

# NEW DECISION MAKER MODEL FOR MULTIOBJECTIVE OPTIMIZATION INTERACTIVE METHODS

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**Abstract.** Decision Maker Model instead of human Decision Maker can be used for testing/comparing multiobjective optimization interactive methods. New Decision Maker Model (called ZuMo) was defined as two criteria multiobjective optimization problem which is solved by using multiobjective evolutionary algorithm NSGA-II. Model can significantly reduce time and workload of testing/comparing experiment design and implementation and easy can be used for ad-hoc methods. Evolutionary algorithms are effective in parallel computing that can reduce experiment time also. The designed model was tested comparing STEM and GUESS interactive methods solving three 2D criteria and three 3D criteria testing problems. Different metrics are used: iteration count, stopping reason, general distance, error ratio, spacing and maximal Pareto error. The GUESS method was more effective in obtaining goal solution than STEM method. The new ZuMo model is universal and can be updated for testing/comparing different multiobjective optimization interactive methods and ad-hoc methods also. Model framework if necessary can be extended by defining additional criteria.

**Keywords:** Decision Maker model, multiobjective optimization, comparative study, interactive method.

## 1 Introduction

Multiobjective optimization (MO) solves problems with two or more conflicting criteria functions. The MO methods are classified: a priori, a posteriori and interactive. Solving optimization problem by the interactive multiobjective optimization method Decision Maker (DM) provides preference information for optimization method before iteration. If the DM is satisfied with the obtained solution optimization stops, otherwise the DM sets the new preference information and method solves problem once again. There are many studies where the interactive optimization methods are compared with group of DM, usually more than 65 peoples. Experiment design and implementing is time consuming, especially for testing/comparing ad-hoc optimization methods. Other reason is DM ability to understand problem nature if it solved more times. Decision Maker Model (DMM) can prevent such problems. The MO problem (MOP) definition is [1]:

$$\begin{aligned} &\text{minimize } \{f_1(x), f_2(x), \dots, f_k(x)\} \\ &g_m(x) \leq 0; h_l(x) = 0; x \in S \end{aligned} \quad (1)$$

where objective functions  $f_i: \mathbb{R}^n \rightarrow \mathbb{R}$ . We denote the objective vector  $f(x) = \{f_1(x), f_2(x), \dots, f_k(x)\}$ . The variable vectors  $x = \{x_1, x_2, \dots, x_n\}$  belongs to the feasible region (set)  $S$  formed by inequality and equality constraints  $g_m(x)$  and  $h_l(x)$ .

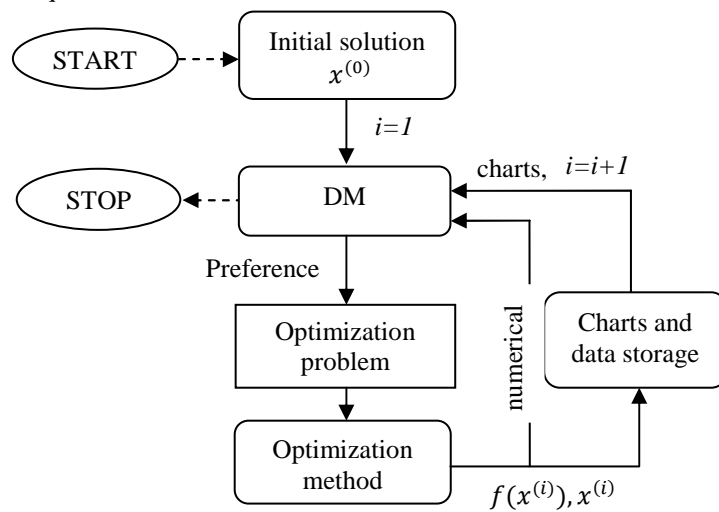
Interactive optimization method (IOM) is a part of interactive optimization procedure (Figure 1.). Optimization starts with initial solution  $x^{(0)}$  and DM at each iteration defines preference information.

Further optimization problem can be updated accordingly to the preference information and method solves (1) optimization problem. If the DM is satisfied with solution, then optimization stops. Otherwise, DM sets  $i = i + 1$  and provides the new preference information to continue optimization process. It should be noted that DM define the new preference information by using values of the previously obtained solutions or graphical information (diagrams). Structure and type of preference information are depended on optimization method. GUESS method has reference point as preference information, while STEM uses grouping of criteria in two classes – improvable and relaxed.

