

## LEVELLING NETWORK CONNECTION BETWEEN LATVIA AND LITHUANIA

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**Abstract.** Unified and continuous national vertical network is back-bone for geodesy, cartography, civil engineering and global positioning. International institutions are working to reach homogenous and unified vertical datum all around globe. Evaluation of levellings on boarder between Latvia and Lithuania is under particularly interested. Connection between vertical networks is made in three places, so connecting lines construct the two first order levelling loops. Joined loop adjustment produce a good base to analyze and evaluate height connection between Latvia and Lithuania not only as between neighbouring countries, but as a part of EVRS also.

**Keywords:** levelling, vertical network, observations.

## 1. Introduction

The existing levelling networks of Latvia and Lithuania are a part of United Precise Levelling Network (UPLN). They do not fit to nowadays requirements of the countries geodetic control. So the project of the Fundamental Vertical Network of Lithuania was prepared (Parseliunas *et al.* 1998; Buga *et al.* 1999; Krikstaponis *et al.* 2007; Zakarevicius *et al.* 2008) The network was observed in 1998–2007.

Projection and construction works of Height Network of Latvia was started in 2000 and continues till 2010. Nowadays the computing of the network is still going.

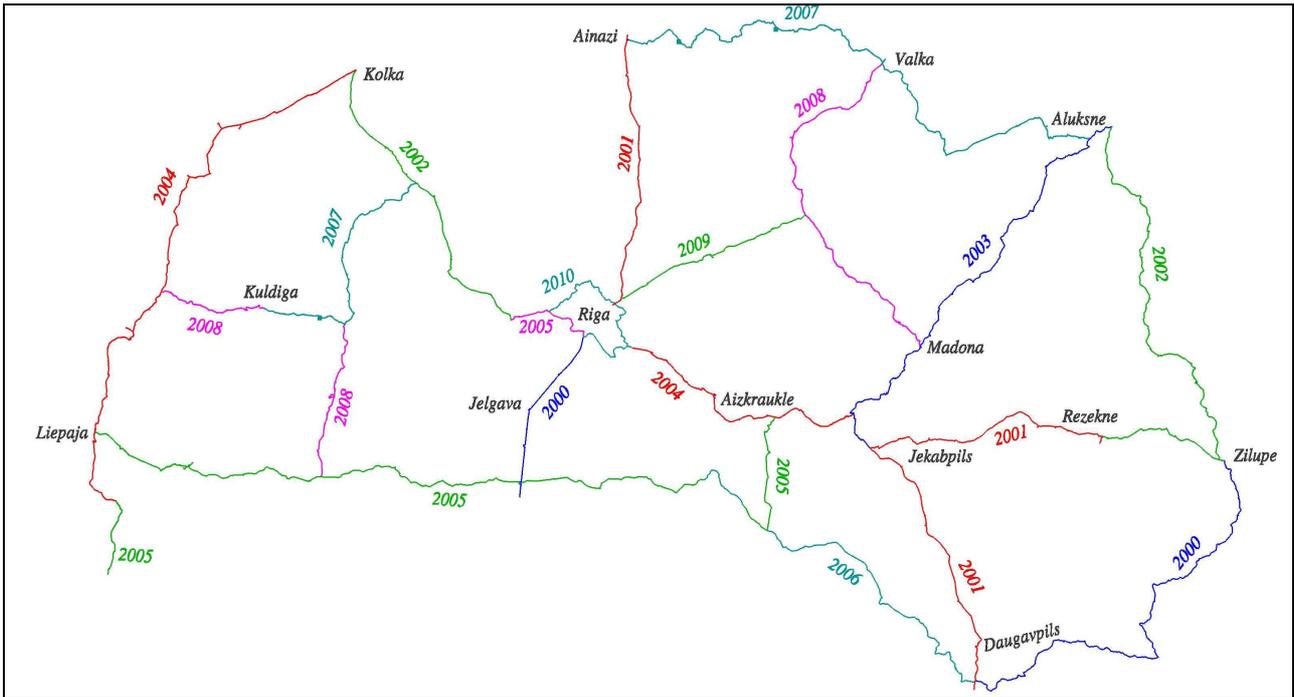
Following the Resolution of the European Reference Frame (EUREF) Symposium adopted in Bad Neuenahr – Ahrweiler in 1998 (Parseliunas *et al.* 1998) requesting to extend and improve the Vertical network around the Baltic Sea, both Latvian and Lithuanian geodesists included the existing state levelling networks to the United European Levelling Network (UELN).

