Abstract. The research focus of the scientific paper is on the problem of performance measurement of credit institutions. The author recommends applying frontier analysis techniques such as Data Envelopment Analysis and Stochastic Frontier Approach to the efficiency analysis of Decision Making Units. Using modern computer technologies, the author has calculated dynamics of efficiency score of ten Latvian banks on the basis of DEA CCR approach, provided recommendations concerning optimal input volumes and established hidden development reserves using SFA method.

Key words: DEA (Data Envelopment Analysis), Decision Making Units (DMUs), efficiency measurement, SFA (Stochastic Frontier Approach)

Mathematics Subject Classification: 90B30, 90B50, 90C08, 90C15, 90C30, 90C90.

Introduction

The remaining uncertainty in development of Latvian national economy and the absence of significant improvement of economical situation impacts activities of all economical subjects. Due to this fact, credit institutions that are performing redistribution of income inside of the country are especially vulnerable. In June 2010 21 banks and seven branches of foreign banks were functioning in Latvia that is indicative of increasing level of competition in the banking sector. Due to the absence of improvement in quality of credit portfolio, the volumes of reserves for non-performing loans in the first current half-year have reached 1,671 million LVL. Operating profits of Latvian banks continue decreasing, mainly because of credit impairments that made 49.3% of total banking losses (430.3 million LVL) in the end of June 2010. These negative macroeconomical trends that have impact on activities of credit institutions are indicative of necessity of strong control over banking performance. Actually the estimation of the level of operating efficiency in the most Latvian banks is realized on the basis of quantitative approach of ratio analysis. Ratios measure the relationship between two variables chosen to provide insights into different aspects of the banks multifaceted operations, such as liquidity, profitability, capital adequacy, asset quality, risk management, and many others.
Although the traditional ratio measures are attractive to analysts due to their simplicity, there are several limitations that must be considered. For example, the analysis assumes comparable units, which implies constant returns to scale (Smith 1990). Each of the indicators yields a one-dimensional measure by examining only a part of the organization's activities, or combining the multiple dimensions into a single, unsatisfactory number. Moreover, the seemingly unlimited number of ratios that can be created from financial statement data are often contradictory, thus ineffective for the assessment of overall performance. This overly simplistic analytical approach offers no objective means of identifying inefficient units and requires a biased separation of the inefficient and efficient levels. [1, 350-351]

Methods of frontier analysis ensure a principally different approach to the problem of efficiency measurement. They provide an opportunity of complex analysis of banking efficiency level for a certain period of time and comparison of it among investigated banks. This multidimensional approach meets the requirements to the banking performance evaluation methodology. The objective of the author’s research is to improve and supplement the methodology of efficiency measurement of Latvian banks on the basis of methods of frontier analysis. In the circumstances of unstable macroeconomical environment and competition, profitability is one of the most important indicators of stability and development of credit institutions. In this connection, the author analyzed the performance of a set of Latvian banks, assuming operating profits as an output. The objects of the research are members of Latvian banking sector; their efficiency level is analyzed over the time period from 2003 to 2009. Evaluating the performance on the basis of frontier approaches, the author included into the set of investigated objects banks that take leading positions on Latvian market (according to the volumes of total assets): JSC “Swedbank”, JSC “DnB Nord Banka”, JSC “Aizkraukles Banka”, JSC “Parex banka” (currently JSC “Citadeles Banka”), JSC “SEB Banka”, JSC “Latvijas Krājbanka”, JSC “Mortgage Bank”, JSC “Rietumu Banka”, JSC “Norvik Banka”, JSC “GE Money Bank” with the exception of branches of foreign banks.

1 Methods of Frontier Data Analysis

The progress of production technology and increase of production volumes have stimulated the development of performance measurement methodology. In the second part of the 20th century there were introduced methods of frontier data analysis that provided a qualitatively different approach to the problem. According to the methodology of methods of frontier data analysis, the efficiency score of investigated DMUs is calculated as a distance from the point that defines the production process of a Decision Making Unit (DMU) to the certain efficiency frontier. Entities that are functioning on the efficiency frontier are considered to be absolutely technically efficient; inefficiency of other DMUs is increasing together with extension of the distance to the efficiency frontier.

