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POISSON PROCESS OF DEFECT INITIATION IN FATIGUE OF A COMPOSITE MATERIAL

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In a new version of the model of fatigue life distribution as a function of both maximum and minimum of cycle stress we take into account that there are several weak micro-volumes (WMV) in which accumulation of fatigue damage takes place and consider a Poisson process of the initiation of these WMV. The cumulative distribution function (cdf) of fatigue life of every specific WMV is calculated using Markov model of fatigue. For the case when it is approximated by a lognormal distribution the specific equation for specimen (as sequence of WMV) fatigue life is obtained. Numerical examples are provided.

Keywords: fatigue, Markov chains, composite

1. Introduction

The paper is a development of models described in previous papers [1-7], devoted to the problem of connection of fatigue life and tensile strength of composite, and specific idea mentioned in [6,7] concerning necessity to take into account the existence of several "weak micro-volumes" (WMV) in which fatigue damage accumulation takes place and failure of any of which results in the failure of a specimen. It is supposed that WMVs do not originate simultaneously but in accordance with a Poisson process. In the new model we take into account the influence of stress ratio of cyclic fatigue loading also.

There is no necessity to convince of importance to study these problems. There are a lot of papers devoted to mathematical models of the S-N curve and its dependence on cycle stress parameters. Short survey of them can be found, for example, in our papers [6,7]. This problem has deep history and wide discussions [8,9,10,11]. So here we shall mention in more detail only specific papers in discussion of specific questions.

2. Theoretical Background

2.1. The lifetime of specimen and microvolumes

As distinct from [6], it is assumed that the fatigue failure of a test specimen occurs after the destruction of any of several weak microvolumes (WMV). In this paper we suppose that the number of WMVs is random function of time. It increases during fatigue loading with intervals X_i, $i = 1,2,3,...$. So X_1, $X_1 + X_2$, $X_1 + X_2 + X_3$, ... are the time moments of initiation of new WMVs. Let us denote by T_j fatigue life of *j*-th specific WMV. Then the life of specimen

$$Y = \min(T_1, T_2 + X_1, T_3 + X_1 + X_2, ...), \tag{1}$$

or

$$Y = \min(T_1, X_1 + Y_1), \tag{2}$$

where $F_Y(y) = F_{Y_1}(y)$.

Solution of the equation (2) for exponential distribution of all the independent random variables (r.v.) $X_1, X_2, X_3, ...$ with parameter μ (mean life is equal to $1/\mu$) is given in [12]:

$$F_Y(y) = 1 - (1 - F_T(y))\exp(-\mu\int_0^y F_T(t)dt)\,. \qquad (3)$$

The function $F_T(t)$ can be found using the Markov model of fatigue described in [6,7].

2.2. Fatigue life distribution calculation using Markov chains theory

As in [6] we suppose that every WMV include two parts: elastic and plastic (Figure 1). The elastic part consists of perfectly elastic (brittle) longitudinal items (LI) (fibres or bundles of fibres (strands)). The plastic part is the matrix itself and all the layers with stacking different from the longitudinal one. During fatigue loading the number of LI decreases. In the matrix, accumulation of plastic deformations takes place.

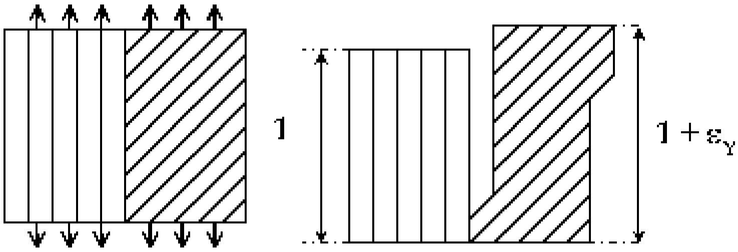

Figure 1. Model of the weak microvolume of a composite under a load and after removal of the load. Explanations in the text

