

# STRESS STATE AND STRENGTH OF THE PIEZOCERAMICS AT STATIC LOAD

## НАПРЯЖЕННОЕ СОСТОЯНИЕ И ПРОЧНОСТЬ ПЬЕЗОКЕРАМИКИ ПРИ СТАТИЧЕСКОЙ НАГРУЗКЕ

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**Abstract:** The strength of piezoelectric transducer at static loading and vibrations was investigated. The theoretical model of strength for piezoceramics stripe glued to structural element was developed. Using the concept of ‘weak chin’ and the results of stress state analysis of three-layered structure with the cracks in one of layers. The general tendencies of effect of this structure parameters were defined. The effect of mechanical loading of piezoceramics transducer to its stress state and destruction can be decreased by rational selection of its geometrical parameters and thickness of coupling layer.

**KEYWORDS:** PIEZOCERAMICS, STRESS, STRENGTH

### 1. Introduction

Structural health monitoring (SHM) of aircraft can be successful, if minimally two requirements are obeyed: 1) SHM system is able reliably detect all kinds of structural damages, and 2) Own reliability of SHM system is high. In ultrasonic SHM system the least defended element is a piezoceramics transducer integrated in structure. Conditions of aircraft operation are very complex: mechanical loading and vibration, environmental degradation, wide range of temperature and others. First of all the transducer is attached to the structural element. It means that is loaded by alternative operational load. It can induce static or fatigue destruction of transducer. For example, Fig. 1 shows a piezoceramics 0.5x10x50mm transducer installed to an Al panel after about 60000 cycles of loading with the alternative stress 150/50 MPa [1]. The transducer was glued on a skin of panel in direction of load action. This level of stress is typical for Al alloy structure of aircraft. At least nine fatigue cracks on a surface of the transducer were detected by penetration. This effect associated with low tensile strength of piezoceramics that equal to 40-80 MPa [2,3]. It is much less than compressive strength (about 600MPa). The crack-resistance of piezoceramics is also low (the toughness about 1-3 MPa·m<sup>0.5</sup>) [4]. There are some structural possibilities to protect piezoceramics from effect of mechanical loading. Pre-stressed transducer described in article [5] is good solution as from overloading as from corrosion. But constraining decreases sensitivity of damage detection.

The influence of transducer parameters and its coupling with structural element to stress and strength is investigated in present paper.



Fig.1. The cracks in piezoceramics after cyclic loading

### 2. Stress state and strength of piezoceramics transducers

#### 2.1. Finite Element Analysis

The own strength of piezoceramics transducer is one of the key problems of SHM implementation and results of its investigation are presented below. First of all the stress state of transducers those are using in the static and fatigue tests of the samples and in the full-scale testing of the Mi-8 helicopter tail beam was analyzed by FEM. The ANSYS academic program was used. The maximal principal stress of one quarter of a scheme “Al sheet – glued

PIC151” is showed in the Fig.2. The 0.5x10x50mm PIC151 is glued by the layer 0.1mm to Al sheet.

The nominal direct stress in sheet is equal to 100 MPa. It can see that the stresses in the plain of symmetry of transducer are distributed in a range 71-86 MPa. The minimal stress 71.35 MPa is on upper surface and maximal one is in points of ‘transducer – glue’ boundary. It is close to strength of material. Therefore the cracking of piezoceramics was observed in spatial static test at the same loading. The cracks were observed before the nominal direct stress in sheet reaches 70 MPa.

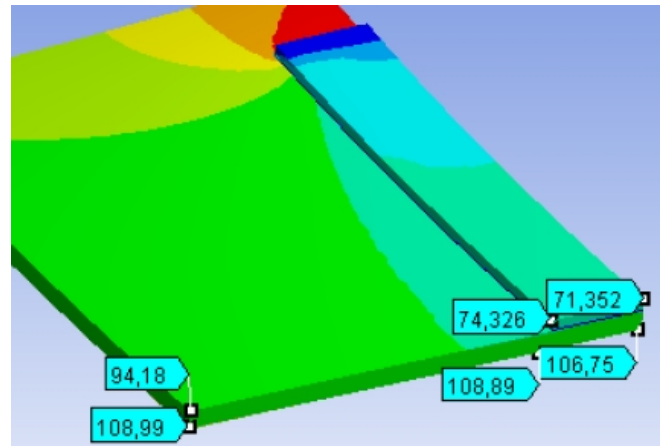


Fig. 2. Stress FEA of the 50mm length transducer (PIC 155)

The Fig.3 illustrates the stresses distribution if a crack occurs in the plain of symmetry of transducer. It can see the stress level in remaining part of transducer decreases. It shows the size of a transducer along the direction of loading influences significantly to stress state.