The base of reliable comparison of IOM is large amount of IOM test results. IOM testing with human DM has two main difficulties: first, necessity of large amount of DM and high workload of experiment design and implementation. 98 DM for CONTEX method testing [2]; 36 DM for GDF, STEM and Trial-And-Error methods comparison[3]; 24 DM for SIMOLP, GUESS and Tchebycheff methods comparison in [4]; 84 DM for ZW, GUESS and SMART methods comparison [5]. Second, after solving a problem several times DM acquires information about MOP properties that reduces reliability of experimental data [6].

As alternative to human DM some works suggest usage of linear and nonlinear value function instead of DM. In [7] work was used nonlinear value function for GDF, STEM and SWT methods comparison; in [8] work was used linear and nonlinear value function for Tchebysheff and SIMOLP method comparison; in paper [9] is used value function together with human DM. Unfortunately a value function no universal for different IOM and

rise many problems in data comparison. Value function has strong decision making behavior then human DM can be sometimes inconsequent.



**Figure 1. Interactive optimization procedure**

The GUESS and STEM methods was used because they has different preference information types. STEM (STEP method) firstly introduced in [2] and GUESS firstly introduced in [10]. Using STEM method DM specify nadir and ideal criteria vectors and divides objectives into two classes  $I^<$  – objectives functions to improve and  $I^>$  – objectives function to relax with upper bounds for each function. Using GUESS method DM specify nadir and ideal criteria vector, upper and lower bound for all criteria and reference point with values between values of nadir and ideal criteria vector.

Depending on MO method testing type – human and mathematical metric differs. In experiments with human DM use: 1) DM satisfaction of last obtained solution; 2) easy of 'handling'; 3) simple to understanding; 4) support of decision making; 5) iteration count; 6) spent time; 7) preference information impact of obtained solutions[11]. In experiments with DM model (DMM) use: 1) iteration count; 2) spent time; 3) Euclidian distance to the goal solution; 4) general distance to the theoretic Pareto front; 5) spacing [12]. Mathematical approach reveals mathematical characteristics of methods, but not gives information about method usefulness or easy to understanding for DM.

The aim of the study is to create a DMM that is universal for different IOM and gives possibilities of understanding IOM methods from mathematical and DM sides. The model is validated by GUESS and STEM methods solving typical optimization problems and six different metrics are used for comparison. The DMM was defined as multiobjective optimization problem, which was solved by a variant of NSGA–II algorithm reported in [13].

## 2 Decision Maker Model

As mentioned before, human DM obtains experience about optimization problem, if it will be solved a lot of times. This fact negatively impact analyses of experimental data. Additionally, in experiments a large amount of the DM, usually more than 65 people, are necessary.

IOM comparison with DMM gives many advantages:

1. not necessary to use human DM;
2. time of experiment design and implementation can be decreased;
3. possible to compare ad–hoc optimization methods;
4. different mathematical metrics can be used;
5. can be easy to compare new optimization methods;
6. can be easy to implement experiments.

DMM (called ZuMo) was defined as two criteria MOP which will be solved by using one of multiobjective evolutionary algorithms. First criterion evaluates Euclidean distance from the current solution  $f(x^{(i)})$  to the goal solution  $z^* = f(x^{(*)})$ . This criterion force DMM to obtain explicitly defined goal  $z^*$ . The second criterion evaluates the preference information correctness accordingly to the optimization method requirements.

ZuMo definition is:

$$\min\{p_1(x^{(i)}, z^*, VRI_T^{(i)}), p_2(VRI_T^{(i)})\} \quad (2)$$

$$x^{(i)}: R^n \in S; z^*: R^k \in D; VRI_T^{(i)} \in S_{VRI}$$

where  $x^{(i)}$  – solution of testing problem (TP) for  $i$  iteration,  $z^*$  – goal solution of TP,  $VRI_T^{(i)}$  – preference information for  $i$  iteration and problem (2) variables;  $S$  – feasible region (set) of TP variables,  $D$  – feasible region (set) of TP criteria functions,  $T$  – testing problem,  $S_{VRI}$  – feasible region (set) of preference information for TP,  $p_1$  – Euclidean distance to the goal solution  $z^*$ ,  $p_2$  – correctness of  $VRI_T^{(i)}$ .