We assume that only the LI can carry a load but the matrix only redistributes the load after failure of some LIs. The initial number of LI in WMV is equal to r_R. If during the cyclic loading this number decreases to zero, the elastic part of the specimen breaks down, which is followed by the failure of the whole specimen. The slanted hatching on Figure 1 symbolically points to the possibility of accumulating an irreversible plastic strain. If it exceeds some level ε_Y, the failure of the WMV and the failure of the specimen occur. We emphasize that this graphic image, as applied to a composite, should be understood symbolically. It is more suitable for metals, where the accumulation of plastic strains is associated with some “act of flow” (for metals – a shift over slip planes).We will assume that an individual act of flow, in the mathematical description of the process, leads to a respective change in the state of Markov chain (MCh), while in the physical description – to the appearance of a constant plastic strain ε_{Y1}. The failure of WMV takes place after the accumulation of a “critical” plastic strain, r_Y, i.e., after the accumulation of such number of “acts of flow” that the relation $\varepsilon_{YC} = \varepsilon_{Y1}\, r_Y$ takes place, where ε_{YC} and r_Y are some model parameters. Since the elastic and plastic parts are integrated in a unit, the accumulation of plastic strains (irreversible deformation of the plastic part) leads to the appearance of residual stresses: tension in the elastic and compression in the plastic part of the specimen.

We associate the process of gradual failure of a specimen with a stationary MCh whose set of states is determined by the number of broken LIs and the number of acts of flow. The matrix of transition probabilities is presented as a totality of (r_Y +1) blocks with (r_R +1) states within each of them. Then, the indices of input and output states, *i* and *j*, respectively, can be expressed in terms of the corresponding local indices i_Y, i_R, j_Y, and j_R by the equations: $i = (r_R + 1)(i_Y - 1) + i_R$; $j = (r_R + 1)(j_Y - 1) + j_R$. Table 1 shows an example of (symbolic) filling of the matrix for the case where $r_Y = r_R$ =2.

Table l. Example of the Matrix Structure of Transition Probabilities

		j_Y	*1*			*2*			*3*		
		j_R	*1*	*2*	*3*	*1*	*2*	*3*	*1*	*2*	*3*
i_Y	i_R	i\j	1	2	3	4	5	6	7	8	9
1	*1*	1	$p_{R0}p_{Y0}$	$p_{R1}p_{Y0}$	$p_{R2}p_{Y0}$	$p_{R0}p_{Y1}$	$p_{R1}p_{Y1}$	$p_{R2}p_{Y1}$	$p_{R0}p_{Y2}$	$p_{R1}p_{Y2}$	$p_{R2}p_{Y2}$
	2	2	0	$p_{R0}p_{Y0}$	$p_{R1}p_{Y0}$	0	$p_{R0}p_{Y1}$	$p_{R1}p_{Y1}$	0	$p_{R0}p_{Y2}$	$p_{R1}p_{Y2}$
	3	3	0	0	1	0	0	0	0	0	0

The continuation of Table 1

	1	**4**	0	0	0	$p_{R0}p_{Y0}$	$p_{R1}p_{Y0}$	$p_{R2}p_{Y0}$	$p_{R0}p_{Y1}$	$p_{R1}p_{Y1}$	$p_{R2}p_{Y1}$
2	***2***	**5**	0	0	0	0	$p_{R0}p_{Y0}$	$p_{R1}p_{Y0}$	0	$p_{R0}p_{Y1}$	$p_{R1}p_{Y1}$
	3	**6**	0	0	0	0	0	1	0	0	0
	1	**7**	0	0	0	0	0	0	1	0	0
3	***2***	**8**	0	0	0	0	0	0	0	1	0
	3	**9**	0	0	0	0	0	0	0	0	1

In this case the destruction of a specimen occurs if two LIs fail (event A), or two acts of flow are accumulated (event B), or events A and B take place simultaneously. To these events the absorbing states of the MCh correspond. In the example considered, there are $(r_Y+1)(r_R+1) = 9$ such states. The symbols p_{R0}, p_{R1}, … designate the probabilities of failure of the corresponding numbers of elastic (rigid) elements; p_{Y0}, p_{Y1}, … are the probabilities of the corresponding numbers of acts of flow (yielding). The example of equations for filling of matrix P is given in [6].

