

# BUCKLING AND VIBRATION ANALYSIS OF STIFFENED COMPOSITE SHELLS

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Due to their high specific modulus compared with a metallic material, laminated composite materials offer more efficient structures for application in the aerospace engineering. Aerospace structures such as wings or fuselages are mostly assemblies of shell structures. Since laminated composites have a low transverse stiffness, the shell theories taking account of transverse shear deformations should be used [1].

In the finite element analysis of stiffened shells, at least two different shell surfaces should be assembled. In order to obtain continuous displacement and rotation fields of the branched shells, a shell theory proposed in [2] can be used. Representation of the displacement field through entire laminate is as follows

$$\mathbf{u}(x^\alpha) = \mathbf{v}(x^\alpha) + x_3 \gamma(x^\alpha). \quad (1)$$

Here  $\mathbf{v}$  is the displacement vector and  $\gamma$  is the rotation vector at the midsurface of the shell. These vectors have three components

$$\mathbf{v} = v^\alpha \mathbf{a}_\alpha + w \mathbf{a}_3, \quad \gamma = \gamma^\alpha \mathbf{a}_\alpha + \gamma \mathbf{a}_3, \quad (2)$$

where  $[\mathbf{a}_\alpha, \mathbf{a}_3]$  is base vectors of the curvilinear coordinates  $[x^\alpha, x^3]$  at the midsurface of the shell. The shell theory developed on the displacement representation (1) is the so-called first order shear deformation theory with six degrees of freedom (FOSDT-6). The present shell theory takes into account not only the transverse shear deformations but also extension of the normal line.

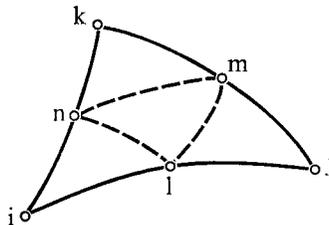


Fig. 1. A curved isoparametric triangular finite element of an arbitrary shell.

A triangular isoparametric finite element based on shell theory (1) and quadratic approximation of displacements  $\mathbf{v}$  and rotations  $\gamma$  has been developed in [3]. For the present element, each node has six degrees of freedom – three displace-

ments and three rotations. Knowing these nodal displacements it is easy to satisfy the continuity conditions for the branched (stiffened) shells.

Numerical examples of buckling of axially loaded stiffened carbon/epoxy cylindrical shells are discussed. Stiffened shells having a blade-type stringers are considered. Depending on geometric parameters of the skin and stringers and layer stacking sequence, different buckling modes were observed. Similarly the vibrations of stiffened composite shells were analysed. The local, global and coupled (stringer-skin) vibration modes were obtained.

### References

1. P. M. Naghdi, "Foundations of elastic shell theory," in: I. N. Sneddon and R. Hill (eds.), *Progress in Solid Mechanics*. Vol 4. Ch. 1, North-Holland, Amsterdam (1963), pp. 1-90.
2. K. Z. Galimov, *The Theory of Shells with Transverse Shear Effects [In Russian]*, Khazan Univ. Press, Khazan (1977).
3. R. Rikards and A. Chate, "Vibration and damping analysis of laminated composite and sandwich shells," *Mech. Compos. Mater. Struct.*, 4, 209-232 (1997).