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PROJECTING ELASTOMERIC SHOCK ABSORBERS WITH ADJUSTABLE STIFFNES

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Abstract In many cases, you must be able to adjust the stiffness of a rubber shock absorber during use, depending on the mode of operation, the workload of the unit or for some other reason. In this paper we propose a model of a rubber shock absorber with adjustable stiffness and the method of calculation depending of type "force – settlement" for such a shock. The solution is obtained by the method of Ritsa using the principle of the minimum complete potential energy of deformation and direct methods using functional considered by V. Prager (1970) and converted to weakly compressible materials. As an example, are reviewed by the method of calculation of a cylindrical rubber shock absorber with adjustable. The resulting dependence of type "force – settlement" for this shock absorber during axial compression.

Keywords: *"force – settlement"*, adjustable stiffness, subareas, elastomeric shock absorber, Ritsa method

1. Introduction

Over the past 20–30 years, thanks to the specific properties of rubber (high elasticity, resistance to environmental influences, good dynamic performance, low compressibility, almost linear relationship between stress and strain at strains up to 15÷20 %), rubber shock absorbers are widely used in mechanical engineering and automobile industry (automobile production). In many cases, you must be able to adjust the stiffness of a rubber shock absorber during use, depending on the mode of operation, the workload of the unit or for some other reason. In this paper we propose a model of a rubber shock absorber with adjustable stiffness.

Variable stiffnes of rubber shock absorber it is suggested to get placing in inward declivous cylindrical shock absorber hard support that can move on in parallel z axes (fig. 1). Stiffness of shock absorber can change, from ordinary declivous rubber shock absorber's stiffness ($k_2 = 0$) (depends on geometrical sizes and brand of rubber) to absolutely hard support ($k_2 = h$).

Thus changing the coordinate of support of κ_2 we can increase or diminish free-form of rubber layer into a shock absorber, to regulate shock absorber's stiffness. Calculation of integral dependence „force – settlement"for the rubber shock absorbers of what construction is a labour intensive task. At the calculation of dependence „force-settlement" we will use methodology of the break fields of moving and tensions with direct methods. Method of breaking up of the investigated area on regular simple subareas at being of integral descriptions of type „force-settlement" allows to simplify the choice of the sought after functions a nd considerably to shorten the volume of calculable work.

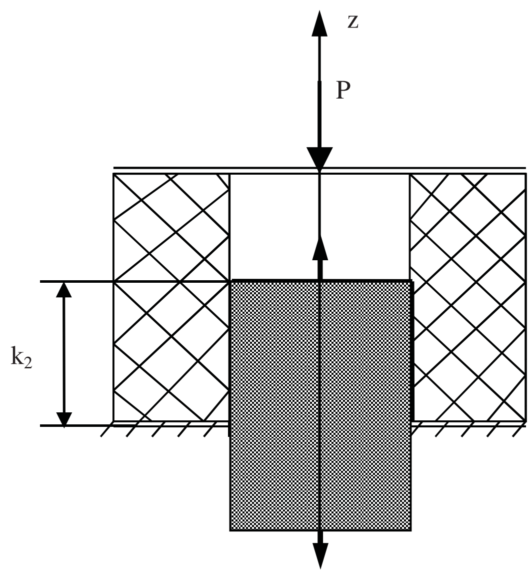

Fig. 1. Principle of operation of elastomeric shock absorbers with adjustable stiffness

2. Decomposition of the volume of the products on a subareas

Let's consider the elastomeric element with the complex geometrical form with volume V and the area of a surface F:

$$V = \sum_{n=1}^{n} V_n \qquad F = \sum_{n=1}^{n} F_n \ , \tag{1}$$

where: V_n – one-coherent regular subareas; N – number of subareas, received as a result of crushing elastomeric element; F – total surface area of all subareas elastomeric element.

The surface limiting n-th subarea, looks like

$$F_n = F_\sigma^n + F_u^n + \Gamma_n \, , \tag{2}$$

where: $F_\sigma{}^n$ – loading surface; $F_u{}^n$ – surface of the fixing; Γ_n – surface area of contact subarea partitioning elastomeric element.

On the crushing surface Γ_n must be satisfied the conditions of continuity:

– of displacements:

$$u_i^n = u_i^{n+1} \, , \tag{3}$$

– and of stresses:

$$\sigma_{ij}^n m_j^n = -\sigma_{ij}^{n+1} m_j^{n+1} \, . \tag{4}$$

Where the index "n" specifies a current issue of a subarea of crushing and m_j^n and m_j^{n+1} – directing cosines normals external, accordingly to V_n and V_{n+1} on Γ_n and Γ_{n+1}, i, j – coordinate system. Here and further on repeated subscripts is carried out the summation, and the comma in the subscript denotes partial derivative.