In order to unify the geodetic datums of Latvia and Lithuania, to have the reliable base for the geodynamic

studies it was decided to connect both levelling networks. So the new first order levelling lines Būtingē–Rucava, Joniškis–Eleja and Turmantas–Demene were observed by Latvian and Lithuanian geodesists in 2007–2010 and adjustment of the common united vertical network was carried out (which consist of all Lithuanian vertical network and seven lines of Latvian first order lines).

## 2. An overview of the Latvian vertical network

Establishment of Latvian National First Order Levelling Network (NFOLN) has begun in 2000 and field measurements were finished in 2010 (Celms, Kaminskis 2005). Geodetic measurements were made by the specialists of the Latvian Land Service from 2000 till 2005 and Latvian Geospatial Information agency from 2006 till now. Establishment and measurements were done following the technical requirements of an “I, II and III classes levelling instruction” (I, II 2000). NFOL consists of 15 loops of precise levelling lines. Five loops around Riga are shown as one loop (Fig 2.1).



**Fig 2.1.** First order lines of the Latvian NFOLN and year of field observations

Geodetic and gravimetric observations data were used for the network establishment. The general requirement for not exceeding RMS error 0.5 mm/km of the measured height differences was followed in the course of establishment of the National First Order Levelling Network.

Digital levels Leica NA3003, Zeiss DiNi12, Zeiss Ni002 and Trimble DiNi0.3 with invar rods with bar code scale Wild GPCL3, Zeiss LD13, Zeiss LD11 and gravimeters *Scintrex CG-3* and *CG-5* were used for measurements. All levelling lines were divided into sections. Sections length is approximate 0.5 km in urban area and approximate 2 km in rural area. Every section was levelled forward and backward.

The section heights differences were corrected by the corrections of levelling rod calibration and temperature corrections. Staff readings were reduced to the staff calibration temperature +20°C. Temperature correction is computed from formula

$$\delta_c = \Delta h \cdot (k_{rod} + k_{thermal} \cdot (t_m^o - t_{cal}^o)), \quad (2.1)$$

where  $\Delta h$  – measured height difference;  $k_{rod}$  – rod length thermal dependency;  $k_{thermal}$  – thermal expansion coefficient;  $t_m^o$  – mean temperature during measurements;  $t_{cal}^o$  – calibration temperature.

Effect of refraction was minimized by equal sight distance from level to rod (sight distance was maximum 40 m but usually 36 m) and carry out measurements at morning and evening times.

Corrections due to non-homogeneity of gravitational field were computed from parameters of real and normal gravitational fields. Accurate gravimetric data and normal gravitational fields of GRS 80 were used for this purpose (Kaminskis, Forsberg 1997). Gravitational acceleration was measured on precise levelling lines. Distance between gravimetrically measured benchmarks is in urban areas approximate 1 km and rural areas approximate 2 km. All gravimetric measurements are connected to Second Order Gravimetric Network, which is built on First (absolute) order network and is realized in IGSN71. No special correction for Moon and Sun caused tidal effect is invited so far.

