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NUMERICAL MODELLING OF PROCESSES IN LOGISTICS FLOW SYSTEMS

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A short review of continuous and discrete modelling methods of processes in flow systems is given. The disadvantages of these methods are pointed out, which limit the breadth of application of problem solving by modelling production and logistics systems. The principles of the mesoscopic approach to flow systems modelling are described. The approach is based on piecewise constant representation of the flow rate and implemented in form of the Discrete Rate Paradigm in mesoscopic simulation package ExtendSim. Multichannel funnels are a mesoscopic model's main component because they properly represent the processes of parallel or sequential processing and storage of several product types and product portions in a real area of operations. Identical types of resources that are arbitrarily allocable among a funnel's different channels are labelled homogenous resources. The paper describes a mesoscopic simulation model of the passenger check-in processes on an airport. Resource allocation strategies based on mathematical formulas are an important part of the model.

Keywords: simulation, material flow, logistics systems, mesoscopic models

1. Introduction

Logistics is often defined as a complex of methods and means used for planning, implementing and controlling flows of goods called material flows. In addition to material flows logistics systems commonly contain information and financial flows. Logistics flow system denotes the complex of one of the three flow types, including the means of creation, transportation, accumulation and transformation of flows. The processes in logistics flow systems can be represented by the flows themselves as well as by the procedures of their creation, transportation, accumulation and transformation. Within the framework of a certain logistics system the three types of flows can interact with each other.

Simulation of flow system processes is traditionally divided into continuous and discrete [1]. As a rule, continuous simulation is applied in form of Forrester's system dynamics models [2]. Discrete simulation is commonly called discrete event simulation [3, 4]. System dynamics models are implemented in the software packages DYNAMO, iThink/STELLA, Powersim, Vensim, already long existing in the market, or with a respective paradigm in comparatively new software AnyLogic. The overwhelming majority of discrete event simulation models are implemented in the AnyLogic, Arena, AutoMod, Enterprise Dynamics, ExtendSim, FlexSim, Plant Simulation, ProModel, QUEST and Simul8 software programs. System dynamics models are very seldom used to reproduce manufacturing and logistics flow systems [5]. This is explained by their high degree of abstractness and low level of detail in the representation of modelled systems, which is insufficient to solve many practical problems. Contrary to system dynamics models, thousands of discrete event models are annually developed worldwide, enabling to efficiently perform the tasks of planning and updating manufacturing and logistics systems. In spite of the fact that nearly any real process (e.g., in a warehouse or transportation system) can be reproduced in discrete event models, the main advantage of a given model type, namely a high level of detail in the representation of the original element, often appears to be their disadvantage. Because of the necessity to incorporate every single object into the model (transportation means, product unit, warehouse cell, worker etc.), its creation and implementation appear to be time and labour consuming. Moreover, many types of information are simply unavailable at the early stages of system planning, that is why a program developer often has to "create" the absent data, which negatively influences model's reliability.

Mentioned above traditional simulation methods have irremovable shortcomings, that is why the problem of finding new methods, which should be more efficient in terms of their practical application under certain conditions, still remains on the agenda. This paper describes the principles of mesoscopic approach to modelling of flow systems [6, 7, 8], which is based on the piecewise constant representation of flow rates. This flow type is called Discrete Rate in the simulation package ExtendSim [9]. The example of a mesoscopic model of passenger check-in processes on an airport illustrates the possibilities of application of analytical methods for dynamic resource allocation in mesoscopic models.

2. Characteristics of a Mesoscopic Approach to Modelling of Flow Systems

In terms of applied modelling of manufacturing and logistics systems, continuous system dynamics models can be called macroscopic, since these models are usually developed in form of abstract generic models that do not support the analysis of concrete material flows but the analysis of business processes on an aggregated level. Common discrete event models can be called microscopic, since they represent changes in the conditions of many separate material (informational and financial) objects. In [6, 7, 8] the authors define a class of mesoscopic models, which are situated between the two model classes described above in terms of the level of modelling detail. On Figure 1 the processes in a single stock are showed, which can be applied in the models of all three classes described above. Stock's input and output flows are set, based on which dynamics of changes in contents is measured.