Methods of frontier analysis may be divided into two groups: parametric (Stochastic Frontier Approach (SFA), Distribution-Free Approach (DFA), Thick Frontier Approach (TFA)) and non-parametric (Data Envelopment Analysis (DEA), Free Disposal Hull (FDH)) methods. In accordance with parametric approaches, the efficiency frontier is constructed on the basis of econometric modeling, usually in form of Cobb-Douglas (log-linear) production function. Econometric analyses include two error components: an error term that captures inefficiency \((u_i)\) and a random error \((v_i)\). Parametric methods have significant advantages – they provide the possibilities to use panel data, to distinguish the random noise from inefficiency and to calculate the standard error of efficiency measurement results. Nevertheless, the stochastic approaches of performance measurement presume
the comparison of investigated DMUs efficiency to the theoretically developed benchmark frontier; therefore the optimal combinations of inputs and outputs sometimes are not achievable practically. The application of parametric methods also requires observance of the restrictions imposed on the distributional assumptions on the inefficiencies and random error. In contrast to the econometric approaches, non-parametric methods are based on the hypothesis that the efficiency frontier is generated from the empirical results of the most efficient DMUs i.e. benchmarks that “float” on the piecewise linear frontier. The level of technical efficiency of these DMUs is 100%. While mathematical, non-parametric methods require few assumptions when specifying the best-practice frontier, they generally do not account for random errors. [8, 93]

1.1 Data Envelopment Analysis (CCR DEA Model)

The CCR DEA model was developed by Charnes, Cooper and Rhodes in 1978 to evaluate the performance of Decision Making Units (DMUs). To allow for applications to a wide variety of activities, the term DMU might be used to refer to any entity that is to be evaluated in terms of its abilities to convert inputs into outputs. These evaluations can involve governmental agencies and non-profit organizations as well as business firms, hospitals and educational institutions. The production process might be aimed either at minimization of resources or maximization of production volumes. The orientation of the model should be aimed at controllable variables. In context of banking, volumes of resources are usually over control of management; therefore only input-oriented model will be examined in the paper.

The measurement of comparative efficiency is based on the assumption that the performance of each DMU is calculated in comparison to \( n \) investigated DMUs. Each DMU consumes varying amounts of \( m \) different inputs to produce \( s \) different outputs. Specifically, \( DMU_j \) consumes amount \( x_{ij} \) of input \( i \) and produces amount \( y_{rj} \) of output \( r \). It is necessary to assume that \( x_{ij} \geq 0 \) and \( y_{rj} \geq 0 \) and further to assume that each DMU has at least one positive input and one positive output value. Primarily the DEA model was expressed in fractional, i.e. ratio-form. In this form the ratio of outputs to inputs is used to measure the relative efficiency of the DMU \( j = DMU_0 \) to be evaluated relative to the ratios of all of the \( j = 1, 2, ..., n \) DMU. The CCR construction can be interpreted as the reduction of the multiple-output/multiple-input situation (for each DMU) to that of a single 'virtual' output and 'virtual' input. For a particular DMU the ratio of this single virtual output to single virtual input provides a measure of efficiency that is a function of the multipliers. In mathematical programming parlance, this ratio, which is to be maximized, forms the objective function for the particular DMU being evaluated. A set of normalizing constraints (one for each DMU) reflects the condition that the virtual output to virtual input ratio of every DMU, including \( DMU_j = DMU_0 \), must be less than or equal to unity. The mathematical programming problem may thus be stated as (1):

\[
\begin{align*}
\max & \quad h_0(u, v) = \sum_r u_r y_{ro} / \sum_i v_i x_{io} \\
\text{subject to} & \quad \sum_r u_r y_{rj} / \sum_i v_i x_{ij} \leq 1 \text{ for } j = 1, ..., n, \\
& \quad u_r, v_i \geq 0 \text{ for all } i \text{ and } r, \\
\end{align*}
\]