Normal height difference in LKS 92 (Latvian Geodetic Coordinate System) is determined in GRS 80 normal field, applying new European gravity system and evaluating GRS 80 normal field equipotential surfaces non-linearity (Moritz 1988). Normal correction computed from formula:

$$f_{ik}^{80} = \frac{1}{\gamma_{80}} (g_{71} - \gamma_{80})_v h_{ik} - \frac{1}{\gamma_{80v}} (\gamma_{80k} - \gamma_{80i}) H_v, \quad (2.2)$$

where  $g_{71}$  – measured gravity value in IGSN71 system;  $h_{ik}$  – measured height difference;  $H_v$  – mean height between points;  $\gamma_{80}$  – GRS 80 normal field gravity value on rotation telluroid surface;

$$\gamma_{80} = \gamma_0 - 0.3086 \cdot H, \quad (2.3)$$

where  $H$  – point height round till meter. GRS 80 normal field gravity value  $\gamma_0$  computed from:

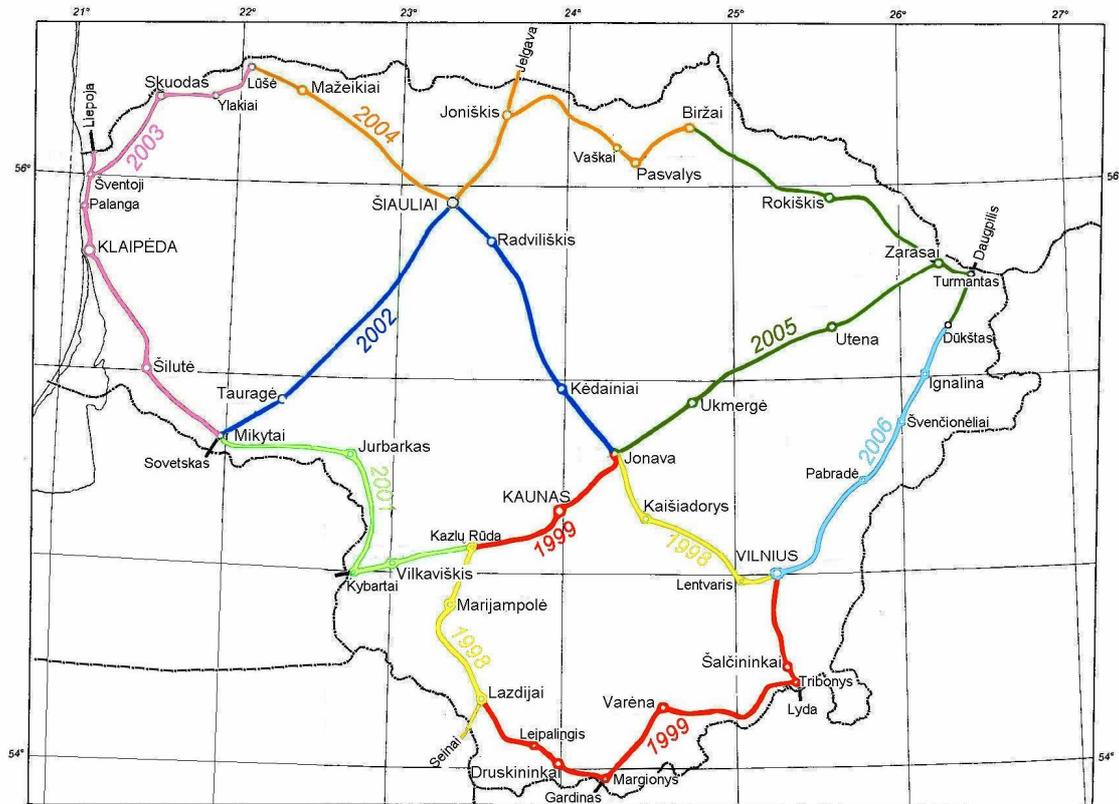
$$\gamma_0 = 97802.7 \cdot (1 + 0.0053024 \cdot \sin^2 B - 0.0000058 \cdot \sin^2 2B) \quad (2.4)$$

where  $B$  – geodetic latitude in GRS80 system;

Observed height differences were corrected by temperature and calibration so the corrected height differences for forward and backward levelling were computed. Mean height difference values were computed and corrected by normal correction  $f_{ik}^{80}$  and final height differences were computed for 3 connection lines and 4 Latvian First Order Levelling network lines near Latvia and Lithuania border area. All network adjustment is still under computing and therefore no new data is integrated in UELN and EVRS.