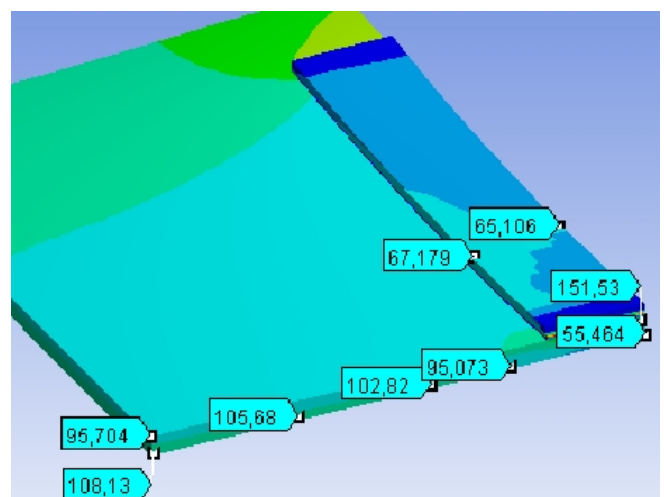


Fig. 3. Stress FEA of the 50mm length transducer (PIC 155) with a crack in the plain of symmetry of transducer

## 2.2. 1-D model

Dynamic model of constrained piezoceramics transducer developed earlier [6] can be modified. The scheme of three-layer structure is shown in Fig.4 (there are some differences in the coordinate system location and layers signing).

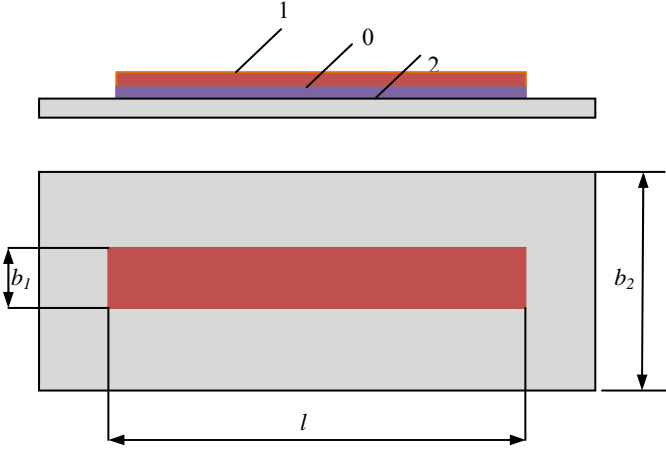


Fig. 4. Piezoceramics transducer 1 with Al alloy overlay 2, glued by Epoxy past layer 0

As a result, the relative axial displacements can be defined by the formulas

$$(1) \quad \bar{u}_1(\xi) = \frac{\alpha_1^2}{\alpha_1^2 + \alpha_2^2} \frac{\sigma}{E_2} \left( \frac{\sinh \lambda \xi}{\lambda \cosh \frac{\lambda}{2}} - \xi \right),$$

$$\bar{u}_2(\xi) = \frac{\alpha_1^2}{\alpha_1^2 + \alpha_2^2} \frac{\sigma}{E_2} \left\{ \left[ 1 + \frac{\alpha_1^2 + \alpha_2^2}{\alpha_1^2} \right] \frac{\sinh \lambda \xi}{\lambda \cosh \frac{\lambda}{2}} + \xi \right\},$$

where the dimensionless parameter

$$\lambda = \sqrt{|\alpha_1^2 + \alpha_2^2|},$$

$$\alpha_1^2 = \frac{l^2 b_2}{(1 + \nu_1) \delta_1^2 b_1} \frac{2G_2 \delta_0}{G_0 \delta_2 D_0},$$

$$\alpha_2^2 = \frac{l^2}{(1 + \nu_2) \delta_2^2} \frac{2G_1 \delta_0}{G_0 \delta_1 D_0}, \quad \xi = \frac{x}{l},$$

$$D_0 = 1 - \left( 1 + \frac{2G_1 \delta_0}{G_0 \delta_1} \right) \left( 1 + \frac{2G_2 \delta_0}{G_0 \delta_2} \right)$$

Here  $u_i, \rho_i, E_i, G_i, b_i, \delta_i$  are axial displacement, density, elastic modulus, shear modulus, cross-section area and its width and thickness for each layer ( $i=0,1,2$ ).

