FLUID FLOW VIBRATION EXCITATION BY THE CONTROL OF INTERACTION SURFACE

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Abstract. The first part of this study investigates the optimization task is by using optimal control theory. The high speed task for energy harvesting from fluid flow is solved for two types of control action limits. One limit type is the boundary value of interacting surface, the second type – transition switching velocity from one surface level to other. It is shown that optimal area control law has one or two limits.

Keywords: optimal control, energy harvesting.

Introduction

One of the classical open-flow problems in fluid mechanics is the analysis of the flow around rigid body if the interaction area can be altered. Here two main tasks are investigated – analysis of system with variable parameters and synthesis of new energy systems by using general optimization procedures. In the report an optimal or quasi-optimal control law is synthesized in a phase plane. Finally, results of the experimental investigations are provided. All experiments were conducted in the "Armfield" wind tunnel.

Equation of motion and optimization

System with one DOF and variable flow has the following motion control equation (1):

$$m\ddot{x} = -cx - b\dot{x} + u(t), \tag{1}$$

where m – mass;
c – stiffness of a spring;
b – damping coefficient;
$u(t)$ – control action like surface alteration;
t – time.

To solve the task of optimization the equation (1) can be written in the following form (2) [1-6]:

$$\begin{aligned} &\ddot{x} = -k^2 x - 2n\dot{x} + U(t), \\ &\dot{x}_1 = x_2; \\ &\dot{x}_2 = -k^2 x_1 - 2nx_2 + U(t); \\ &\dot{x}_3 = -k^2 x_2 - 2n[-k^2 x_1 - 2nx_2 + U(t)] + \dot{U}(t), \end{aligned} \tag{2}$$

where $k^2 = \frac{c}{m};\ 2n = \frac{b}{m}; U(t) = \frac{u(t)}{m};$

x_3 – additional phase coordinate that takes into account derivation $\dot{U}(t)$ limit of control action $U(t)$

Two limits in form (3) exist in this report:

$$U_1 \le U(t) \le U_2; \quad -D_1 \le \dot{U}(t) \le +D_2, \tag{3}$$

where U_1, U_2, D_1, D_2 – positive constants for limits of control and its derivation.

Hamiltonian is (4) [2-5]:

$$H = \psi_1 x_2 + \psi_2[-k^2 x_1 - 2nx_2 + U(t)] + \psi_3\{-k^2 x_2 - 2n[-k^2 x_1 - 2nx_2 + U(t)] + \dot{U}(t)\}, \tag{4}$$

where ψ_1, ψ_2, ψ_3 – adjoin variables.

System of equations (5) exist for adjoint variables:

$$\begin{aligned}\dot{\psi}_1 &= -\frac{\partial H}{\partial x_1};\\ \dot{\psi}_2 &= -\frac{\partial H}{\partial x_2};\\ \dot{\psi}_3 &= -\frac{\partial H}{\partial x_3}.\end{aligned} \quad (5)$$

where $\dot{\psi}_1$, $\dot{\psi}_2$, $\dot{\psi}_3$ – derivation of adjoin variables from time *t*.

From equations (4) and (5) can be used to find system (6):

$$\begin{aligned}\dot{\psi}_1 &= \psi_2 k^2 - \psi_3 2nk^2;\\ \dot{\psi}_2 &= -\psi_1 + \psi_2 2n + \psi_3 (k^2 - 4n^2);\\ \dot{\psi}_3 &= 0.\end{aligned} \quad (6)$$

Scalar multiplication of two vector functions ψ and X in (4) any time (Hamiltonian H) must be maximum [4-6]. To have such maximum, control actions $U(t)$ and $\dot{U}(t)$ must be within limits (3), depending only from the sign of functions ψ_1, ψ_2.

Its allows synthesising new control actions for mechatronic systems when the control action swiching velocity from one state to other is limited (3) [1-6].

Synthesis of control action as like time function

Results of modeling in MathCAD taking into account control switch velocity limits are shown in Fig. 1-4 Feedback system does not exist, so, motion is unstable and stops (Fig. 4).