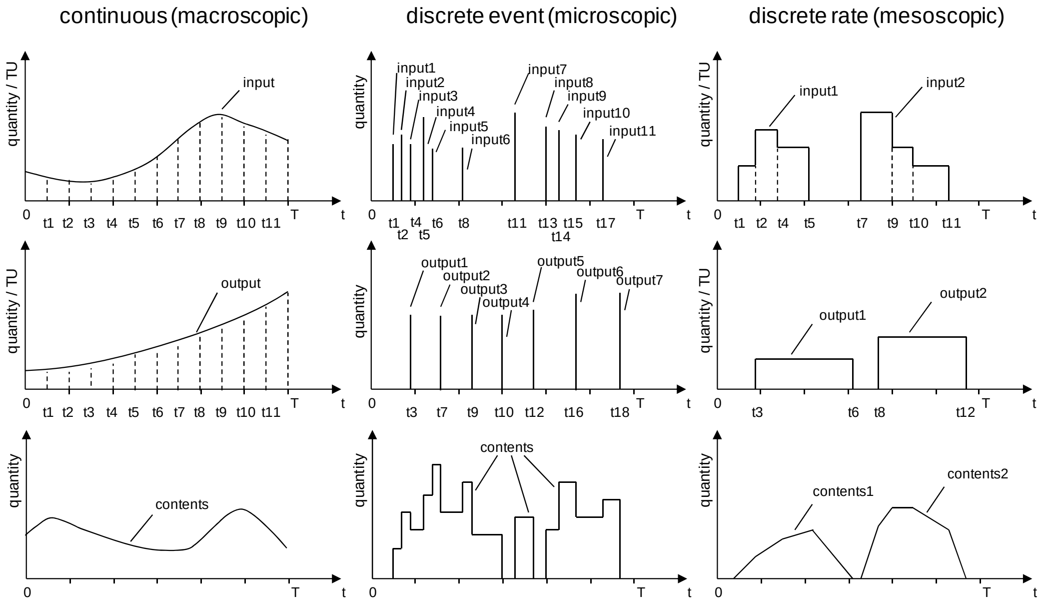

Figure 1. Processes in a single stock in different types of modelling

Orientation of mesoscopic models on piecewise constant flow rates is of major importance since the changes of cumulative flows and contents levels under these conditions correspond solely to piecewise linear functions. In the same way as in piecewise linear aggregates [10], the points of time can be forecast when the model's state variables represented by such piecewise linear functions reach the set "critical values". In other words, mesoscopic models enable to plan events (in the same way as in discrete event models) for continuous processes, characteristics of which do not change between these events.

Figure 2 presents a mesoscopic model with multi-channel funnels instead of simple ones. Exactly the application of parallel channels in a funnel enables to treat separately product portions simultaneously stored in a funnel. Funnel's channels (see funnels *stock 1* and *stock 2* on Figure 2) are given numbers, which correspond to the numbers of parallel product flows (see products *Pr1* and *Pr2* on Figure 2). Products in a mesoscopic model can be represented by all types of a substratum transported in flows.

Multi-channel funnel can be regarded as the main structural component of a given mesoscopic model. Figure 3 depicts the performance principle of a multi-channel funnel and the main components of its mathematical model. The channel's input flow $\lambda_i^{in}(t)$, its maximum throughput $\mu_i(t)$, and the level of contents $S_i(t)$ are assumed to be given. Figure 3 shows that the channel's output flow $\lambda_i^{out}(t)$ at each point of time can be determined in one of the three possible ways, depending on the given conditions.

A multi-channel funnel in a model can represent a workstation, manufacturing area, warehouse or even an entire manufacturing or logistics enterprise. A transport element (see element *transport 1* on Figure 2) is used in a model to represent all the planned delays, which arise, for instance, during the product's transportation or storage processes and are important for modelling. A delay in a funnel can be caused solely by formation of jam, which leads to accumulation of contents. No additional time is needed to for the funnel's output flows. (The process can be pictured as a free passage of bulk products through the funnel, with zero contents in a funnel). An example can be the situation when the check-in capacity on an airport is 500 passengers per hour, whereas only 300 arrive during a certain hour. These passengers leave the check-in area with the same speed, what they arrive with. A several minute delay in serving a single passenger can be omitted in mesoscopic modelling.

