It is assumed that end customer demands arrive with fixed time-intervals, and their value is variable and is derived from a normal distribution. A constant lead time between replenishment is considered. No order processing delay is taken into account, so all demand events are treated immediately by the inventory system. We will also assume no capacity constraints for supplier of the inventory system. In this case, stockouts will lead to backorders, not lost sales.

One of simulation techniques usage assignments in modelling and analysing inventory systems is analytical model validation and verification (Banks and Malave 1984). It is the most important part of a simulation study that enables determining whether an analytical model of an inventory management system performs as intended and is an accurate representation of the real-world system under study. The simulation model was developed using the ARENA 12.0 simulation modelling environment. Simulation is used to analyse and evaluate the increase in variability of placed orders in the described supply chain and to validate the results of the analytical solution (Petuhova and Merkuryev 2007).

The model was run for 1 replication. Replication length is defined as 2000 time periods. The warm-up period is avoided by setting the initial inventory level equal to the lower control limit called the reorder point *s*. Numerical results of comparison of the statistics and simulation based methods are given in Table 1.

Table 1: Validation Results of the Analytical, Statistics-Based Method for the Evaluation of the Bullwhip Effect Value

$STD(Q)_{an}$	$STD(Q)_{sim}$	χ^2_{fact}	$\left[\chi^2_{1-0.95,389};\ \chi^2_{0.95,389}\right]$	result
29.89	17.93	233.28	[344.29; 435.99]	disagree
32.01	17.71	215.23	[344.29; 435.99]	disagree
35.13	17.89	198.08	[344.29; 435.99]	disagree
39.79	18.63	182.15	[344.29; 435.99]	disagree
47.36	20.74	170.35	[344.29; 435.99]	disagree

To compare results given by analytical and simulation models the χ^2 test is used. The validation indicates that results given by the analytical model proved to be in disagreement with those given by the simulation model with a probability of 95%. The variance of placed orders calculated by analytical model is much greater than actual variance of placed orders derived from the simulation model (see Table 1). The reason for the inadequate bullwhip effect quantification by the analytical model is an existing dependence between a number of periods when an order is placed *v* and realisations of the end customer demand X_i. In other words, the proposed formulae (5) and (6) assume independence of *v* and *X*, but in the described inventory management system they are dependent in the way of conditional probability of *v* occurrence $p_v=P(X_1+X_2+\ldots+X_v>\Delta / X_1+X_2+\ldots+X_{v-1}<\Delta)$.

The proposed statistics-based method could be used for the evaluation of the bullwhip effect value based on the numerical measures of the customer demand distribution in the case, when a period of order placement is independent of the value of the received demand. For example, if period of the order placement *v* is an independent random variable with known theoretical distribution, e.g., normal distribution with mean equal to 4 and standard deviation equal to 1, then performed experimental study confirms that analytically obtained results agree with those given by simulation (see Table 2).

Table 2: Validation Results of the Analytical, Statistics-Based Method for the Evaluation of the Bullwhip Effect Value in Case of Order Time Period Independency

$STD(Q)_{an}$	$STD(Q)_{sim}$	χ^2_{fact}	$\left[\chi^2_{1-0.95,245};\ \chi^2_{0.95,245}\right]$	result
53.85	58.19	264.72	[209.76;282.51]	agree
75.39	81.46	264.70	[209.76;282.51]	agree
96.93	104.68	264.58	[209.76;282.51]	agree
118.47	127.95	264.59	[209.76;282.51]	agree
140.01	151.20	264.58	[209.76;282.51]	agree

The validation indicates that results given by the analytical model agree with those given by the simulation model with a probability of 95%.

4.2. Probability theory based method

To consider a correlation between demand value X_i and time period of order placement v, a probability density function of the order size Q should be defined. It is obtained by integration of a total probability formula of the sum of received demand values ($S_Q=X_1+X_2+...+X_v$). The distribution function of the random variable S_Q with regard to the total probability formula is:

$$\begin{aligned} F_Q(x) &= P(S_Q < x) = P(X_1 + X_2 + ... + X_v < x) = \\ &= P(S_Q < x/v = 1)\cdot P(v = 1) + P(S_Q < x/v = 2)\cdot P(v = 2) + \\ &+ P(S_Q < x/v = 3)\cdot P(v = 3) + ... = P(X_1 < x)\cdot P(v = 1) + \\ &+ P(X_1 + X_2 < x)\cdot P(v = 2) + P(X_1 + X_2 + X_3 < x)\cdot P(v = 3) + ... \end{aligned} \quad (7)$$

where

$P(v=i)$, $i=1\div\infty$ – probability that the order will be placed in the i^{th} time period;

$P(S_Q<x/v=i)$ – probability that the order size will be less than x in time period v.