Let us recall [6] that if the matrix of transition probabilities, P, is written in the form

$$P = \begin{bmatrix} Q & R \\ 0 & I \end{bmatrix}, \tag{4}$$

where I is the unit matrix and 0 is the matrix consisting of zeros then, if π is the distribution of initial states, the cumulative distribution function (cdf) of the number of steps in the MCh before absorption, T, is given by

$$F_T(t,S) = \pi P^t b, \quad t = 1,2,3,.... \tag{5}$$

where b is a column vector of type (0, … 0,…,1,…,1)′, with the number of unities equal to the number of absorbing states. For example if the process starts from first state the vector π has the form (1, 0, 0, ... 0). If we have only one absorption state the vector b is equal to $(0,...,0,1)$. Let us underline that in the following the time, t, is measured by the number of steps of MCh if not stated otherwise. A proportional connection between the number of steps of MCh and corresponding number of fatigue cycles is assumed: $n = k_M t$.

Precise calculation of P^t for large t, needed for calculation of (5), and then (3) is not too easy even if one uses a modern PC, and it takes a lot of time. In any case it is useful to have a simple approximation of (5). In [3] it is shown that the lognormal approximation can be used for this purpose. The approximation can be made on the bases that the mean and standard deviation of T with cdf (5) should be equal to the corresponding values of lognormal distribution.

Using the formulas of the theory of finite MCh [13], we obtain that the column vector of average number of steps needed to reach any absorbing state from different initial transient states is $\tau = N \cdot \xi$, where $N = (I-Q)^{-1}$, ξ is a column vector filled with unities. The corresponding vector of variances is $\tau_2 = (2N - I)\tau - \tau_{sq}$, where $\tau_{sq}(i) = (\tau(i))^2, i \in I_T$, and I_T is the set of indices of transient states.

Let vector π' gives the initial probabilities for the transient states. Then for the mean and variance of T we have equations

$$E(T) = \pi'\tau; \; Var(T) = \pi'(2N - I)\tau - (\pi'\tau)_{sq}. \tag{6}$$

If (5) is approximated by a lognormal distribution then [12]

$$F_Y(y) = 1 - \Phi(-z)\exp(-\mu(y\Phi(z) - \exp(\mu + \theta_{1L}^2/2) * \Phi(z - \theta_{1L}))), \tag{7}$$

where $\Phi(.)$ is cdf of the standard normal distribution, $z = (\log(y) - \theta_{0L}) / \theta_{1L}$, the parameters of the lognormal distribution at the load defined by S: $\theta_{0L} = \ln(E(T)) - \ln(r^2 + 1) / 2$, $\theta_{1L} = (\ln(r^2 + 1))^{1/2}$, $r^2 = Var(T) / (E(T))^2$.

It can be assumed also that the rate of the Poisson process is a function of the load; for example: $\mu = \mu_1 F(S_{max})$, where μ_1 is the factor of proportionality (a parameter of the model), $F(.)$ is the cdf of the static strength, $S_{\max}$ is the maximum of cyclic stress.

3. Probability Fatigue Diagram, the Influence of Stress Ratio

The number of cycles $n_p(S)$ corresponding to the probability of failure *p* under fatigue load with the maximum stress *S* (the *p*-quantile fatigue curve) and mean fatigue curve are defined by equations

$$n_p(S) = k_M F_Y^{-1}(p; S), \tag{8}$$

$$E(N(S)) = k_M \int_0^\infty (1 - F_Y(y, S)) dy, \tag{9}$$

where, let us remind that k_M is the number of cycles (a parameter of the model), corresponding to one step of the MCh. Let us note that the knowledge of cdf (8) or (9) allows us to estimate the parameters of the model (including k_M) by using of either the nonlinear least squares method or the maximum likelihood method if we have corresponding test data. This question is treated in [3].