First criterion can be defined:

$$p_1(x^{(i)}, z^*, VRI_T^{(i)}) = \sqrt{\sum_{j=1}^k (z_j^* - u_j)^2}, \quad (3)$$

$$x^{(i)}: R^n \in S; u, z^*: R^k \in D$$

where  $x^{(i)}$  – solution of testing problem (TP) for  $i$  iteration,  $z^*$  – goal solution of TP,  $VRI_T^{(i)}$  – preference information for  $i$  iteration and variables of (2) problem,  $k$  – number of TP functions,  $n$  – number of TP variables,  $u_j$  – value of  $j$  criterion of TP. The values of  $u_j$  can be obtained by solving optimization problem:

$$u = F_T(f(x^{(i)}), VRI_T^{(i)}), \quad (4)$$

$$x^{(i)}: R^n \in S, f(x^{(i)}): R^k \in D,$$

$$VRI_T^{(i)} \in S_{VRI}; g(x^{(i)}) \leq 0; h(x^{(i)}) = 0$$

where  $f(x^{(i)})$  – TP criteria functions,  $VRI_T^{(i)}$  – preference information for  $i$  iteration,  $F_T$  – transformation function which updates optimization problem (1) accordingly to the IOM problem definition type.

Figure 2 present architecture of ZuMo. As mentioned before variables  $x^{(i)}, z^*, \varepsilon, i_{max}$  are input parameters for this DMM. DN – stopping criteria decion node.

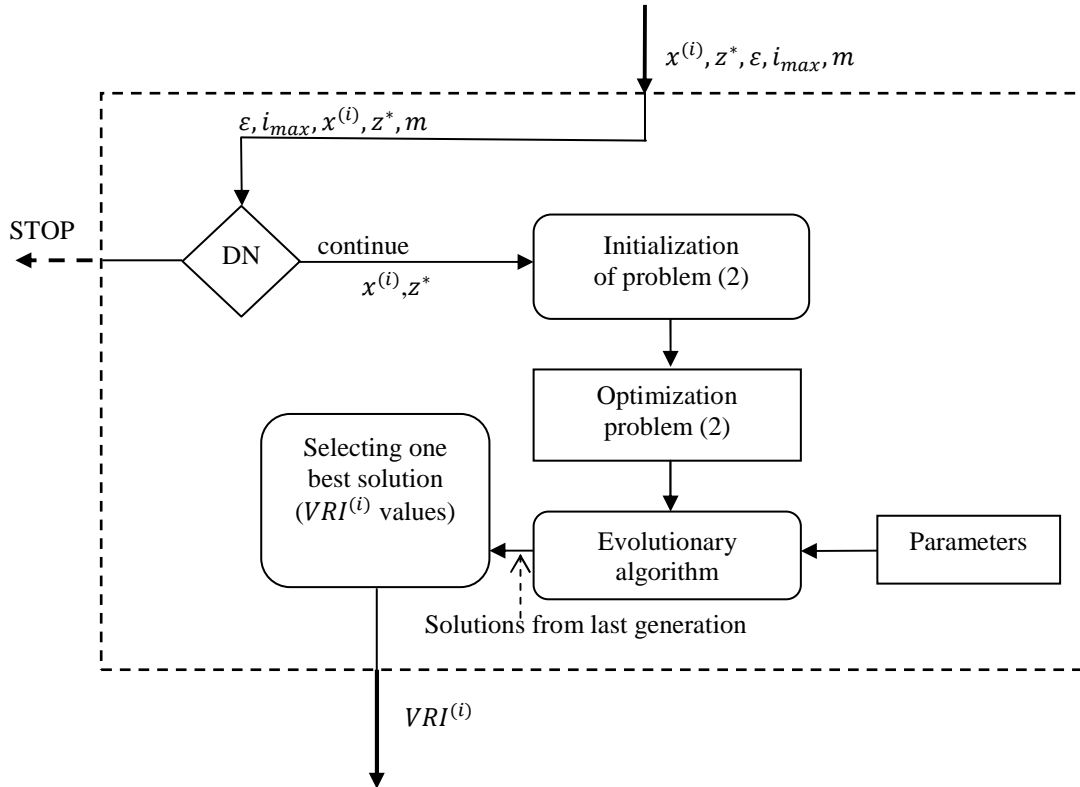


Figure 2. The Decision Maker model (called ZuMo) architecture

Problem (2) can be solved by using multiobjective evolutionary algorithm and  $VRI$  values are (2) problem variables, but  $x^{(i)}, z^*$  – constants.