where

- \( h_0 \) – the function of virtual output and virtual input ratio of DMU\(_0\);
- \( u_r \) – the output multiplier of DMU\(_0\);
- \( v_i \) – the input multiplier of DMU\(_0\);
The above ratio form yields an infinite number of solutions; if \((u^*, v^*)\) is optimal, then \((\alpha u^*, \alpha v^*)\) is also optimal for \(\alpha > 0\). However, the transformation developed by Charnes and Cooper (1962) for linear fractional programming selects a representative solution \((u, v)\) for which \(\sum_i v_i x_{io} = 1\) and yields the equivalent linear programming problem in which the change of variables from \((u, v)\) to \((\mu, \nu)\) is a result of the Charnes-Cooper transformation (2):

\[
\max z = \sum_{i=1}^{m} \mu_i y_{io} \\
\text{subject to} \quad \sum_{i=1}^{m} \mu_i y_{ij} - \sum_{j=1}^{n} \nu_j x_{ij} \leq 0 \quad \sum_{j=1}^{n} \nu_j x_{io} = 1 \quad \mu_i, \nu_j \geq 0,
\]

\(\mu_i\) – the output multiplier of DMU0;  
\(\nu_i\) – the input multiplier of DMU0;  
\(y_{io}\) – the output of DMU0;  
\(x_{io}\) – the input of DMU0;  
\(y_{ij}\) – outputs of 1,2…n DMUs; 
\(x_{ij}\) – inputs of 1,2…n DMUs.

Model that is expressed by (2) can be solved by its dual problem (3):

\[
\theta^* = \min \theta \\
\text{subject to} \quad \sum_{j=1}^{n} x_{ij} \lambda_j \leq \theta x_{io} \quad i = 1,2,..., m; \\
\sum_{j=1}^{n} y_{ij} \lambda_j \geq y_{ro} \quad r = 1,2,..., s; \\
\lambda_j \geq 0 \quad j = 1,2,..., n,
\]

where  
\(\theta^*\) – the optimal value of dual variable \(\theta\) of DMU0;  
\(\theta, \lambda_j\) – dual variables of DMU0;  
\(y_{ro}\) – the output of DMU0;  
\(x_{io}\) – the input of DMU0;  
\(y_{ij}\) – outputs of 1,2…n DMUs; 
\(x_{ij}\) – inputs of 1,2…n DMUs.
By virtue of the dual theorem of linear programming we have $z^* = \theta$. Hence either problem may be used. One can solve the dual linear program, to obtain an efficiency score. Setting $\theta = 1$ and $\lambda_k^* = 1$ with $\lambda_k = \lambda_k^*$ and all other $\lambda_k^* = 0$, a solution of dual problem (see Formula 3) always exists. Moreover this solution implies $\theta^* \leq 1$. The optimal solution, $\theta^*$, yields an efficiency score for a particular DMU.

The process is repeated for each DMU. i.e., solving the model, expressed by Formula 3, with $(X_o, Y_o) = (X_k, Y_k)$, where $(X_o, Y_o)$ represent vectors with components $x_{ik}, y_{rk}$ and, similarly $(X_k, Y_k)$ has components $x_{ok}, y_{rk}$. DMUs for which $\theta^* < 1$ are inefficient, while DMUs for which $\theta^* = 1$ are boundary points. Some boundary points may be "weakly efficient" because we have non-zero slacks. This may appear because alternate optima may have non-zero slacks in some solutions, but not in others. However, we can avoid this effect by invoking the following linear program in which the slacks are taken to their maximal values (4).

$$\max \sum_{i=1}^{m} s_{i}^- + \sum_{r=1}^{s} s_{r}^+$$

subject to

$$\sum_{j=1}^{n} x_{ij} \lambda_{j} + s_{i}^+ = \theta^* x_{io} \quad i = 1, 2, ..., m;$$

$$\sum_{j=1}^{n} y_{rj} \lambda_{j} - s_{r}^- = y_{ro} \quad r = 1, 2, ..., s;$$

$\lambda_j, s_i^-, s_r^+ \geq 0 \forall i, j, r,$

$s_r^+$ = output slacks;

$\theta^*$ – the optimal value of dual variable $\theta$ of DMU$_0$;

$\lambda_j$ – the dual variable of DMU$_0$;

$y_{r0}$ – the output of DMU$_0$;

$x_{io}$ – the input of DMU$_0$;

$y_{rj}$ – outputs of 1, 2…n DMUs;

$x_{ij}$ – inputs of 1, 2…n DMUs.