The considered model, as the model described in [6], is appropriate for pulsating cycle. With some level of "accuracy" any other cycle can be "recalculated" to an equivalent pulsating cycle using the energy method described in [14,15]. We propose a modification of this method. In [14] the range of a normalized strain energy density of a linear elastic material is shown to be proportional to ($S_{\max}\varepsilon_{\max} - S_{\min}\varepsilon_{\min}$) and, therefore, proportional to $(S_{\max}^2 - S_{\min}^2) = S_{\max}^2(1 - R^2)$. But taking into account that in general case there may be some deviation from the Hooke's law, $S = \varepsilon E$, we suppose that for two equivalent cycles with different stress ratio the following relation holds:

$S_{\max 1}^m (1 - R_1^m) = S_{\max 2}^m (1 - R_2^m)$. So for the cycle with $S_{\max 2}, R_2$ the value of $S_{\max 1}$ of equivalent cycle with R_1 is defined by equation

$$S_{\max 1} = S_{\max 2}((1 - R_2^m) / (1 - R_1^m))^{1/m}. \tag{10}$$

Indeed, for the processed fatigue data we obtain $m < 2$ (see section 5 below).

Let us note that after estimation of the parameters of the model using data for a specific stress ratio R, equation (10) allows construction of the whole probability fatigue diagram.

4. Residual Stress in Two Stress Level Fatigue

First again, let us consider the case of one WMV. Fatigue loading by n_1 cycles with maximum stress S_1 we denote by (S_1, n_1). The corresponding number of steps of a MCh t_1 is equal to n_1 / k_M. The new probability distribution on the states of MCh is

$$\pi_{S_1 n_1} = \pi (P(S_1))^{t_1}, \tag{11}$$

where π is a priori distribution.

The last $(r_Y + 1 + r_R)$ components of this vector define the probability of failure in the corresponding absorbing states. Of course, the residual strength and residual life are related only with transient states, the number of which is equal to $m' = (r_Y + 1)(r_R + 1) - (r_Y + 1 + r_R)$.

For relevant conditional distribution we have

$$\pi'_{S_1 n_1}(k) = \pi_{S_1 n_1}(k) / \sum_{m=1}^{m'} \pi_{S_1 n_1}(m), \qquad (12)$$

where $\pi_{S_1 n_1}(k)$, $k = 1,...,m'$, are components of the vector $\pi_{S_1 n_1}$.

Let $\pi^*_{S_1 n_1}$ be such a vector (of the size $(r_R + 1)(r_Y + 1)$) that the first m components of it coincide with the components of vector $\pi'_{S_1 n_1}$ but all the other are equal to zero.

Now the cdf of stress, $\sigma_{S_1 n_1}$, at which in one step of MCh the failure takes place of the specimens which are intact after fatigue loading (S_1, n_1)

$$F_{\sigma_{S_1 n_1}}(x) = \pi^*_{S_1 n_1} P(x) b.$$

If the actual number of cycles corresponding to one step of MCh, k_M, does not differ too much from unit then this cdf is approximately equal to the cdf of residual strength of the specimens which are still intact after fatigue loading (S_1, n_1).