After solving problem (2) we need to choose best solution from last generation:

7. find in last generation solution with minimal value of  $p_2$  denote it by  $R_{min}$ ;
8. create solution set L1 with solutions where  $p_2=R_{min}$ ;
9. choose solution from L1 with minimal  $p_1$  value.

Second criterion of problem (2) return 0, if VRInformation is correct. Otherwise return value, according to penalty range for IOM.

The DMM before solving (2) problem decides to stop optimization or continue. Stopping criteria are:

1. The goal solution  $z^*$  was obtained with precision less or equal to  $\varepsilon$  tolerance;
2. Euclidean distance for last  $m$  iterations increase;
3. Solutions fluctuate around the same solution with small deviations less or equal to  $\varepsilon$  tolerance for last  $m$  iterations;
4. Solutions obtained from last  $m$  iterations converge to the goal solution and maximal Euclidean distance from first and last of them less or equal to  $\varepsilon$  tolerance.
5. Exceeded maximal iteration count  $i_{max}$ .

Initialization node prepares TP accordingly to the IOM requirements and return definition of the optimization problem (2) which will be solved by using evolutionary MO algorithm with predefined parameters (for example, generation count, population size, selection operator etc.). Finally from last generation of evolutionary algorithm select one best solution as new preference information  $VRI^{(i)}$ .

### 3 Experiment methodology

#### 3.1 Experiment conditions

Optimization problem (2) was solved by using MATLAB (7.7.0.471) Genetic Algorithm and Direct Search Toolbox function *gamultiobj()* (a variant of NSGA-II algorithm [13]) with predefined parameters: 1) maximal generation count 100; 2) population size 40; 3) stall generations 20 (algorithm stops if there no improvement in the best fitness value after 20 generations); 4) selection operator –Tournament; 5) crossover operator– Intermediate; 6) mutation operator –Adaptive Feasible [14].

The DMM solves each TP with each IOM (STEM and GUESS) 10 times, maximal iteration count  $i_{max} = 15$  of solving TP, stopping criteria include all cases mentioned before, where  $m = 4$  iterations ( $m$  – last iteration count for stopping criteria checking),  $\varepsilon$  – tolerance defined individually for each TP. Problem (4) definition depends on TP and IOM. For each iteration the following metrics was evaluated: 1) *Spacing*(SP) – determine how obtained solutions (in current and previous iterations) are distributed by self in Pareto front – *PF* [15]; 2) *Error Ratio* (ER) – determine how obtained solutions belongs to theoretical Pareto front – *PF\** [15]; 3) *Generational Distance* (GD) – determine how far obtained solutions are located from the *PF\** [15]; 4) *Standard Deviation from the General Distance* (SDGD) – determine how obtained solutions are distributed from the *General Distance* [15]; 5) *Maximal Pareto Front Error* (MPE) – determine maximal Euclidean distance from obtained solutions to the *PF\** [15]; 6) Euclidean distance (ED) from current solution to the goal solution  $z^*$ ; 7) Iteration count (IC). Variables, objectives and preference information values were recorded into experiment data base. After solving each problem iteration count and stopping reason was recorded also.

Preference information was coded for NSGA-II algorithm as a set of variables which was transformed into structure of preference information accordingly of IOM requirements after selecting best solution (see Figure 2). For each TP the goal solution  $z^*$  was determined, initial solution  $x^{(0)}$  in variable space that belongs to Pareto set,  $f(x^{(0)})$  in Pareto front and empirically obtained  $\varepsilon$  tolerance.

Criterion  $p_2$  value for the STEM method was evaluated by checking these cases: 1) add 1 if TP criteria index presents in  $I^<$  (objectives to improve) or more times (duplicates) in  $I^>$  (objectives to relax); 2) add 2 if TP criteria index presents in  $I^<$  and  $I^>$  at the same time; 3) add 3 if TP criteria index not presents in  $I^<$  and  $I^>$ ; 4) add 4 if  $I^< = \emptyset$ . For the GUESS method: 1) add 2 if criteria upper bounds are less or equal to lower bounds; 2) add 1 if criteria upper bounds are greater than nadir vector values; 3) add 1 if criteria lower bounds are less than ideal vector values; 4) add 2 if reference point values are not between nadir and ideal vectors values.