The direct stresses in the middle plain of layers are defined by those formulas:

$$(2) \quad \frac{\sigma_1(\xi)}{\sigma} = \frac{\alpha_1^2}{\alpha_1^2 + \alpha_2^2} \frac{E_1}{E_2} \left( 1 - \frac{\cosh \lambda \xi}{\cosh \frac{\lambda}{2}} \right),$$

$$\frac{\sigma_2(\xi)}{\sigma} = \frac{\alpha_1^2}{\alpha_1^2 + \alpha_2^2} \left\{ \left[ 1 + \frac{\alpha_1^2 + \alpha_2^2}{\alpha_1^2} \right] \frac{\cosh \lambda \xi}{\cosh \frac{\lambda}{2}} + 1 \right\}$$

The direct stresses on external surfaces are defined by those formulas:

$$(3) \quad \frac{\sigma_{1(e)}(\xi)}{\sigma} = (1 + \beta_{12}) \frac{\sigma_1(\xi)}{\sigma} - \beta_{12} \frac{E_1}{E_2} \frac{\sigma_2(\xi)}{\sigma},$$

$$\frac{\sigma_{2(e)}(\xi)}{\sigma} = -\frac{E_2}{E_1} \beta_{21} \frac{\sigma_1(\xi)}{\sigma} + (1 + \beta_{21}) \frac{\sigma_2(\xi)}{\sigma}$$

$$\text{Here } \beta_{11} = - \left[ \left( 1 + \frac{2G_2 \delta_0}{G_0 \delta_2} \right), \frac{2G_1 \delta_0}{G_0 \delta_1} \right] \frac{1}{D_0},$$

$$\beta_{12} = -\frac{2G_2 \delta_0}{G_0 \delta_2} \frac{1}{D_0}, \quad \beta_{21} = -\frac{2G_1 \delta_0}{G_0 \delta_1} \frac{1}{D_0},$$

$$\beta_{22} = - \left[ \left( 1 + \frac{2G_1 \delta_0}{G_0 \delta_1} \right), \frac{2G_2 \delta_0}{G_0 \delta_2} \right] \frac{1}{D_0}$$

Some results of stress analysis by developed model are presented in Fig.5 and Fig.6. The direct stress (as a part of nominal external stress in sheet) in cross-section of transducer and sheet are showed for points of middle plane of these elements. It is seen the length of a transducer significantly influences to stress state.

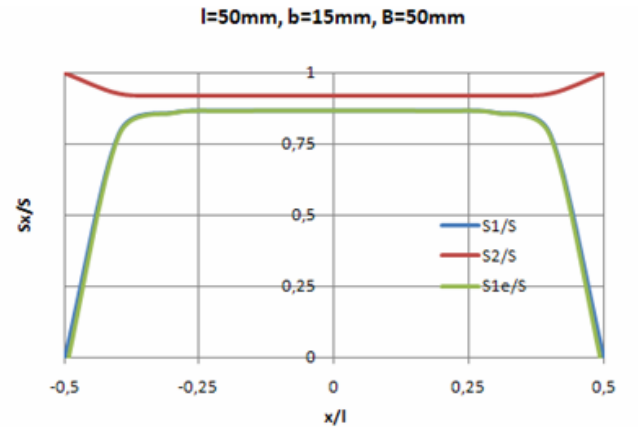


Fig. 5. Stress distribution in middle plane of a large length transducer and sheet

The predicted stresses are close to FEA and to results of experimental measurement (error not more than 4%).

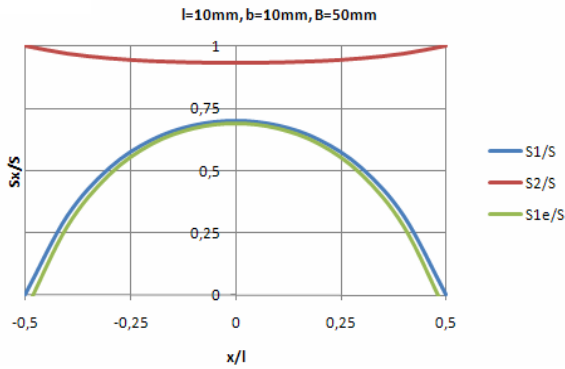


Fig.6. Stress distribution in middle plane of a small length transducer

It allows conclude the model can be used for parametrical analysis of effect of geometrical and mechanical properties to stress in piezoceramics transducer. In particular the effect of length is seen from Fig. 7.