If you use only the external normal to V_n, which at Γ_{n+1} for V_{n+1} is internal, then $m_j^{n+1} = -m_j^n$ and the condition (4) reads:

$$\sigma_{ij}^n m_j^n = \sigma_{ij}^{n+1} m_j^n \tag{5}$$

Hereinafter, for brevity, over repeated lower indices are summed, and the comma denotes a partial derivative.

Contact surface Γ_n may be artificial in the geometric decomposition of the elastomeric element, or natural, if the physical and mechanical characteristics of the material (G, μ) at the contact surface change abruptly, that is, a volume V composed of different materials.

Partition of the elastomeric element in the subareas extends the permissible class of unknown functions in the piecewise smooth and piecewise continuous functions with piecewise smooth and piecewise continuous derivatives (Prager 1970, Kurant 1951). Discontinuity of displacements and efforts on the surface Γ_n denote:

$$u_i^n - u_i^{n+1} = \{u_i{}^n\},$$
$$\sigma_{ij}^n m_j^n - \sigma_{ij}^{n+1} m_j^n = \{\sigma_{ij}^n m_j^n\} \, . \tag{6}$$

If the surface areas Γ_n of subareas V_n displacement components and efforts (all or only some) does not satisfy the conditions of continuity (3) and (4), then, using the designation (6), we can write:

$$\left\{\sigma_{ij}^n m_j^n u_i^n\right\} = \left\{\sigma_{ij}^n m_j^n\right\}^{\mathrm{I}} u_i^n + \sigma_{ij}^n m_j^n \left\{u_i^n\right\}^{\mathrm{II}} \, , \tag{7}$$

where the indices I and II, respectively, indicate that the summation extends only to the displacement components and efforts that do not satisfy the conditions on the surface Γ_n.

Suppose that in each subareas partitioning the have the required properties of continuity and differentiability.

Variational principle for discontinuous functions during the fragmentation of the field on a subareas is given in (Prager 1970). Using the method of Lagrange multipliers, can be generalized to the case of weakly compressible and incompressible materials, which include majority of elastomeric materials.

3. Mathematical model

When solving boundary value problems of static elasticity theory for incompressible and weakly compressible materials easier as the unknown functions to choose displacement u_i and the function of hydrostatic pressure s, which, for small strains, leads to a mathematical model (Lavendel 1976, Gonca 1970):

Equation of equilibrium:

$$G\left[\nabla^2 u_i + \frac{3}{2(1+\mu)} s_i\right] + f_i = 0 \, . \tag{8}$$

Volumetric deformation:

$$u_{j,j} = \frac{3(1-2\mu)}{2(1+\mu)} s \, . \tag{9}$$

Deformations:

$$\varepsilon_{ij} = 0.5\left(u_{i,j} + u_{j,i}\right). \tag{10}$$

Stress:

$$\sigma_{ij} = G\left[u_{i,j} + u_{j,\mathrm{I}} + \frac{3\mu}{(1+\mu)s\delta_{ij}}\right]. \tag{11}$$

Forces boundary conditions:

$$G\left[u_{i,j} + u_{j,\mathrm{I}} + \frac{3\mu}{(1+\mu)s\delta_{ij}}\right]n_j = P_i \, . \tag{12}$$

Displacements boundary conditions:

$$u_i = u_{0i} \, . \tag{13}$$

When partition the element into a subarea to a mathematical model (8)–(13) to add conditions for docking (3) and (4).

In determining the integral characteristics, of type "force – settlement", of the elastomeric element boundary problem (8)–(13) (without crushing the elastomeric element in the subareas) easier to solve a variational method using the Ritz procedure for the functional (Lavendel 1976, Gonca 1970):

$$J(u_i^n, s^n) = G\int_V [\frac{1}{2}\left(u_{i,j}^n u_{j,i}^n + u_{i,j}^n u_{j,i}^n\right) + \frac{3\mu}{1+\mu} s^n u_{i,i}^n - \frac{9\left(1-2\mu_n\right)}{4\left(1+\mu_n\right)^2} s^{n2}]dV . \quad (14)$$