#### 3.2 Testing problems

Methods GUESS and STEM was tested with six testing problems: 3 2D and 3 3D, where Pareto set connected and Pareto front convex or concave. In all selected TP theoretic Pareto set is continuous. Problems was divided into four classes: A –*PF\** continuous and concave, B –*PF\** continuous and convex, C –*PF\** nonsymmetric and continuous, D –*PF\** symmetric and concave. Problem names noted TP1–BINH1 [16], TP2 –

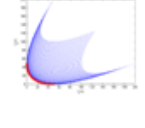
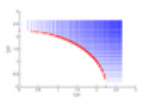
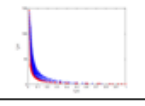
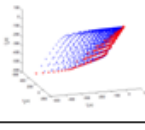
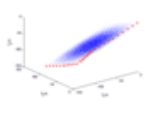
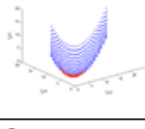
HANNE [17], TP3 – RENDON[15], TP4 – TAPPETA[18], TP5 – DOWNING[19], TP6 – VINNET[20]. See Table 1.

All testing problems was reduced to single criterion optimization problem and solved with MATLAB (7.7.0.471) Optimization Toolbox function *fmincon()*.

### 3.3 Data analyses

Obtained data was analyzed after experiment using descriptive statistics and frequency analyses of stopping reason. Data of each TP was grouped by stopping reason: 1) solutions fluctuate around the same value; 2) the goal was obtained. Then data was compared using descriptive statistics: mean, max, min, standard deviation and variance for all metrics described in previous subsection. Obtained solutions were graphically compared with theoretical Pareto front.

**Table 1. Testing problems definitions.**

Problem name	Definition/Constraints	Constraints	Pareto front
<b>TP1 (A)</b> Tol=0.065	$F(X) = (f_1(x), f_2(x))$ $f_1(x) = x_1^2 + x_2^2$ $f_2(x) = (x_1 - 5)^2 + (x_2 - 5)^2$	$x_1 \geq -5,$ $x_2 \leq 10,$	
Initialization	$x^{(0)} = [5, 5]; f(x^{(0)}) = [50, 0]; z^* = [0.13, 45.13]$		
<b>TP2 (B)</b> Tol=0.1	$F(X) = (f_1(x), f_2(x))$ $f_1(x) = \sqrt{x_1},$ $f_2(x) = \sqrt{x_2}.$	$x_1 \geq 0,$ $x_2 \geq 0,$ $x_1 + x_2 \geq 5.$	
Initialization	$x^{(0)} = [0.4, 4.6]; f(x^{(0)}) = [0.63, 2.14]; z^* = [2.13, 0.67]$		
<b>TP3 (A)</b> Tol=0.07	$F(X) = (f_1(x), f_2(x))$ $f_1(x) = 1/(x_1 + x_2 + 1)$ $f_2(x) = x_1 + 3x_2 + 1$	$x_1 \geq -3,$ $x_2 \leq 3$	
Initialization	$x^{(0)} = [6, 1]; f(x^{(0)}) = [0.026, 40]; z^* = [0.23, 4.72]$		
<b>TP4 (C)</b> Tol=0.12	$F(X) = (f_1(x), f_2(x), f_3(x))$ $f_1(x) = 25 - (x_1^3 + x_1^2(1 + x_2 + x_3) + x_2^3 + x_3^3)/10$ $f_2(x) = 35 - (x_1^3 + 2x_2^3 + x_2^2(2 + x_1 + x_3) + x_3^3)/10$ $f_3(x) = 50 - (x_1^3 + x_2^3 + 3x_3^3 + x_2^2(2 + x_1 + x_2))/10$	$x \geq 0$ $12 - x_1^2 - x_2^2$ $-x_3^2 \geq 0$	
Initialization	$x^{(0)} = [0.8, 0, 6.4]; f(x^{(0)}) = [-1.74, -8.73, -44.26]; z^* = [-123.19, 83.71, -335.74]$		
<b>TP5 (C)</b> Tol=0.5	$F(X) = (f_1(x), f_2(x), f_3(x))$ $f_1(x) = x_1 + 9x_2 + 10x_3 + x_4 + 3x_5$ $f_2(x) = 9x_1 + 2x_2 + 2x_3 + 7x_4 + 4x_5$ $f_3(x) = 4x_1 + 6x_2 + 7x_3 + 4x_4 + 8x_5$ $3x_1 + 9x_2 + 9x_3 + 5x_4 + 3x_5 \leq 1039$ $-4x_1 - x_2 + 3x_3 - 3x_4 - 2x_5 \leq 94$ $3x_1 - 9x_2 - 9x_3 - 4x_4 \leq 61$ $5x_1 + 9x_2 + 10x_3 + x_4 - 2x_5 \leq 942$ $3x_1 - 3x_2 + x_4 + 5x_5 \leq 420$	$x_1, x_2, x_3$ $\geq 0$ $x_4, x_5 \geq 0$	
Initialization	$x^{(0)} = [0, 0, 1, 0, 0]; f(x^{(0)}) = [10, 2, -7]; z^* = [-118, 102, -137]$		
<b>TP6 (D)</b> Tol=0.04	$F(X) = (f_1(x), f_2(x), f_3(x))$ $f_1(x) = x_1^2 + (x_2 - 1)^2$ $f_2(x) = x_1^2 + (x_2 + 1)^2 + 1$ $f_3(x) = (x_1 - 1)^2 + x_2^2 + 2$	$x_1 \geq 2,$ $x_2 \leq 2$	
Initialization	$x^{(0)} = [0.8, 0.4]; f(x^{(0)}) = [1, 3.6, 2.2]; z^* = [1.96, 1.36, 3.16]$		