(4)

It shall be noted that the choices of $s_i^-$ and $s_r^+$ do not affect the optimal $\theta^*$ which is determined from model expressed by (3). These developments lead to the following definitions of DEA efficiency:

**DEA Efficiency:** The performance of DMU$_0$ is fully (100%) efficient if and only if both (i) $\theta^* = 1$ and (ii) all slacks $s_i^-, s_r^+ = 0$.

**Weakly DEA Efficiency:** The performance of DMU$_0$ is weakly efficient if and only if both (i) $\theta^* = 1$ and (ii) $s_i^- \neq 0$ and/or $s_r^+ \neq 0$ for some $i$ and $r$ in some alternate optima. [2, 8-12]

The CCR efficiency score is indicative of the overall efficiency level of investigated DMUs.

2 The Stochastic Frontier Approach

The Stochastic Frontier Approach (SFA) is the econometric method of efficiency estimation. It presumes a certain functional form for the description of the process of production. The performance of investigated banks is estimated on the basis of stochastic Cobb-Douglas production function. The SFA method allows estimating the level of technical efficiency. Assuming that the production function is depending on several factors $x_1, ..., x_n$, its functional form is $y = F(x_1, ..., x_n) \times \text{Exp}(-u)$, and the functioning bank using a similar volume of resources can produce at least the same volume of production if: $y = F(x_1, ..., x_n) \times \text{Exp}(-u) \leq F(x_1, ..., x_n)$ where $u > 0$. \text{Exp}(-u) expresses the level of
technical inefficiency of investigated objects. Due to the competition in the banking sector, the author recommends to use the stochastic model with time-varying technical efficiency for panel data (5):

\[
\ln y_{it} = \beta_{0i} + \sum_{n} \beta_{n} \ln x_{nit} + v_{it} - u_{it},
\]

(5)

where
- \( y_{it} \) – panel data of production (output) volumes;
- \( \beta_{0i} \) – frontier intercept (constant);
- \( \beta_{n} \) – vector of technological parameters;
- \( x_{nit} \) – panel data of resources (input) volumes;
- \( v_{it} \) – random error term for panel data;
- \( u_{it} \) – inefficiency error term for panel data.

The Stochastic Frontier Approach model includes two error components: an error term that captures inefficiency (\( u_{it} \)) and a random error (\( v_{it} \)). It is impossible to calculate the precise value of inefficiency, because of its composite structure. Due to this, the result of efficiency measurement is the conditional expectation of its value (6):

\[
\hat{u}_{it} = E(u_{it} \mid v_{it} - u_{it} = \hat{e}_{it})
\]

(6)

where
- \( \hat{u}_{it} \) – modeled inefficiency error term for panel data;
- \( u_{it} \) – inefficiency error term for panel data;
- \( v_{it} \) – random error term for panel data;
- \( \hat{e}_{it} \) – modeled random error term for panel data without inefficiency error term.

