A SIMPLE MODEL FOR THE FATIGUE LIFE OF COMPOSITE MATERIALS

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A composite material of a simple structure consisting mainly of n parallel rigid strands is considered. Let $G_i = E_i L_m / E_m L_i$ be the relative rigidity of the i-th strand (here we take into account the actual and mean values of the elastic modulus and strand length). For n large enough, we can suppose that there exist a density distribution $f_G(g)$. Let $F_{W|g}$ be a conditional cumulative distribution function (c.d.f.) of the ultimate strength W at a given rigidity. Then, the mean relative number of unbroken strands in N cycles at a nominal stress S_N will be

$$\psi = \int [1 - F_{W|g}(gS_N \,|\, g)] f_G(g)\, dg.$$

The new stress value S_{N+1} after N cycles will be equal to S_0/ψ. The stress increment in one cycle, $S_{N+1} - S_N$, can be considered as a derivative dS/dN. Then, the number of cycles, N^*, needed to increase the stress from S_0 to S^* corresponding to failure, will be

$$N^* = \int_{S_0}^{S^*} \frac{dN}{dS}\, dS = \int_{S_0}^{S^*} \left\{ \frac{1}{S_0 / \int [1 - F_{W|g}(gS_N \,|\, g)] f_G(g)\, dg - S} \right\} dS.$$

The formula defines a fatigue curve — the mean value of N^* as a function of the initial stress S_0. We can also get the statistical characteristics of the fatigue life if, instead of $F_{W|g}$ and f_G, we consider their statistical estimates $\hat{F}_{W|g}$ and $\hat{f}_G$ corresponding to actual collections of $w_1, \ldots, w_n$ and $g_1, \ldots, g_n$. For example, if $W|g$ has a log-normal distribution, then, instead of the c.d.f.

$$\Phi\,[(\ln w - \theta_0)/\theta_1],$$

we should use

$$\Phi\,[(\ln w - \hat{\theta}_0)/\hat{\theta}_1],$$

Where $\hat{\theta}_0$ and $\hat{\theta}_1$ are "estimates" of the parameters θ_0 and θ_1 corresponding to the collection $w_1, \ldots, w_n$. It is worth noting that a great variance in the fatigue life of a composite means that the stress distributions in some cross section is not uniform. It can be assumed uniform only in some relatively small region. Disturbance of this region means the beginning of a "chain reaction" and disturbance of composite as a whole. The "size" of the critical volume, n, is not too large, because the variation coefficient should be approximately proportional to $1/\sqrt{n}$.