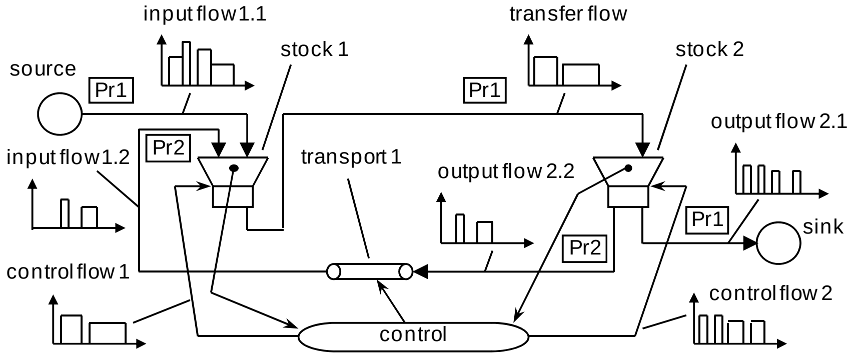

Figure 2. Example of a mesoscopic model structure

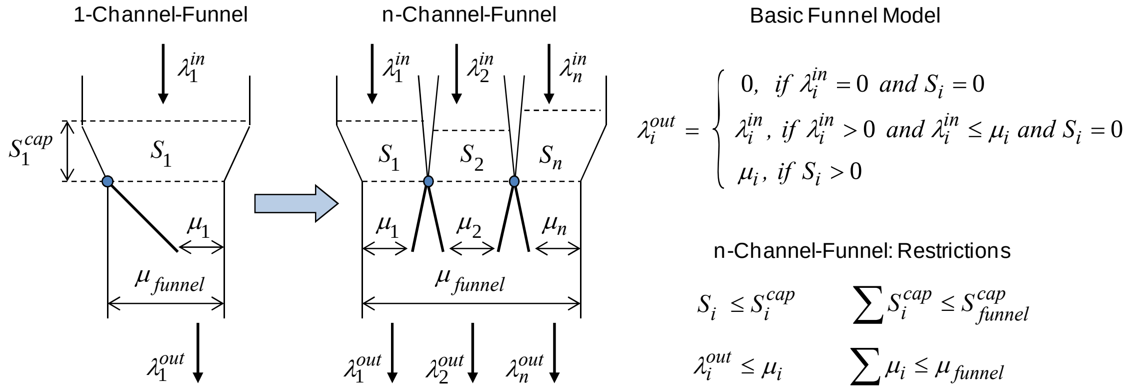

Figure 3. Model of the multi-channel funnel

Throughout capacity of each funnel directly depends on resources used in processing the product portions. Resources are divided into consumable (fuel, water etc.) and reusable ones (staff, technical tools etc.). Both consumable and reusable resources can be limited and unlimited.

Resource allocation strategies in mesoscopic modelling can be described in form of plain algorithms. A special interest, however, is aroused by a possibility to directly apply analytical methods, one of which will be analysed in this paper further.

3. Resource Allocation Strategies

Let a service station (a multi-channel funnel of the mesoscopic model) has m resource holders at its disposal, each of which is characterized by potential performance $pl_k\left(k=1,...,m\right)$, measured in "amount of allocated resource per unit of time". The total potential performance of resource holders is

$$PL=\sum_{k=1}^{m}pl_k. \tag{1}$$

Let the resource demand, i.e. the required throughput of resource consumption, of each input flow in relation to the performance of the station be $bd_i\left(i=1,...,n\right)$, where n is the number of input flows. The total resource demand, i.e. the total required throughput of resource consumption of all input flows equals

$$BD=\sum_{i=1}^{n}bd_i. \tag{2}$$

The variables bd_i and BD, as well as pl_k and PL are measured in "amount of allocated resource per unit of time". The variable bd_i can be set based on any principle, nevertheless, it is most commonly determined by the required throughput capacity of a station μ_i^* for flow $i\left(i=1,...,n\right)$, measured in "flow volume per unit of time". The variable μ_i^* can be, for instance, set equal to the intensity of input flow λ_i^{in}. The variables bd_i and μ_i^* are linearly dependant with the so-called coefficient of resource consumption vk_i. The coefficient vk_i is equal to the amount of resource consumed by a flow unit during its processing at the service station.