3 The Application of Multistage Approach to the Efficiency Measurement of Latvian Banks

3.1 Methodology of the Research

It is possible to view stochastic frontier regressions as competing with DEA. Carried to an extreme the two approaches, DEA vs. Stochastic Frontier Regressions, can be regarded as mutually exclusive – as in Schmidt (1985). An alternative view is also possible in which the two approaches can be used in complementary fashion. Ferreira and Lovell (1990), for example, use two approaches to cross-check each other. In this approach, the objective is to avoid what Charnes, Cooper and Sueyoshi (1988) refer as „methodological bias“. Indeed, going a step further, it is possible to join the two approaches in the multistage methodology of efficiency evaluation. [6, 292-293]

The problem of keeping profitability is especially actual and important in the circumstances of unstable macroeconomical environment. In this connection, there is developed a concept of efficiency measurement of Latvian banks in the research, assuming operational profit to be an output while interest expense, personnel costs and credit impairments are defined as inputs. The first stage of the performance evaluation will be completed on the basis of DEA CCR approach that allows calculating overall efficiency score and optimal volumes of inputs. The second stage of the research is realized using SFA method that provides a possibility of cross-checking and identification of hidden reserves of development.
3.2 Efficiency Measurement Results of Latvian Banks on the Basis of CCR DEA Approach

The results of banking performance evaluation on the basis of CCR input-oriented model, assuming operating profit as an output, are represented in Figure 1.

![Fig. 1. Dynamics of banking CCR efficiency score, (%)](image)

The application of DEA approach requires the determination of assumptions, concerning orientation measures of the model and the concept of returns to scale (RTS). The banking production process may be aimed either at minimization of resources (input-oriented) or maximization of production volumes (output-oriented). It is emphasized in the international researches that the orientation of the model should be aimed at controllable variables. Usually volumes of resources are considered to be over control of banking management, therefore there is applied the assumption of input orientation in the research. Since the constant returns to scale CRS approach represents the total (overall) efficiency level, CCR DEA model is considered to be the basic concept of the research.

The results of the efficiency measurement approve that the average level of performance of investigated banks has diminished to 76.54% in 2008. The highest overall efficiency (84.58%) was observed in 2006 that is concerned with the increase of crediting activity in the banking sector. The redistribution of leaders’ positions among investigated objects is among important trends that are characterizing the dynamics of the efficiency score.

JSC “Rietumu Banka” has demonstrated the best result, operating on the efficiency frontier during all periods of the observation. The long-term stability of the efficiency level of JSC “Rietumu Banka” is indicative of its ability to maximize the volume of output using minimal volumes of inputs and to ensure optimal proportions of output and inputs in the process of production, thus of both 100% technical and scale efficiency in comparison to the set of investigated banks. The maximal overall efficiency of “Rietumu Banka” is confirmed by its successful strategy and consistent management activities. The target customer group of JSC “Rietumu Banka” consists of legal entities and private customers with high level of income, mainly nonresidents. Other financial indicators justify high efficiency level of JSC “Rietumu Banka”: its profit for the nine months of 2010 is 4 million euro, the capital adequacy ratio is 18.17% and liquidity ratio – 53.85%.

To improve the performance of inefficient banks, it is important to determine, which proportions of resources will maximize the overall efficiency level (see Table 1).

Table 1
JSC “Swedbank” and JSC “Latvijas Krājbanka” have improved their performance during the investigated period significantly. In 2008 both credit institutions were functioning on the efficiency frontier, having 100% CCR efficiency score. Despite of growing interest expenses and impairment losses JSC “Swedbank” and JSC “Latvijas Krājbanka” have succeeded in keeping and increasing volumes of operating revenues. JSC “Aizkraukles Banka” and JSC “Norvik Banka” were fully efficient in 2003 and 2005, but both credit institutions have lost their leading positions by the end of the investigation period.

The DEA approach provides a possibility to calculate the volumes of virtual optimal inputs. According to the data in Table I, the CCR projection requires significant reduction in inputs, especially for JSC “DnB Nord Bank”, JSC “Latvijas Krājbanka”, JSC “Mortgage Bank” and JSC “Parex Banka”. It is necessary to emphasize that interest expense and impairment losses have a stronger impact on the overall efficiency level than personnel costs. The average efficiency in the year 2008 has remained on the same level as in 2003. Nevertheless, the improvement of profitability of two banks could be achieved only after dramatic decrease of input volumes: JSC “Parex Banka” should cut its interest expense and impairment losses by 61,34% and 68,51% respectively in order to operate on the efficiency frontier (see Table 2). However, the optimization of JSC “GE Money Bank” input volumes is concerned with diminishing of its personnel costs by 71,72% and credit impairment costs by 61,96%.