If we consider two stress level fatigue and after fatigue loading (S_1, n_1)maximum stress of the cyclic loading becomes equal to S_2 then the cdf of residual life, i.e. the cdf of number of steps to absorption (failure of specific WMV), $T_{S_1 n_1}$

$$F_{T_{S_1 n_1}}(x) = \pi^*_{S_1 n_1} (P(S_2))^x b \quad , x = 1, 2, ...$$

But now let us recall that the j-th WMV is originated in ("calendar") time moment (number of MCh-steps) $Z_j = X_1 + X_2 + ... + X_j$ so its real random fatigue loading is ($S_1, (n_1 - k_M Z_j)$). Let us denote the relevant cdf of residual strength by $F_{\sigma_j}(x)$. Then for specific observations $(x_1,...,x_k)$ of the vector $X_{1,K} = (X_1, X_2, ..., X_K)$ where K is r.v, such that $k_m Z_K \le n_1 < k_m Z_{K+1}$ we have the following random cdf-s of residual strength of specimen which is considered as a series system (series of WMVs)

$$F_{\sigma, x_{1,k}}(s) = 1 - \prod_{j=1}^{k} (1 - F_{\sigma_j}(s)) \text{ and } F_{T, x_{1,k}}(t) = 1 - \prod_{j=1}^{k} (1 - F_{T_j}(t)).$$

Let $f_{X_{1,k}|K=k}(x_1,...,x_k \mid K = k)$ be the conditional pdf of $X_{1,k}$ at $t_1 = t$ and fixed $K = k$:

$$f_{X_{1,k}|K=k}(x_1,...,x_k \mid K = k) = \mu^k e^{-\mu(x_1 + ... + x_k)} e^{-\mu(t - x_1 - ... - x_k)} / (e^{-\mu t} (\mu t)^k / k!) = k! / t^k .$$

It is a uniform distribution in the subspace of $R^{k+1} : x_i > 0,\ i = 1,...,k,\ \sum_{i=1}^{k} x_i \le t < \sum_{i=1}^{k+1} x_i$.

Mean cdf-s of residual strength is defined by equation

$$F_\sigma(s) = \sum_{k=1}^{\infty} p_k \left(\int_{R^{k+1}} F_{\sigma, x_{1,k}}(s) f_{X_{1,k}|K=k}(x_1,...,x_k \mid K = k) dx_1 ... dx_k \right),$$

It can be assumed that at the beginning of fatigue loading there is one WMV (in the weakest cross section) and the random number of additional WMV, which appears during n_1 cycles, $K_P = K - 1$, has Poisson distribution with the parameter $\Lambda = \mu n_1 / k_m$.

An approximate value of the multivariate integral is calculated very easily by the Monte Carlo method: $F_\sigma(s) = \sum_{j=1}^{N_{MC}} F_{\sigma, x_{1,k}}(s) / N_{MC}$, where $\sum_{i=1}^{k_j} x_i \le n_1 / k_m < \sum_{i=1}^{k_j + 1} x_i$, x_i, $i = 1,...,k_j$, are observations of rv with exponential distribution, with pdf $\mu \exp(-\mu x)$, $x \ge 0$.

5. Processing of Test Data and Discussion

Based on the formulas obtained, let us try to explain the data of tests which were performed in the Institute of Polymer Mechanics on specimens of a composite material. The specimens were made from material which consisted of layers of unidirectionally reinforced (UD) (more precisely Udo UD ES 500/300 - SGL epo GmbH with resin LH 160 of „Composites HAVEL"; the thickness 0.49 mm) stacked in the following way: [0/45/0]. The total length of a specimen is equal to 250 mm, the "working" length – 60 mm. Mean static strength (21 specimens) was found to be equal to St = 487.56 MPa. Fatigue tests were performed with the maximum cycle stress equal to 0.5; 0.6; 0.75; 0.7 of the ultimate strength at two levels of stress ratio, *r*: 0.1 and 0.5 (see Table 2,3).

Table 2. Fatigue lifes for *r* = 0.1

N	Stress (MPa)	Number of cycles
1	292.535	70 000;52000;
2	341.29	16084; 14300;
3	390.05	2243; 1150; 895; 890; 750; 650; 400.

Table 3. Fatigue life for *r* = 0.5

N	Stress (MPa)	Number of cycles
1	341.29	265000;
2	365.67	101800; 146500;
3	390.05	4500; 9200; 10000;
4	414.42	4100; 3650.