### An overview of the Lithuanian vertical network

Process of Lithuanian National Geodetic Vertical First Order Network (NGVN) establishment was going on from 1998 till 2007 (Parseliunas *et al.* 1998; Buga *et al.* 1999; Krikstaponis *et al.* 2007). Contracting authority for network establishment was National Land Service under the Ministry of Agriculture. Lithuanian National Geodetic Vertical Network was established following technical regulation of requirements „Lithuanian National Geodetic Vertical Network“. Latest requirements on vertical networks establishment were considered (European 2000; European 2000a; Ihde, Augath 2000). The NGVN consists of 5 loops of precise levelling lines (Fig 3.1).



**Fig 3.1.** First order lines of the Lithuanian NGVN and year of field observations

Data of the geodetic and gravimetric observations were used for the network establishment. The geopotential heights of the points were determined from results of the precise levelling and gravimetric data. The ellipsoidal heights of the network points were obtained by means of GNSS positioning.

The general requirement for not exceeding RMS error 0.5 mm/km of the measured height differences was followed in the course of establishment of the National Geodetic Vertical First Order Network.

Digital levels *Leica NA3003*, invar precise staffs bar coded staffs *Wild GPCL-3*, GPS receivers *Ashtech Z12*, *Z-Surveyor*, *Trimble 5700* and gravimeters *La Coste & Romberg* were used for measurements. All levelling lines

were divided into sections. Every section was levelled forward and backward. The field measurements of heights differences were corrected by the corrections of staffs' calibration, and temperature corrections. Effect of refraction was also taken into account. Corrections due to non-homogeneity of gravitational field were computed from parameters of real and normal gravitational fields. Sufficiently accurate gravimetric data and normal gravitational fields of Helmert and GRS 80 were used for this purpose. Gravimetric acceleration measurements in control gravimetric first order network were performed with *La Coste & Romberg* gravimeters. Gravimetric observations were tied to Lithuanian National Zero Order Gravimetric Network, at the stations of which absolute

gravitational acceleration was measured (Parseliunas *et al.* 2010).

Tides are caused due to Moon and Sun tidal effect (Petroskevicius 2000; Petroskevicius 2004; Torge 1989; Petroskevicius *et al.* 2008). Due to this a periodical change of height difference between Earth surface points is going on. Height difference change due to Moon is possible from  $-0.18$  mm to  $0.18$  mm and due to the Sun from  $-0.07$  mm to  $0.07$  mm on the Lithuanian territory for the points separated by 2.5 km. There are two maximums and minimums during the day time. The largest effect of both celestial bodies is during the full and young Moon periods. Tidal corrections  $\delta_{MS}$  for the height differences were computed using formulas:

$$\delta_{MS} = \delta_M + \delta_S, \quad (3.1)$$

where  $\delta_M$  – correction due to the Moon and

$$\delta_M = v_M S \cos(A_M - A), \quad (3.2)$$

$\delta_S$  – correction due to the Sun:

$$\delta_S = v_S S \cos(A_S - A), \quad (3.3)$$

$S$  – line between points of vertical network in km,  $v_M$  and  $v_S$  – deflection of vertical due to Moon and Sun,  $A_M$  and  $A_S$  – azimuths of Moon and Sun,  $A$  – azimuth between the points.

Staff readings were reduced to the staff calibration temperature  $+20^\circ\text{C}$  (Putrimas 1999; Skeivalas 2000; Skeivalas *et al.* 2009; Zakarevicius, Puziene 2010; Krikstaponis 2001; Krikstaponis 2002; Parseliunas *et al.* 2010). Temperature correction is computed from formula

$$\delta_t = a_m \times \alpha / 2,93, \quad (3.4)$$

where  $a_m$  – staff reading,  $\alpha$  – equation of temperature dependency of staff invar strip of 2.93 m, which common expression is

$$\alpha = k_1 (t_m - 20^\circ) + k_2, \quad (3.5)$$

where  $k_1$  and  $k_2$  – are coefficients of equation of staff length thermal dependency, determined at Finnish Geodetic Institute,  $t_m$  – temperature of invar strip during the levelling.