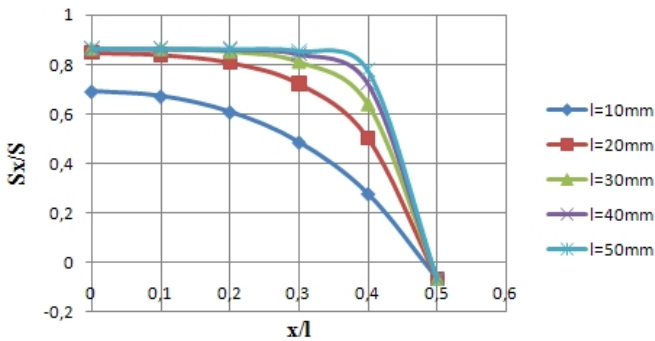


Fig.7. Stress distribution in middle plane of a transducer for five values of a length

Using the developed model of constrained transducer the effect of thickness of a glue layer was estimated. Maximal stress on external surface of transducer as a part of nominal stress in a cross-section of the sheet as a function of relative thickness of a glue for five values of a length is showed in Fig.8. It is seen the thickness is some significant parameter which increasing can decrease the direct stress in transducer.

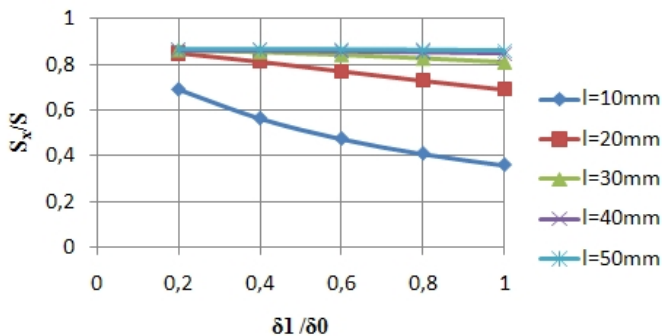


Fig. 8. Maximal stress on external surface of a transducer as a function of relative thickness of glue for five values of a length

FEA shows the size of a transducer along the direction of loading influences significantly to stress state. The developed 1-D model of constrained PZT can be used for analysis of the elastic and geometrical parameters effect to properties of piezoceramics transducer, and for structural health monitoring of element with possible damage. First practical application is showed if Fig.9. The

combined piezoceramics transducer is used in final dynamic test of SHM system integrated to the Mi-8 helicopter tail beam. Use of array of small sizes (1x6.35x6.35mm) transducers allows significantly to decrease stresses and increase their lifetime.

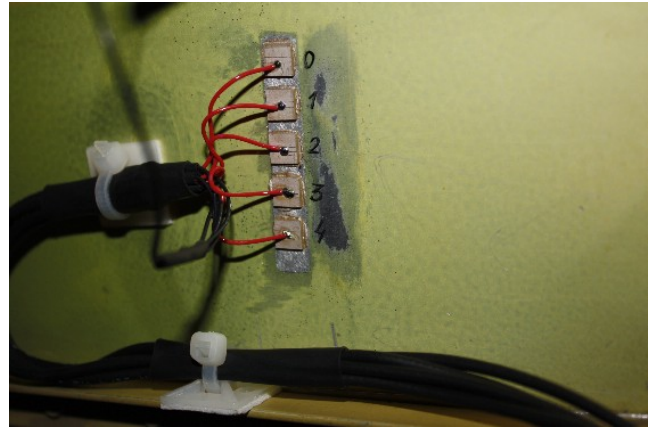


Fig. 9. Combined piezoceramics transducer installed to the skin of the Mi-8 helicopter tail beam

### 3. Discussion about the further model of strength

The results of stress analysis allow to build the theoretical model of strength of piezoceramics transducer. Both static and cyclic models have the same base. Constraints between transducer and loaded structural element define specific destruction. After first crack in some cross-section the transducer will divided to two separate parts. It can conclude the stress level for each of them decreases, but strength is more than stresses before first cracking.

The function of strength distribution of unique small part of a transducer should be the base of a model.

The maximal direct stress after each local destruction decreases and can be estimated for each current configuration. Using the concept of 'weak chin' the history of destruction can be predicted. Stress state analysis of three-layers structure with the crack in one of layers allows to select optimal geometrical sizes of constrained transducer.

Important practical recommendation is the combined transducer of excitation/sensing in SHM system that consists from the array of small size transducers much less affected from mechanical loading.

### 4. Conclusions

- 1) FEA shows the size of a transducer along the direction of loading influences significantly to stress state.
- 2) The developed 1-D model of constrained PZT can be used for analysis of the elastic and geometrical parameters effect to properties of piezoceramics transducer, and for structural health monitoring of element with possible damage.
- 3) The combined transducer as an array of small length of excitation/sensing at the final stage of full-scale test of the Mi-8 helicopter tail beam.

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