Then for different duration of the preliminary fatigue loading and different maximum cycle stress loading (at *r* = 0.1) the residual strength was tested (see Table 4, column „Resid.strength(MPa)").

Table 4. Residual strength for *r* = 0.1

N	Stress (MPa)	Number of cycles	Resid.strength(MPa)	Mean resid.strength(MPa)
1	243.78	265000	399.8	399.8
2	292.53	60000	465.84; 432.04; 425.13; 414.73; 408.84; 387.55;	422
3	390.05	900	481.58; 478.39; 477.78; 474.16; 456.54; 451.85.	470

We see great scatter of the residual strength, so in the following the calculation of mean values of residual strength was performed and these data were used in data processing (see Table 3, column „Mean resid. strength (MPa)").

In this paper we did not consider the problem of finding the model parameters which provide the best fitting of test results. If this model is considered as a nonlinear regression model then the number of unknown parameters is very large. Moreover, this problem is very difficult and should be considered in a separate study. In this paper we apply the estimates of parameters of the static strength.

Processing of the test data was done in the following way. First, the parameters of the model were chosen to ensure fitting of the fatigue life data at *r*=0.1. Then these parameters were used for prediction of the fatigue life at the *r*=0.5 by "translation" of the stress at *r*=0.5 into an "equivalent" stress at *r*=0.1 by equation (10). These results of fitting and prediction are shown on Figure 2

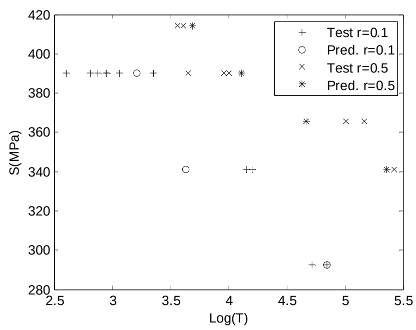

Figure 2. Results of fitting for *r*=0.1 and prediction for *r*=0.5 of fatigue life

We see an acceptable fitting for r=0.1 and a reasonable quality of prediction for r=0.5 of fatigue life.

In this case the fitting parameters were chosen in such a way that only the failure of the rigid components could be the reason of the failure of specimen because the matrix of the tested specimen was very weak. It was supposed that the rigid component carry 95% of load and that the logarithm of strength of rigid component has a normal distribution with cdf $F_{LR}(x) = \Phi((x-\theta_{0LR})/\theta_{1LR})$, $\theta_{0LR} = 6.188$, and $\theta_{1LR} = 0.15$. (recall that exp(6.188)=487.56 is the mean static strength of specimens). It was assumed that the number of rigid items in WMV is equal to 5. It is not the optimal estimate of the parameter r_R but rather a practical one. Increasing this value does not provide an essentially better fitting but a calculation problem appears with matrix inversion.

On Figure 3 and Figure 4 we see results of investigation of residual strength for $\mu_1 = 1$ and $\mu_1 = 20$ (processing of the data of residual strength was performed only for r=0.1). On Figure 3a the values of a discrete analog of probability density function (dapdf) of the residual strength (RS) are given for preliminary loading with maximum stress $S_{pr} = 243.78 MPa$:

$$\Delta_F(s) = F_\sigma(s_i) - F_\sigma(s_{i-1}), \quad i = 1,\ldots,k_\Delta,$$

where $s_0 = S_{pr}$, $s_j = S_{pr} + j(S_\infty - S_{pr})/k_\Delta$, $j = 0,1,2,\ldots,k_\Delta$, S_∞ is a large number (approximately, $F_\sigma(S_\infty) = 1$), k_Δ is the number of intervals. It was chosen : S_∞=1000 MPa, k_Δ=20.

On Figure 3b the mean test value of RS (+), prediction of it (*), and confidence limits of it (▲ ,▼) are shown. On Figure 3c and on Figure 3d we see the same but for preliminary cycling loading with $S_{pr} = 390 MPa$. We see that the predicted mean RS is higher (for $S_{pr} = 243.78 MPa$) or approximately equal (for $S_{pr} = 390\ MPa$) to the test values but in both cases the test values are within the confidence limits.