Staff readings corrected by staff calibration corrections

$$\delta_k = k_3 + k_4 a_m + k_5 a_m^2 + \dots + k_N a_m^{N-3}, \quad (3.6)$$

Height difference at the station

$$h'_s = (a_m + \delta_t^a + \delta_k^a) - (p_m + \delta_t^p + \delta_k^p), \quad (3.7)$$

where  $a_m$  and  $p_m$  – backsight and foresight staffs readings,  $\delta_t^a$  and  $\delta_t^p$  – temperature corrections for backsight and foresight readings,  $\delta_k^a$  and  $\delta_k^p$  – calibration corrections for backsight and foresight readings.

Height differences were corrected for refraction

$$\delta_r = A \Delta t S^2 h'_s, \quad (3.8)$$

where  $A$  – coefficient,  $\Delta t$  – temperature difference between heights  $Z_2$  and  $Z_1$  above earth,  $S$  – length of collimation line,  $h'_s$  – height difference at the station.

Coefficient  $A$  computed:

$$A = \frac{4,76 \cdot 10^{-4}}{Z_2^c - Z_1^c} \left\{ \frac{p^{c+1} - a^{c+1}}{c+1} - Z_0^c (p-a) \right\}, \quad (3.9)$$

where  $c$  – coefficients,  $Z_0$  – levelling instrument height. Values used  $Z_0 = 1,5$  m,  $Z_1 = 1,0$  m,  $Z_2 = 2,0$  m. Coefficient  $c$  was taken from. Intermediate values derived from second order conformal transformation.

For determination of point's normal heights differences it is needed to evaluate normal field equipotential surfaces non-parallelity, real and normal field non-coincidence. For this purpose normal corrections for height differences determined by levelling in real gravity field are computed (Petroskevicius 2004; Torge 1989; Petroskevicius *et al.* 2008). Gravity value  $g_{71r}$  of European system at the marks height of first order network points computed from Bouguer anomalies  $(g_p - \gamma_H)_{2,3}$ , taken from gravity map, scale 1:200 000. Gravity value  $g_{71z}$  at the surface derived from the formula:

$$g_{71z} = (g_p - \gamma_H)_\delta + \gamma_H^0 - 0,3086 H_z + 0,0419 \delta H_z - 14, \quad (3.10)$$

where  $g_p$  – free fall acceleration in Potsdam system;  $\gamma_H$  – Helmerts field normal gravity value at the teluroid;  $H_z$  – approximate earth surface normal height,  $\delta = 2,3$  g/cm<sup>3</sup> – density of Earth crust,  $\gamma_H^0$  – normal gravity value at the ellipsoid surface, from Helmerts formula:

$$\gamma_H^0 = 978030(1 + 0,005302 \sin^2 B_{42} - 0,000007 \sin^2 2B_{42}), \quad (3.11)$$

where  $B_{42}$  – geodetic latitude in 1942 coordinate system. Gravity value  $g_{71r}$  at the mark height  $H$  computed from:

$$g_{71r} = g_{71z} + dg. \quad (3.12)$$

If  $H_z > H$ , then

$$dg = 0,3086dh - 2 \cdot 0,0419\delta dh, \quad (3.13)$$

where  $dh = H_z - H$ .

If  $H_z < H$ , then

$$dg = -0,3086dh, \quad (3.14)$$

where  $dh = H - H_z$ .