Using the variational principle of V. Prager (1970) and applying the method of undetermined Lagrange multipliers with a functional (14) be the boundary value problem (8)–(13) with the conditions of the joining (3) and (4) replaced by the variational problem with discontinuous function of demand on the surfaces of the partition Γ_n for the functional:

$$J^*(u_i, s) = \sum_{n=1}^{N} J(u_i^n, s^n) - \sum_{n=1}^{N-1} G_n \int_{\Gamma_n} [(u_{i,j}^n + u_{j+i}^n) + \frac{3\mu_n}{1+\mu_n} s^n \delta_{i,j}] m_j \left\{u_i^n\right\}'' d\Gamma_n , \quad (15)$$

where in each subare: u_i^n – displacements; s^n – function of hydrostatic pressure; G_n – modulus of elasticity in shear; μ_n – Poisson's ratio.

Using the functional (15), the choice of displacement functions u_i^n only need to follow geometric boundary conditions (13), as a function of hydrostatic pressure s_n in each subarea can be selected independently, not caring about its continuity at the boundary of the partition Γ_n.

The examined shock absorber we break up on two parts on the border of support, in parallel axes of or (fig. 2.)

At use functional $J^*(u_i, s)$ (15), choosing u_i^n and s_n, it is enough to satisfy to geometrical boundary conditions on external surface F_u.

Shock absorber is divided into two subareas (see Figure 2). All functions with an index "1" it is carried to a subarea I, and with an index "2" to a subarea II. We believe that the geometry of the elastomeric layer can not take into account the compressibility of the elastomer, that is, believe that the Poisson coefficient μ = 0,5.

The main boundary conditions will be:

$$u_1(r,-k_2) = u_2(r,k_1) = 0$$
$$u_1(a,z)\big|_{0\ge z\ge -k_2} = w_1(a,z)\big|_{0\ge z\ge -k_2} = 0$$
$$w_1(r,-k_2) = 0 \quad (16)$$
$$w_2(r,k_1) = -\Delta ,$$

where: the functions u_i and w_i – displacement, respectively, on the axis of r and z.

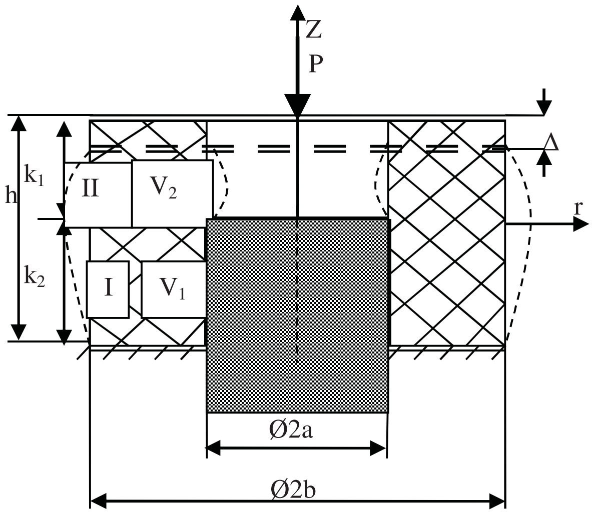

Fig. 2. Elastomeric shock absorbers with adjustable stiffness calculate model

Dependence "force – settlement" it is defined from the equation of balance of the top base of the absorber:

$$2\pi\int_a^b \sigma_{2zz|z=k_1} r dr = -P . \quad (17)$$

On a surface of splitting of a condition of ideal contact piece look like:

$$u_1(r,0) = u_2(r,0)$$
$$w_1(r,0) = w_2(r,0)$$
$$\sigma_{1rz}(r,0) = \sigma_{2rz}(r,0) \quad (18)$$
$$\sigma_{1zz}(r,0) = \sigma_{2zz}(r,0)$$

We choose conveyances u_n, w_n and function s_n whenever possible in the most simple kind with the account only conditions (11) and prospective character of deformation.

$$u_2 = B_1 rz(z-k_1) + B_2(r-a)(z-k_1)$$
$$w_2 = -\frac{\Delta z}{k_1} + A_1(r-a)(z-k_1)$$
$$s_2 = C_2 \quad (19)$$
$$u_1 = B_3(r-a)(z+k_2)$$
$$w_1 = A_2(r-a)(z+k_2)$$
$$s_1 = C_1$$

where A_1, A_2, B_1, B_2, C_1, C_2, Δ – unknown constants.