If a service station provides for every flow the throughput capacity $\mu_i \left(i = 1,...,n\right)$, then the total resource consumption equals

$$R = \sum_{i=1}^{n} \left(\tau_i \cdot \mu_i\right), \tag{3}$$

Solving the problem of resource allocation means finding the values $\mu_i \left(i = 1,...,n\right)$ subject to the condition $R \le PL$.

In terms of strategy to run: the throughput capacity μ_i in relation to every incoming flow $i\left(i = 1,...,n\right)$ should be proportional to its requirement, i.e. to the variable bd_i. We introduce a so far unknown variable X and state this condition as follows:

$$\mu_i = X \frac{bd_i}{BD}. \tag{4}$$

Since the condition $R = PL$ holds (all available resources are allocated) the following applies:

$$R = \sum_{i=1}^{n} \left(vk_i \cdot \mu_i\right) = \frac{X}{BD} \cdot \sum_{i=1}^{n} \left(vk_i \cdot bd_i\right) = PL, \tag{5}$$

based on which we define variable X as

$$X = \frac{PL \cdot BD}{\sum_{i=1}^{n} \left(vk_i \cdot bd_i\right)} \tag{6}$$

and obtain the final result:

$$\mu_i = bd_i \cdot \frac{PL}{\sum_{i=1}^{n} \left(vk_i \cdot bd_i\right)} \tag{7}$$

A variant of this strategy is applied in the example later in this paper. Whereas a flow unit is a discrete object (e.g., a passenger), the resource consumed for its processing is the operating time of the staff, and the amount of resource consumed when processing one object is the average check-in time of one flow unit, i.e. $vk_i = \tau_i (i = 1,...,n)$. Since the resource consists of time itself, exactly one resource unit can be allocated by one resource holder (an employee) per one time unit of the check-in process. In this case all values $pl_k (k = 1,...,m)$ are equal to 1 and the following equality holds:

$$PL = \sum_{k=1}^{m} pl_k = m. \tag{8}$$

Since resource demand of each input flow bd_i is the "amount of allocated resource per unit of time", in this case it is equal to the number of operating time units, which should be allocated to a given flow during one time unit of a process. Under this condition, each operating time unit corresponds to one staff unit, thus, the following equality holds:

$$bd_i = m_i, \tag{9}$$

where m_i is the required number of staff units for flow $i\left(i = 1,...,n\right)$.

As a result the formula (7) is obtained, which is applied in the modelling of a check-in process of passengers on an airport described later in this paper:

$$\mu_i = m_i \cdot \frac{m}{\sum_{i=1}^{n}(\tau_i \cdot m_i)}. \quad (10)$$

Flawless functioning of this model proves the applicability of the described strategy of resource allocation in a multi-channel service station.

4. Modelling Task

The check-in process is being modelled. The modelled time is 240 minutes (4 hours), within which the check-in of passengers for 3 flights is completed. The model represents a group of passengers of one flight as a flow. Table 1 shows parameters of the three modelled passenger flows.

Table 1. Parameters of the modelled flows

Parameter	Flow 1 (Pax 1)	Flow 2 (Pax 2)	Flow 3 (Pax 3)
Quantity of passengers in the flow	600	300	150
Starting time of incoming passenger flow [min]	0	30	60
Duration of incoming passenger flow [min]	180	180	180
End time of incoming passenger flow [min]	180	210	240
Starting time of check-in [min]	90	120	150
Duration of check-in [min]	90	90	90
End time of check-in [min]	180	210	240
Average check-in duration of 1 passenger [min]	1,0	0,75	0,5
Working time for check-in [man-min]	600	225	75

For modelling the dynamics of the three incoming passenger flows the same scheme for distribution of passengers in the time shown on Figure 4, is applied. It is assumed that real statistical data of the passenger arrival process at check-in can be presented in this mesoscopic form. Every step of the *differential* function refers to 10 minute interval of the passenger arrival process and shows the percentage of the total number of passengers of a flow that is expected to arrive within the 10 minutes. The piecewise linear *cumulative* function shows the result of integration of the differential function. The result of application of the *differential* function for the generation of the three passenger flows in Table 1 is the overall passenger arrival process at the check-in with duration of 240 minutes as shown on Figure 5.