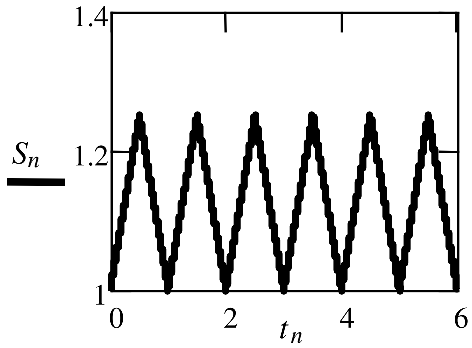

Fig. 1. **Surface (S_n) control in time (t_n) domain**

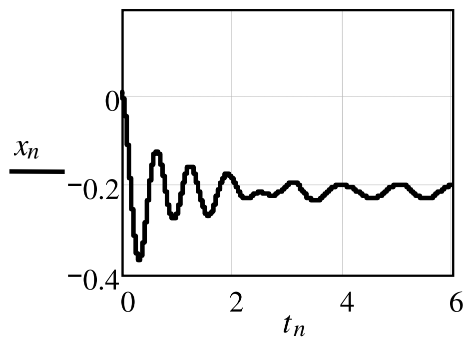

Fig. 2. **Displacement in time domain**

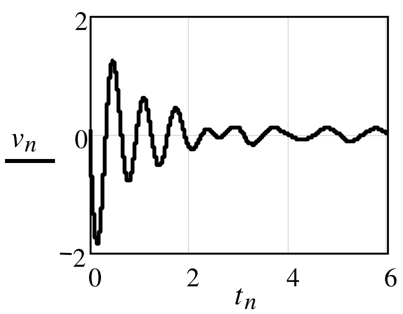

Fig. 3. **Velocity in time domain**

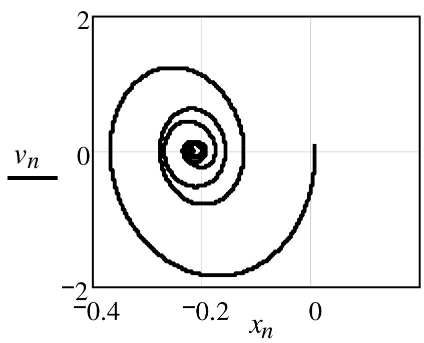

Fig. 4. **Motion in phase plane**

Synthesis of adaptive control action

Results of modeling are shown in Fig. 5-6.

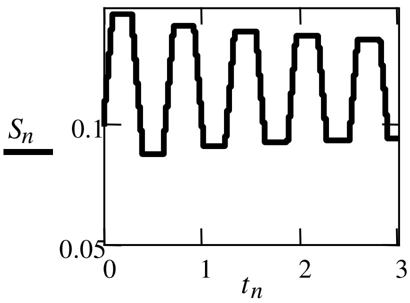

Fig. 5. **Surface change in time domain takinge into account bounds of switching velocity and surface limits**

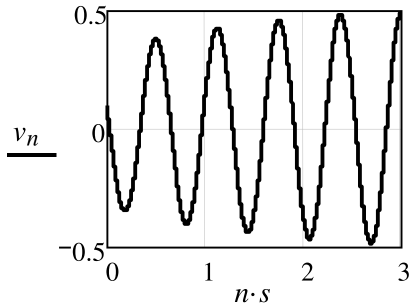

Fig. 6. **Velocity in time domain**

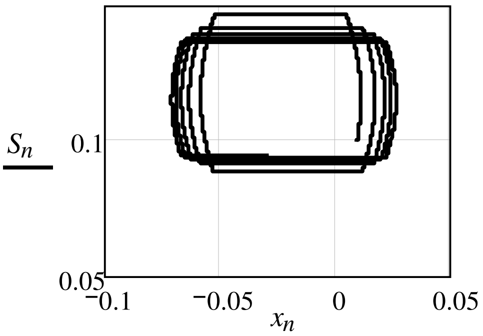

Fig. 7. **Surface exchange as function of the displacement**

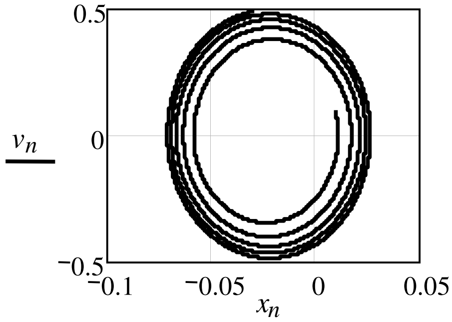

Fig. 8. **Motion in phase plane**

Results and discussions

First attempts to determine force F acting on a plate located in perpendicular to flow were made long time ago. Consideration was that fluid particles fully lose kinetic energy after hitting obstacle. Kinetic energy of fluid volume (Fig. 2) is:

$$\rho \cdot L \cdot S \cdot \frac{V^2}{2} \quad . \tag{7}$$

Plate stops this volume, thus revealing useful work A is:

$$A = L \cdot F \, , \tag{8}$$

where ρ – density;
V – flow velocity;
S – specific area.