For all TP was obtained  $PF^*$ .

#### 4 Results and discussions

Experiment data was grouped by stopping reason for each TP and IOM. Table 2 shows frequencies of stopping reason for all IOM and TP, total TP solving repeat number – 10.

**Table 2. Stopping reason frequencies (analyzing last 4 iterations)**

Stopping reason	TP1		TP2		TP3		TP4		TP5		TP6	
	G	S	G	S	G	S	G	S	G	S	G	S
1. The goal was obtained	-	-	10	-	5	-	-	-	-	-	10	5
2. Euclidean distance increase	4	-	-	2	-	-	-	6	2	-	-	-
3. Solutions fluctuate around the same value	1	10	-	8	4	10	10	2	-	10	-	5
4. Solutions conv., to the goal slowly	-	-	-	-	1	-	-	-	-	-	-	-
5. Exceeded max iteration count	5	-	-	-	-	-	-	2	8	-	-	-

\* G – GUESS method, S – STEM method.

The goal solution by GUESS method was obtained (with predefined tolerance) solving TP2, TP3, and TP6. STEM method obtained the goal solution only solving TP6. Solutions fluctuates at the same value for GUESS solving: TP1, TP3 and TP4; STEM solving: TP1, TP2, TP3, TP4, TP5 and TP6. Table 3 shows results of case when solutions fluctuate around the same value and their metrics values. The metrics values are the same for all repeating times where dispersion and variance are equal to zero. Table 4 provides information of metrics values for cases when solutions fluctuate around the same value and the goal was obtained.

The DMM solving TP4 by GUESS provided absolutely correct VRI information but values of upper and lower bounds of objectives and reference point values was small in all repeating times, this strategy is not appropriate for GUESS method.

**Table 3. Metric values are the same for all iterations (case – solutions fluctuate around the same value)**

Method–Problem	GD*	MPE	SP	ED	IC
GUESS–TP4	0.0	0.0	0.0	329.02	3
STEM–TP5	38.25	12.37	8.95	196.38	4
STEM–TP2	0.0	0.01	0.68	1.10	4
STEM–TP4	0.41	0.97	54.88	412.78	4

\*see chapter 3.1 for legend

The DMM solving TP5 by STEM was provided wrong VRI information (for all repeating times)– objectives indexes are repeated in the class  $I^<$  but the class  $I^>$  was mainly empty. The DMM solving problem TP2 provided empty class  $I^>$  but solving problem TP4 situation was similar to TP5.