Normal height difference in LKS 94 (Lithuanian Coordinate System of 1994) is determined in GRS 80 normal field, applying new European gravity system and evaluating GRS 80 normal field equipotential surfaces non-linearity (Moritz 1988). Normal correction computed from formula:

$$f_{ik}^{80} = \frac{1}{\gamma_{80v}} (g_{71} - \gamma_{80})_v h_{ik} - \frac{1}{\gamma_{80v}} (\gamma_{80k}^0 - \gamma_{80i}^0) H_v, \quad (3.15)$$

where  $g_{71}$  – gravity value in European system;  $\gamma_{80}$  – GRS 80 field gravity value on teluroid surface;  $h_{ik}$  – measured height difference; GRS 80 normal field gravity value  $\gamma_{80}^0$  computed from :

$$\gamma_{80}^0 = \gamma_{80e}^0 \frac{1 + k_{80} \sin^2 B_{94}}{\sqrt{1 - e_{80}^2 \sin^2 B_{94}}}, \quad (3.16)$$

where  $B_{94}$  – geodetic latitude in LKS 94 system; normal gravity value at equator on equipotential ellipsoid surface  $\gamma_{80e}^0 = 978032.67715$  mGal;  $e_{80}$  – first eccentricity of ellipsoid;  $e_{80}^2 = 0.00669438002290$ ; coefficient  $k_{80} = 0,001931851353$ .

Mean normal gravity value between ellipsoid and teluroid for Lithuanian territory  $\gamma_{80v} = 981500$  mGal.

Free air gravity anomaly of vertical network points:

$$(g_{71} - \gamma_{80}) = g_{71} + \delta g_a - \gamma_{80}^0 - \Delta \gamma_{80}, \quad (3.17)$$

here atmospheric gravity correction (Wenzel 1985)

$$\delta g_a = 0,874 - 0,99 \cdot 10^{-4} H + 0,356 \cdot 10^{-8} H^2, \quad (3.18)$$

height correction ( $H$  in metres) (Torge W. 1989):

$$\Delta \gamma_{80} = -0,30877(1 - 0,00142 \sin^2 B_{94})H + 0,75 \cdot 10^{-7} H^2 \quad (3.19)$$

Observed height differences were corrected by temperature, calibration, refraction and tidal corrections, so the corrected height differences for forward and backward levelling were computed. Mean height difference values were computed and corrected by normal correction  $f_{ik}^{80}$  and final height differences were computed.

In summer of 2007 the NGVN was integrated into UELN (Fig 3.2) (ernsperger 1986; Lang, Sacher 1995; 1996; 1997; Sacher *et al.* 1998; Sacher *et al.* 1999)

The data preparation was divided into some steps:

- Computing the gravity values of the benchmarks and heights differences,
- Computing the geopotential height differences and geopotential heights of benchmarks,
- Preliminary control and adjustment of single Lithuania levelling network,
- Detecting the connections with levelling network of neighbouring countries,
- Encoding the nodal benchmarks according to coding system of UELN,
- Adjustment of total UELN,
- Calculation of normal heights of benchmarks,
- Comparison of the received normal heights with normal heights of national height system.

Later on the NGVN of Lithuania was adjusted using the program HOENA too (Schoch 1995).

Allowable misclosure of closed loops are computed from formula (Table 3.1)

$$f_h = 2 \cdot m_0 \sqrt{L}, \quad (3.20)$$

where  $m_0$  – apriori standard deviation of points heights in mm,  $L$  – loop perimeter in km.

**Table 3.1.** Misclosures of the loops

Loop No.	Loop perimeter, km	Actual misclosure, mm	Allowable misclosure, $m_0=1.0$ mm
1	491,2	-4.33	43.4
2	525.2	+14.80	44.9
3	575.4	-39.94	47.0
4	452.0	-9.97	41.6
5	509.9	+7.79	44.2

All actual misclosures are below allowable ones and this proves right method of normal height difference determination and fieldwork was done with highest quality.

Here are the parameters of the adjustment of the new NGVN of Lithuania:

- Number of fixed points: 1,
- Number of unknowns: 1359,
- Number of measurements: 1364,
- Degrees of freedom: 5,
- Standard deviation: 0.70 kgal×mm/km,
- A-posteriori standard deviation referred to a levelling distance of 1km: 0.83 kgal×mm,
- Mean value of the standard deviation of the adjusted geopotential differences: 1.53 kgal×mm,
- Mean value of the standard deviation of the adjusted geopotential heights: 7.3 kgal×mm,
- Biggest value of the standard deviation of the adjusted geopotential heights: 9.1 kgal×mm,
- Average redundancy: 0.017.