From (7) and (8) yields:

$$F = S \cdot \frac{\rho \cdot V^2}{2} \quad . \tag{9}$$

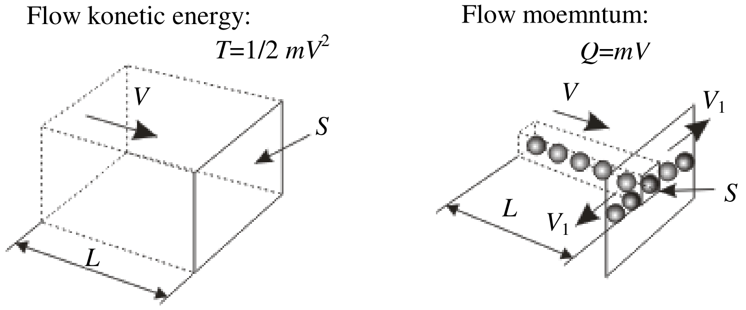

Fig. 9. **Two approaches to determine force acting on plane**

We consider it is not entirely correct to use the theorem of kinetic energy exchange, because particles after hitting the surface, don't actually stop, but continue moving in all directions, thus keeping some energy. Kinetic energy is scalar, and it can not be projected on flow direction. Instead, we recommend to use theorem of momentum exchange:

$$m \cdot v_2 - m \cdot v_1 = \int_0^{\tau} \overline{F} \cdot dt \quad . \tag{10}$$

Where $m = \rho \cdot L \cdot S$ – elementary volume mass, $\tau = L/V$ – time interval in which mass hits the plate. Consider here $v_2 = 0$ and $v_1 = V$. Then the following can be yielded:

$$F = S \cdot \rho \cdot V^2 \quad . \tag{11}$$

The only difference between (9) and (11) is coefficient "1/2".

However, using (9) or (11) in practice don not give corresponding results. Even a simple object like plate gives 20 % error. That is because drag and lift coefficients are used. They show how much real force differs from calculated with (9). Some examples of drag coefficients for several basic shapes are shown below.

So, it is not important if the "1/2" coefficient is used or not, but one must remember, that all experimentally obtained aerodynamic coefficients were calculated using (9), that includes this coefficient. Likely, formula (9) is used in practice, because it accents the force dependency on flow kinetic energy – $1/2\ \rho v^2$.

Formula (11), however, gives satisfying results when calculating force acting on a wall, when water stream from hose is hiting it.

Consider a more general case, where momentum exchange theorem is used (Fig. 10.). For elementary area *dx dy* the following can be written:

$$dm \cdot \overline{V_1} - dm \cdot \overline{V_0} = d\overline{F} \cdot dt\,, \tag{12}$$

where $dm = \rho\ V_0\, dt\ dx\ dy \cos\gamma$.

If the velocity of the particle after impact is proportionally linear to the velocity before, interaction follows as:

$$V_1 = k1 \cdot V_0 \cdot \sin\gamma \tag{13}$$

Here, coefficient $k1$ includes boundary layer interaction forces (and viscous damping forces too).

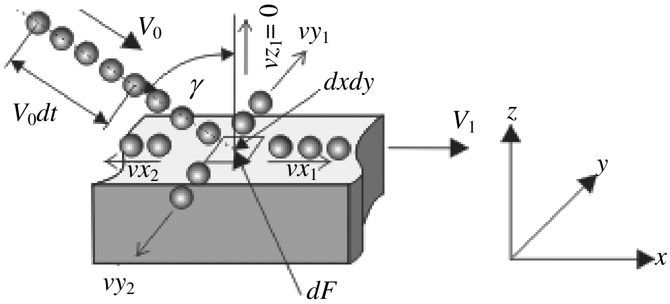

Fig. 10. **Finding a force acting on a solid body in general case**

Projecting the force to axis we get:

$$x: \quad dF_x = -\rho \cdot V_0^2 \cdot dx \cdot dy \cdot \cos\gamma \cdot \sin\gamma \cdot (1 - k1) \quad . \tag{14}$$

$$y: \quad dF_y = 0 \quad . \tag{15}$$

$$z: \quad dF_z = \rho \cdot V_0^2 \cdot \cos^2\gamma \cdot dx \cdot dy \quad . \tag{16}$$

Equations (14-16) can be used for global fluid environment resistance calculation. In this case integrals along surface $f = f(x, y, z)$ must be found. Take into account, that angle γ along streamlines may be function of x, y. Additionally, velocity V_0 may vary too.

Experimental investigations

All experiments were conducted in Armfield "wind tunnel" (Fig. 11). The main problem was to find independent surface control without special sensors and actuators. Experiments shows that it is possible only in system with two DOF when centre mass of body mass centre moves by loop in a closed trajectory.