### Table 2

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<tr>
<th>CCR virtual input volumes in 2003 (thsd. LVL)</th>
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<td>------------------------------------------------</td>
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<tr>
<td>Latvijas Krājbanka</td>
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<td>Swedbank</td>
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### 3.3 Efficiency Measurement of Latvian banks on the Basis of Stochastic Frontier Approach

The first stage of the performance analysis provides information concerning both DEA overall efficiency scores and optimal input volumes. In accordance with the approach of Data Envelopment Analysis, the efficiency frontier is generated from the empirical results of the most efficient Decision Making Units i.e. benchmarks that “float” on the piecewise linear frontier; therefore some of investigated objects are 100% efficient. Nevertheless, it is important to identify hidden efficiency reserves even for fully CCR-efficient DMUs. It is achievable on the basis of Stochastic Frontier Approach technique. Due to this method, the efficiency frontier is constructed using principles of econometrical modeling that allows estimating the performance in comparison to the theoretically developed efficiency frontier.

The application of SFA method requires specifying the functional form of the efficiency estimation model. The author has used the log-linear model specification, as in (5). In accordance with the basic concept of the research, operational profits are presumed to be an output while interest expense, personnel costs and credit impairments are defined as inputs. Due to the composite structure of the model error, there are applied Ordinary Least Squares (OLS) and Maximum Likelihood Estimation (MLE) methods in the efficiency analysis. \( v_i \) and \( u_i \) are assumed to be normal and half-normal distributed, respectively. The calculations are accomplished using the FRONTIER 4.1. program (see Table 3).

#### Table 3

Coeficients of the efficiency measurement model on the basis of Stochastic Frontier Approach

<table>
<thead>
<tr>
<th>Parameters of the model</th>
<th>Coefficient</th>
<th>Standard errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beta 0 (constant)</td>
<td>0.28</td>
<td>0.70</td>
</tr>
<tr>
<td>Beta 1 (personnel costs)</td>
<td>-0.77</td>
<td>0.12</td>
</tr>
<tr>
<td>Beta 2 (interest expense)</td>
<td>0.88</td>
<td>0.69</td>
</tr>
<tr>
<td>Beta 3 (impairment losses)</td>
<td>0.72</td>
<td>0.13</td>
</tr>
</tbody>
</table>

On the one hand, it is possible that on the basis of MLE algorithm due to distributional assumptions concerning composite error terms there is calculated a local extremum of the log-likelihood function. On the other hand, the results of efficiency measurement correspond to the economical intuition: after optimization of input volumes 100% CCR-efficient banks proved to be 83-86% efficient on the basis of SFA method, demonstrating minimal volatility of the efficiency score during the period of the research (see Figure 2).
Conclusions

The scientific paper is devoted to the performance measurement problem of Latvian credit institutions. The standard methods of efficiency measurement, such as quantitative ratio analysis, ratings and regression analysis do not provide the possibility of multidimensional efficiency evaluation. The methodology of frontier methods is considered to be a sophisticated tool for efficiency measurement that allows the investigation of complex production processes among a set of Decision Making Units (DMUs). According to the information that is available to the author, Latvian banks to the efficiency measurement currently do not apply methods of frontier data analysis.

The author has implemented the multistage approach, analyzing efficiency scores of a set of Latvian banks using CCR DEA and SFA techniques. JSC “Rietumu Banka” has demonstrated the best result, operating on the efficiency frontier during all periods of the observation. There were calculated volumes of weighted optimal inputs that provide the possibility to maximize the overall efficiency score, using DEA technique. Using the Stochastic Frontier Approach, it was stated that even after optimization of input volumes there is still a possibility to improve the banking performance by 17% in the year 2009.

According to the obtained results of the research, the author recommends to improve the methodology of efficiency measurement of Latvian banks on the basis of multistage application of frontier analysis methods.

References


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