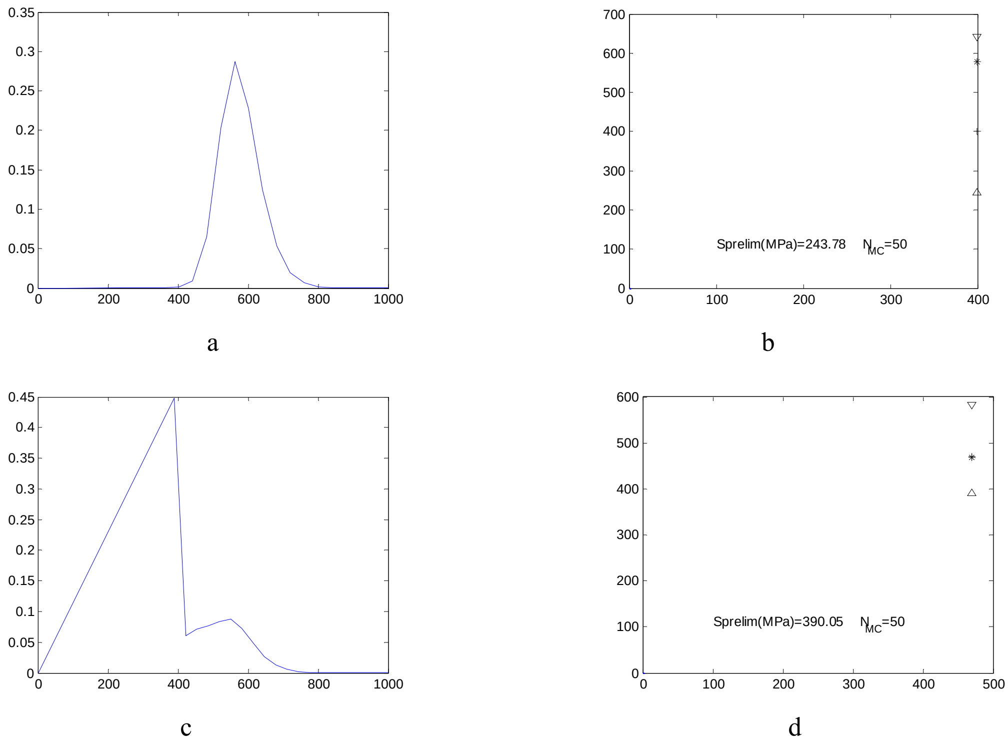

Figure 3. Results of investigation of residual strength for $\mu_1 = 1$

The same conclusion can be made also for the case with $\mu_1 = 20$ (see Figures 4a, b,c,d).

Now we provide additional explanation for figures with letters a and c. On Figure 4c we see an example (of only one Monte Carlo trial!) of four dapdf-s corresponding to the case when in specimen appear one (the lowest line), two, three, four (the uppermost line) WMV-s modelled by the Monte Carlo method: first dapdf of RS_1 was calculated for the case of only one WMV; then the second WMV occasionally appeared and adpdf was calculated for RS_2 and for $\min(RS_1, RS_2)$,..., then for $\min(RS_1, RS_2, RS_3, RS_4)$ (it is upper line). For calculations of mean RS 20 Monte Carlo trials were used. It appeared sufficient for an illustrative example. On Figure 4c the results of only one trial are shown. In other trials the number of WMV-s changes from one to five.

On Figures 3a, 3c and 4a adpdf for only one WMF is shown. In these cases additional WMV-s did not appear in all 20 Monte Carlo trials. This is the reason of small influence of μ_1 on the fatigue curves (on Figure 2 only fatigue curve for μ_1 =1 is shown but for μ_1 =20 we have nearly the same curve).