**Fig 3.2.** The Lithuanian NGVN in UELN (red ellipses show the datum points for NGVN)

The adjustment of geopotential heights differences of enlarged UELN (solutions 20/02 – with new NGVN data, and 95/15), including levelling networks of Poland (new levellings are not included) and three Baltic states, was performed as an unconstrained adjustment linked to the reference point 13600 in Amsterdam, geopotential height

of which was set to 0.70259 kgal×m, and normal height to 0.71599 m. The datum points of NGVN are presented in Table 3.2. These datum points were used in the adjustment of the united network of Latvia and Lithuania.

**Table 3.2.** Datum points of Lithuanian NGVN

No.	Name	National code	UELN code	LKS94 coordinates	Geopotential number, $m^2 \cdot s^{-2} \cdot 10^{-1}$	Accuracy of geopotential number in UELN network, $m^2 \cdot s^{-2} \cdot 10^{-1}$	Normal height, m	LSS07 gravity acceleration, $m \cdot s^{-2}$
1	ŠIAULIAI	55S-0128	2412001	55°54'48,78202" 23°22'17,18605"	138,795	0,0127	141,402	9,815339
2	VILNIUS	73S-0271	2412002	54°39'11,30417" 25°17'55,19158"	211,797	0,0128	215,801	9,814334
3	MOLAS	25S-1522	2412004	55°43'47,23801" 21°04'58,88606"	4,590	0,0136	4,676	9,815498
4	ŽELVIAI	26V10300	2412015	56°00'41,96954" 21°06'51,86654"	9,126	0,0138	9,297	9,815762
5	MIKYTAI	34V10201	2412020	55°07'54,06812" 21°57'34,81749"	16,370	0,0116	16,678	9,814947
6	JONAVA	64V--217	2412023	55°05'55,95392" 24°16'20,64503"	67,575	0,0122	68,848	9,814745
7	KAZLAI	53V12421	2412030	54°44'43,61659" 23°28'14,25382"	63,884	0,0112	65,090	9,814756
8	LAZDIJAI	52V-1021	2412038	54°13'18,96189" 23°30'43,65627"	129,529	0,0105	131,981	9,814077
9	PETRŪNIŠKIS	85V-0739	2412055	55°43'08,70335" 26°14'41,29362"	142,250	0,0136	144,924	9,815321
10	RADIKIAI	56V---11	2412065	56°12'13,21889" 23°34'03,21221"	59,636	0,0134	60,754	9,815793

### 3. Adjustment of the United Vertical Network of Latvia and Lithuania

In order to unify the geodetic datums of Latvia and Lithuania, to have the reliable base for the geodynamic

studies it was decided to connect both levelling networks. So the new first order levelling lines Būtingē–Rucava, Joniškis–Eleja and Turmantas–Demene were observed by Latvian and Lithuanian geodesists in 2007–2010.

**Table 4.1.** Data on the benchmarks of connecting levelling lines

Benchmark code	Approx. normal height, m	<i>B</i>	<i>L</i>
Būtingē–Rucava			
26V-6237	9.36900	56 03 09.97802	21 07 09.22163
26V10238	11.72000	56 04 16.70929	21 07 19.04867
21L-1684	11.02613	56 04 50.4464	21 07 22.58237
Joniškis–Eleja			
56V10051	39.89800	56 20 52.57184	23 38 30.49239
56S--335	38.89700	56 21 42.24637	23 39 28.86839
02L-0718	39.24405	56 21 55.1571	23 40 26.33
Turmantas–Demene			
03L-0331	137.83819	55 42 44.9058	26 28 14.77
03L-2285	138.82556	55 42 13.6655	26 28 03.76
95V-0053	139.20100	55 41 29.00782	26 27 36.01