**Table 4. Descriptive statistics for experiment metrics**

	Solutions fluctuate around the same value					The goal was obtained				
	min	max	avg	$\sigma$	$\sigma^2$	min	max	avg	$\sigma$	$\sigma^2$
	<i>STEM–TP1</i> ( $\varepsilon=0.065$ ), N=10					<i>GUESS–TP2</i> ( $\varepsilon=0.1$ ), N=10				
GD	0.000	0.029	0.006	0.012	0.000	0.000	0.000	0.000	0.000	0.000
MPE	0.000	0.356	0.007	0.147	0.022	0.000	0.070	0.036	0.023	0.001
SP	17.673	22.361	21.423	1.976	3.907	0.000	0.000	0.000	0.000	0.000
ED	26.566	34.896	33.23	3.509	12.320	0.000	0.070	0.036	0.023	0.001
IC	4	7	4.600	1.265	1.600	1	1	1.000	0.000	0.000
	<i>GUESS–TP3</i> ( $\varepsilon=0.07$ ), N=4					<i>GUESS–TP3</i> ( $\varepsilon=0.07$ ), N=5				
GD	0.009	0.012	0.011	0.001	0.000	0.000	0.014	0.006	0.007	0.000
MPE	0.154	0.205	0.174	0.021	0.000	0.010	0.244	0.122	0.108	0.120
SP	11.690	15.940	14.842	2.102	4.421	0.000	20.357	10.096	9.477	89.820
ED	0.089	0.169	0.127	0.040	0.002	0.009	0.040	0.025	0.014	0.000
IC	4	8	5.000	2.000	4.000	1	5	2.600	1.817	3.300

	Solutions fluctuate around the same value					The goal was obtained				
	min	max	avg	$\sigma$	$\sigma^2$	min	max	avg	$\sigma$	$\sigma^2$
	STEM-TP3 ( $\varepsilon=0.07$ ), N=10					GUESS-TP6 ( $\varepsilon=0.04$ ), N=10				
GD	0.000	0.006	0.001	0.002	0.000	0.000	0.035	0.007	0.013	0.000
MPE	0.000	0.173	0.031	0.065	0.004	0.000	0.264	0.777	0.107	0.012
SP	11.262	17.877	16.754	2.412	5.820	0.000	1.932	0.532	0.859	0.739
ED	1.471	3.799	3.447	0.789	0.623	0.000	0.039	0.016	0.014	0.000
IC	4	9	4.800	1.751	3.067	1	3	1.400	0.699	0.489
	STEM-TP6 ( $\varepsilon=0.04$ ), N=5					STEM-TP6 ( $\varepsilon=0.04$ ), N=5				
GD	0.012	0.016	0.014	0.002	0.000	0.010	0.014	0.012	0.001	0.000
MPE	0.191	0.191	0.191	0.000	0.000	0.191	0.247	0.201	0.025	0.001
SP	1.185	1.200	1.191	0.000	0.000	0.898	1.236	1.105	0.139	0.019
ED	0.170	1.276	0.833	0.606	0.367	0.000	0.008	0.003	0.004	0.000
IC	4	5	4.400	0.548	0.300	4	8	5.20	1.643	2.700

In a case when solutions fluctuate around the same value, the DMM solving TP1 by STEM mainly provided empty class  $I^>$  and objectives indexes not repeated in  $I^>$ . Solving TP3 by STEM the DMM mainly provided empty class  $I^>$  and repeated objectives indexes in the  $I^>$ . The DMM solving TP3 by GUESS was provided absolutely correct VRI values. In case when the goal was obtained the DMM solving TP6 by STEM mainly provided empty class  $I^>$  and objectives indexes repeated in class  $I^>$ . If the  $I^>$  class is empty then we can't set upper bounds  $\varepsilon_j$  for relaxed objectives and search process converge to the same solution every time.

Analyzing values of general distance (GD) and maximal Pareto front error (MPE) we concluded that obtained solutions belongs to the  $PF^*$  or not. Less values near 0 are preferred, but a greater shows that obtained solutions are far from  $PF^*$ . GUESS and STEM methods as well as others IOM are sensitive of solutions  $x^{(i)}$  belonging to  $PF^*$ , otherwise methods cannot obtain Pareto-optimal solutions, but only weakly Pareto-optimal. Spacing metric (SP) gives comprehensive of obtained solutions standard deviation. If SP value is near 0, then obtained solutions are very close each to other that is typical for solutions fluctuating around the same value. Otherwise SP shows that IOM has obtained different solutions (in different places of  $PF^*$ ).