Functions (19) have on section height approximately following appearance for conveyances at $r = a$; b:

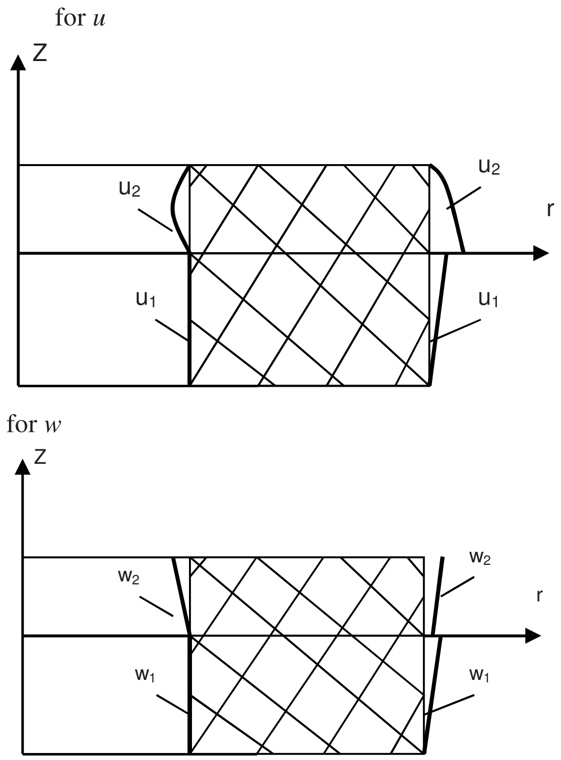

Fig. 3. Expected character of deformation

After integration it is received that functional J^* depends only on unknown constants:

$$J^* = J(A_1, A_2, B_1, B_2, B_3, C_1, C_2, \Delta). \qquad (20)$$

From a condition stationarity

$$\frac{\partial J^*}{\partial(A_1, A_2, B_1, B_2, B_3, C_1, C_2, \Delta)} = 0. \qquad (21)$$

We receive system of the algebraic equations. For dependence "force – settlement":

$$\Delta = \frac{Pk_1}{2\pi G a^2 (1-\alpha^2)} \frac{D_1}{D}, \ \alpha = \frac{a}{b}, \qquad (22)$$

where: D, D_1 – determinants of algebraic equations (21), an expression which, due to the complexity of writing, are not given.

Then the stiffness of the shock absorber according to k_2 (because $k_1 = (h - k_2)$) can be calculated by the formula:

$$c = \frac{P}{\Delta} = \frac{2\pi \cdot G \cdot a^2 (1-\alpha^2) D}{(h - k_2) D_1}. \qquad (23)$$

For absorber: $h = 30$ mm, $a = 20$ cm, $b = 40$ mm, $G = 7 \cdot 10^{-2}$ kg/mm^2, in Figure 3 shows the results (using MatCad):

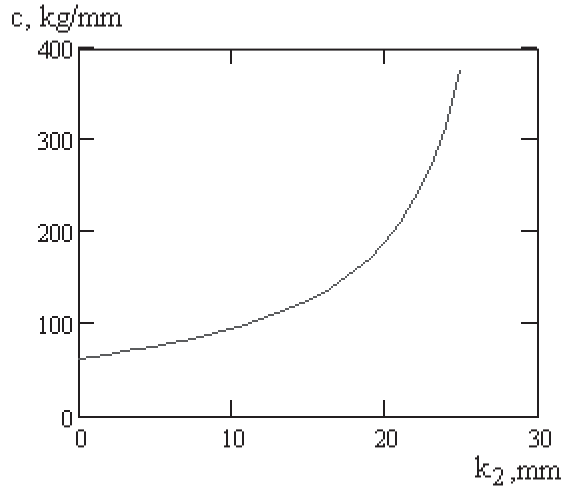

Fig. 4. The stiffness of the shock absorber according to k_2

6. Conclusions

The model of rubber shock absorber offered in-process with adjustable stiffness can have a large range of stiffness regulation. Stiffness of shock absorber can change, from ordinary declivous rubber shock absorber's stiffness (depends on geometrical sizes and brand of rubber) to absolutely hard support. The chart of possible stiffness is in-process got for the shock absorber of concrete sizes. From a chart evidently, that stiffness of such shock absorber can change from a 70 kg/mm ($k_2 = 0$) to absolutely hard support ($k_2 = h$). A construction of shock absorber is not difficult and easily realized. The proposed calculate method allows in case of partition of the investigated area V on subarea V_n, using functional (15) and discontinuous functions sought to obtain integrated dependences of type "force – settlement".

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