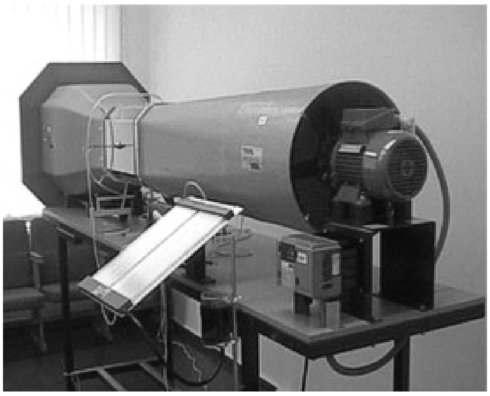

Fig. 11. **Armfield wind tunnel**

Here we present several results, obtained using "Armfield" subsonic wind tunnel. Drag forces, acting on different shapes, were measured and drag coefficients calculated using (17):

$$F_d = C_d \cdot S \cdot \frac{\rho \cdot V^2}{2} \; . \qquad F_l = C_l \cdot S \cdot \frac{\rho \cdot V^2}{2} \; . \tag{17}$$

where ρ – density,
V – flow velocity,
S – specific area,
C_d and C_l – drag and lift coefficients.

These coefficients depend on bodies' geometry, orientation relative to flow, and non-dimensional Reynolds number

All experiments were done at constant air flow velocity in the range of 10-20 m·s^{-1}. This corresponds to R_e value about 40 000.

Drag coefficient for a plate perpendicular to flow depends on plate's length and chord ratio. For standard plates C_d is about 1.2. For infinitely long plates C_d seeks to 2.

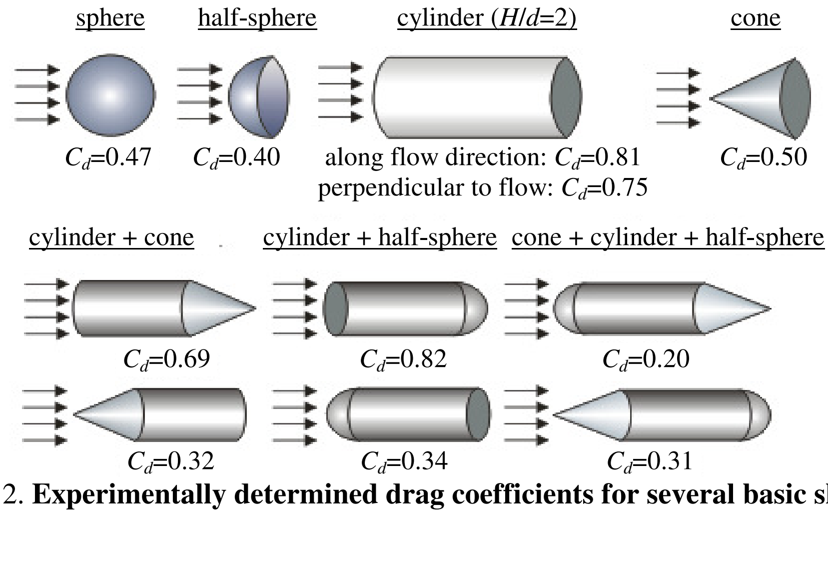

Fig. 12. **Experimentally determined drag coefficients for several basic shapes**

Conclusion

Is shown that body surface alteration may be used for vibration excitation. Control action needs two sensors – displacement and velocity. Stable motion can be achieved only in adaptive systems with feedback control [7; 8]. Experiments shows that vibration excitation without special actuators is possible only in system with two DOF.

References

1. Lavendelis E., Viba J. Individuality of Optimal Synthesis of vibro impact systems. Book: Vibrotechnics". Kaunas. Vol. 3 (20), 1973. pp. 47-54. (in Russian).
2. Viba J. Optimization and synthesis of vibro impact systems. Zinatne, Riga, 1988. 253. p. (in Russian).
3. Lavendelis E., Viba J. Methods of optimal synthesis of strongly non-linear (impact) systems. Scientific Proceedings of Riga Technical University. Mechanics. Vol. 24. Riga, 2007. pp. 9-15.
4. Boltyanskii V.G., Gamkrelidze R.V., Pontryagin L.S. On the Theory of Optimum Processes (In Russian), Dokl. AN SSSR, 110, No. 1, 1956. pp. 7-10.
5. Boltyanskii V.G. Mathematical Methods of Optimal Control, Nauka, Moscow. 1969. 408. p. (in Russian).
6. Sonneborn L., Van Vleck F. The Bang-Bang Principle for Linear Control Systems, SIAM J. Control 2, 1965, pp. 151-159.
7. Tipans I., Viba J., Fontaine J.G., Kruusmaa M., Megill W., Cifanskis S. Theory of robot fish plane motion. Scientific Proceedings of Riga Technical University. Mechanics. Vol. 31. Riga, 2009. pp. 8-18.
8. Kulikovskis G., Abele M., E. Kovals., Tipans I., Cifanskis S., Kruusmaa M., Fontaine J.G., Viba J. Robotic fish tail motion excitation by adaptive control. Scientific Proceedings of Riga Technical University. Mechanics. Vol. 33. Riga, 2010. pp. 15-20.