Figure 3 shows obtained solutions in Pareto front and Euclidean distance to the goal solution. Solutions in iterations 2 – 4 has equal values of objectives, the same situation for iteration 6 and 7 solutions. Provided preference information by the DMM was different for all iterations. The DMM obtained the goal solution very precisely (tolerance=0.004 and Euclidean distance equal to 0.0005) at iteration 8.

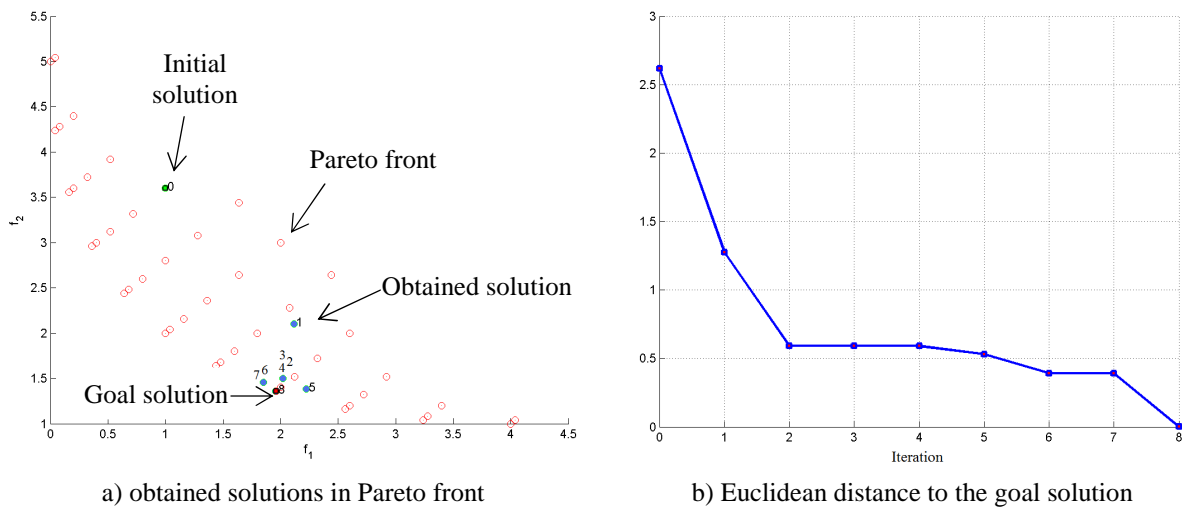


Figure 3. The DMM solving TP6 by STEM (the goal was obtained)

Comparison of results showed that the DMM for GUESS method works better than STEM. On the other hand STEM method can't bring close to the goal solution because values of VRI provided by the DMM was incorrect. In future to solve this problem we need to precisely define the second criterion of the (2) problem.

## 5 Conclusions

The new DMM called ZuMo obtained goal solution of different TPs by GUESS method more times than by STEM. The goal solution by GUESS method was obtained solving TP2 (10 cases of 10), TP3 (5 of 10), and TP6 (10 of 10), while STEM only solving TP6 (5 of 10). Solving TPs by STEM the DMM many times provided wrong preference information (indexes of objectives are repeated in the class  $I^<$ ) for 3D problems. STEM method preference information rules allow keeping empty class  $I^>$  and not defining  $\varepsilon_j$  – upper bounds of relaxed objectives. The DMM (2) problem second criterion fulfill the conditions of STEM method. Each IOM has its own best strategy for solving problems. For this reason criterion definition for complicated strategy will be difficult. For better solution results the definition of second criterion should be as precise as possible.

Metrics values were taken into account to evaluate IOM effectiveness and the DMM behavior. Values of those metrics depends on how precisely  $PF^*$  solutions are calculated, not only on IOM effectiveness. ZuMo provides mathematical approach of IOM testing/comparison. Computation time can be reduced using parallel computation technologies. NSGA-II and others evolutionary algorithms has best opportunities on parallel computation. Researchers can design and implement experiments of testing/comparison IOM without human DMs reducing time and workload. ZuMo can be easy used for ad-hoc IOM comparison. The TP  $PF^*$  front and goal solution can be determined before the DMM start optimization.

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