Analytically estimating the probability of the period when order is placed $P(v=1, 2, 3, ...)$ and probability when the sum of the demand values reaches the Δ level $P(X_1+X_2+X_{3+...}<x)$ it is possible to define the following distribution function of the order size $F_Q(x)$:

$$\begin{aligned} F_Q(x) &= \int_0^x f(x_1)dx_1 \cdot \left(1-\int_0^\Delta f(x)dx\right) + \\ &+\int_0^x f(x_1)\int_0^{x-x_1} f(x_2)dx_2dx_1 \cdot \int_0^\Delta f(x_1)\int_{\Delta-x_1}^\infty f(x_2)dx_2dx_1 + \\ &+\int_0^x f(x_1)\int_0^{x-x_1} f(x_2)\int_0^{x-x_1-x_2} f(x_3)dx_3dx_2dx_1 \cdot \int_0^\Delta f(x_1)\int_0^{\Delta-x_1} f(x_2)\cdot \\ &\cdot \int_{\Delta-x_1-x_2}^\infty f(x_3)dx_3dx_2dx_1 + \\ &+\int_0^x f(x_1)\int_0^{x-x_1} f(x_2)... \int_0^{x-x_1-...-x_{i-1}} f(x_i)dx_idx_{i-1}...dx_1 \cdot \\ &\cdot \int_0^\Delta f(x_1)\int_0^{\Delta-x_1} f(x_2)... \int_{\Delta-x_1-x_2-x_{i-1}}^\infty f(x_i)dx_idx_{i-1}...dx_1 \end{aligned} \quad (8)$$

where

$f(x_i)$ – probability density function of the customer demand distribution;

Δ – the difference between target inventory level S and reorder point s;

i – the number of random variable values; the more addends are available, the more precisely the distribution describes the random variable Q behaviour.

To define a probability density function of the order size $f_Q(x)$, its distribution function $F_Q(x)$ (8) should be derived:

$$\begin{aligned} f_Q(x) &= [F_Q(x)]' = f(x)\cdot\left(1-\int_0^\Delta f(x)dx\right) + \\ &+\int_0^x f(x_1)f(x-x_1)dx_1 \cdot \int_0^\Delta f(x_1)\int_{\Delta-x_1}^\infty f(x_2)dx_2dx_1 + \\ &...+\int_0^x f(x_1)\int_0^{x-x_1} f(x_2) \int_0^{x-x_1-...-x_{i-2}} f(x-x_1-...-x_{i-1})dx_{i-1}dx_{i-2}...dx_1 \cdot \\ &\cdot \int_0^\Delta f(x_1)\int_0^{\Delta-x_1} f(x_2)... \int_{\Delta-x_1-x_2-x_{i-1}}^\infty f(x_i)dx_idx_{i-1}...dx_1 \end{aligned} \quad . (9)$$

Knowing the random variable probability density function (9) it is possible to define analytically its numerical characteristics: expected value (10) and variance (11):

$$E(Q) = \int_\Delta^\infty x f_Q(x)dx, \quad (10)$$

$$D(Q) = \int_\Delta^\infty x^2 f_Q(x)dx - [E(Q)]^2. \quad (11)$$

where $f_Q(x)$ – probability density function of the order size.

The variance of order size Q characterises the bullwhip effect value in the supply chain. Knowing distribution function of the customer demand and its numerical characteristics it is possible to estimate the variance of the order size Q by applying the developed probability theory based method.

Practical application and validation of the proposed probability theory based methods is a subject of future research.

5. CONCLUSIONS

Two methods for the analytical evaluation of the bullwhip effect value in the supply chain are discussed in this paper. The statistics-based method could be used for evaluating the order size variance based on the distribution parameter values of the customer demand if the period when order is placed v does not depend on the customer demand value X. In its turn, the probability theory based method could be used when the dependence does exist.

Obviously, the simulation supports evaluation of the bullwhip effect in all supply chain configurations and it could be taken as a general technology for supply chain operation analysis and planning. Analytical approaches to analysing the supply chain operation do not need any special software; however mathematical methods are not always able to describe complicated dynamical and stochastic systems. The research performed allows one to conclude about the simulation technology advantages for analysing dynamical systems that operate in a stochastic environment.

With the aid of the developed statistics and probability theory based methods it is possible to conclude about supply chain operation stability and to verify the adequacy of the created supply chain simulation model. There are developed a number of simulation models during the research, and various experiments are performed with them in order to demonstrate practical applications of the developed statistics-based method in supply chain management area. The practical application of probability theory based methods and its validation by simulation is a subject for further research.

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