**Table 4.2.** Data on height differences of the connecting lines

Start point	End point	<i>D</i> , km	<i>h</i> , m (Lithuanian measurements)	<i>h</i> , m (Latvian measurements)
Būtingē–Rucava				
26V-1561	26V-6237	1.76	3.20416	
26V-6237	26V10238	2.08	2.39474	2.3948
Joniškis–Eleja				
56V10049	56V10051	1.43	0.56092	
56V10051	56S--335	1.88	-1.00096	-1.00014
56S--335	02L-0718	1.50		-0.3766
Turmantas–Demene				
03L-0331	03L-2285	1.21		0.98737
03L-2285	95V-0053	1.60		0.31740
95V-0053	95S--295	0.35	1.54210	1.5421

All Lithuanian National Geodetic Vertical First Order Network with 3 connection lines and 4 lines of Latvian National First Order Levelling Network was adjusted using the program HOENA (Schoch 1995) developed by the Leipzig department of the Bundesamt für Kartographie und Geodäsie. The adjustment of

geopotential heights differences of the united levelling network was performed as a constrained adjustment linked to the datum points presented in Table 3.2.

The misclosures of the loop are presented in Table 4.3

**Table 4.3.** Misclosures of the loops

Loop	Number of points	Loop perimeter, km	Actual misclosure, kGal×mm	Allowable misclosure, $m_0=1.0$ mm
1	357	491.5	4.29	43.45
2	446	640.7	6.12	49.61
3	352	548.2	1.83	45.89
4	376	525.4	-14.65	44.93
5	412	576.3	-32.83	47.05
6	345	510.0	11.66	44.26
7	320	452.0	3.04	41.67
8	377	569.5	14.71	46.77

All actual misclosures are below allowable ones and that proves the right method of the normal height difference determination and fieldwork was done with highest quality in both networks. Adding seven new lines to all Lithuania network gives small improvement in loop misclosure that indicated high accuracy of measurements and right treatment of difference gravimetric data.

Here are the parameters of the adjustment of the connected vertical network of Latvia and Lithuania:

- Number of fixed points: 10,
- Number of unknowns: 1738,
- Number of measurements: 1823,
- Degrees of freedom: 85,
- Standard deviation: 0.617 kgal×mm/km,

- A-posteriori standard deviation referred to a levelling distance of 1km: 0.3121 kgal×mm,
- Mean value of the standard deviation of the adjusted geopotential differences: 0.37 kgal×mm,
- Mean value of the standard deviation of the adjusted geopotential heights: 1.78 kgal×mm,
- Biggest value of the standard deviation of the adjusted geopotential heights: 2.74 kgal×mm,
- Average redundancy: 0.047.

The normal heights of the border points are presented in Table 4.4.

**Table 4.4.** Normal heights of border benchmarks

Point code	Geopotential number kGal×mm	Normal height m	Standard deviation kGal×mm
26V10238	11.82887	12.05065	0.83
21L-1684	10.97168	11.17738	0.89
56S--335	38.29802	39.01526	1.35
02L-0718	38.66769	39.39185	1.39
03L-2285	136.43932	139.00456	1.31
95V-0053	136.75090	139.32215	1.26

#### 4. Conclusions

1. First step is done to prepare establishment the United Vertical Network of Latvia and Lithuania. The levelling data of both countries fit to each other better than 1 mm accuracy.
2. The accuracy of Latvian and Lithuanian united levelling network in first iteration (standard deviation is 0.617 kgal×mm/km) is at the same level as that of the vertical networks of biggest part of other participating in UELN project countries.
3. The data of Latvian vertical network should be entered to the UELN data base as soon as possible. It will make possible to transfer the European Vertical System to Latvia and Estonia (European 2000).
4. The differences between Latvian-Lithuanian levelling network and UPLN height systems at the border points are about 15 cm.
5. Further job must be done for full Latvian and Lithuanian levelling network common adjustment and unification of the verticals (heights) systems of the both countries.

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