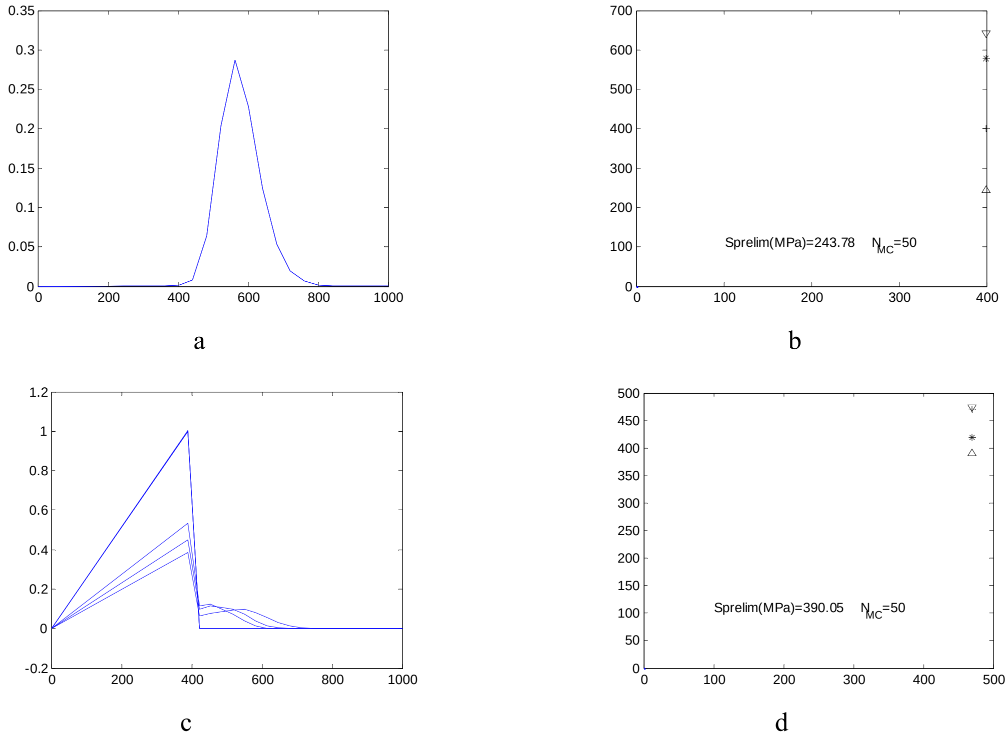

Figure 4. Results of investigation of residual strength for $\mu_1 = 20$

It should be noted also that on Figures 3c, 4c a large jump of dapdf is seen that takes place already at the first increase of load in residual strength test (as a result of calculation with application of matrix P with $S = s_1$).

Comparison of the upper confidence limits (approximately, 575 MPa and 475 MPa for $\mu_1 = 20$ and $\mu_1 = 1$, respectively, at $S_{pr} = 390MPa$) shows the influence of μ in combination with the influence of S_{pr}. At $S_{pr} = 243.78MPa$ we do not see this influence. This stress is not sufficiently large to initiate additional WMV.

6. Conclusions

Using the fatigue model [6,7] we can estimate fatigue life only for a pulsating cycle, while using the new version of the model , tension-tension fatigue loading with any stress ratio can be treated. If we set $\mu_1 = \mu_{1L} L$, where L is the length of the specimen, then the model can be used for prediction of the size effect on fatigue life.

In this paper the residual strength was studied but in a similar way the cdf of residual life in two stress level fatigue can be predicted as follows:

$$F_T(t) = \sum_{k=1}^{\infty} p_k \left(\int_{R^{k+1}} F_{T,x_{1,k}}(s) f_{X_{1,k}|K=k}(x_1,...,x_k \mid K = k) dx_1 ... dx_k \right).$$

But testing of the new model for the size effect and residual life in two stress level fatigue against experimental data is the subject of further study.

The main conclusion of this paper is that the model offered deserves to be studied more substantially.

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