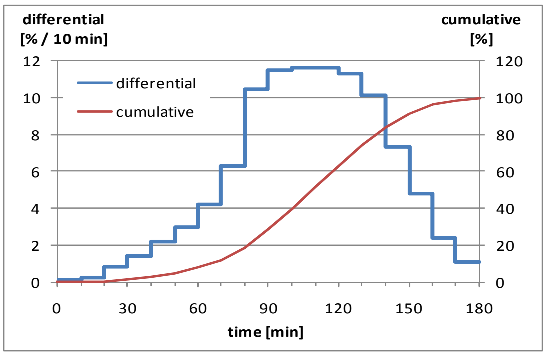

Figure 4. The basic model of the incoming passenger flow

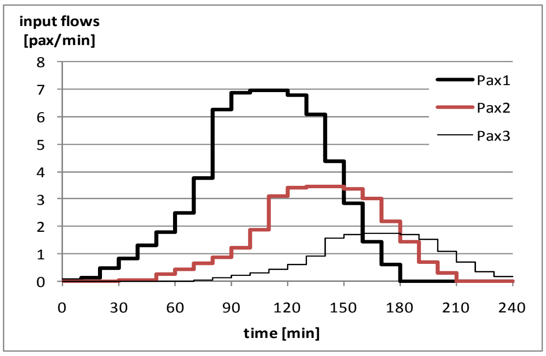

Figure 5. The rates of the three passenger flows

The first characteristic of organizing the check-in process is the assumption that every passenger group forms an own queue. The check-in desks working at the same time can serve passengers from all the three queues in the defined proportions (see queuing system structure on Figure 6). The second characteristic lies in the fact that the number of check-in desks working at the same time m is variable, representing the total amount of resources allocated for serving the passengers. Thus, the task of assessing the required amount of resources consists in determining m. The resource allocation task consists in determining the passenger serving rules for the m check-in desks working at the same time. Regardless of its characteristics, every concrete resource allocation algorithm combining these two tasks, should be oriented on completing the check-in of the passengers of every flow $i(i=1,2,3)$ by the point of time t_i^{end}, shown in Table 1. The mesoscopic approach assumes that the resource allocation algorithm is activated only at the so-called decision points. The given model suggests that these decision points occur in correspondence with the schedule, i.e. regularly with a 10 minutes interval.

5. Solution of the Modelling Task

The main target of the presented model is the illustration of application of the mesoscopic modelling principles, in particular, in terms of resource allocation of the analysed system. The given model can be implemented using simulation software for both continuous simulation (e.g. Vensim) and discrete-event simulation (e.g. AnyLogic or ExtendSim). It can also be created on the basis of a universal programming language, e.g. in MS Excel-VBA. The principle of flexible distribution of passengers between *m* concurrently operating check-in desks is considerably easier to implement with continuous simulation tools. A "real" mesoscopic model (i.e. a model with dynamic event planning for piecewise continuous processes) is best implemented with the Discrete Rate paradigm in the ExtendSim package or with the special mesoscopic simulation package MesoSim (Figure 7).

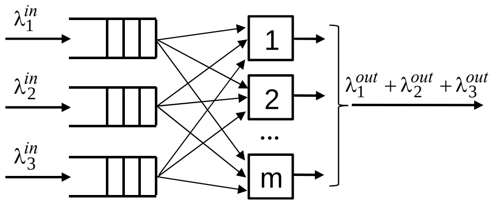

Figure 6. Conceptual model of the passenger check-in process

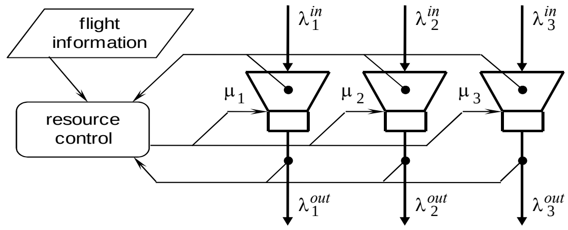

Figure 7. Executable mesoscopic model in MesoSim notation

In the following part of this paper two variants of the model will be shown, which are based on the principle of "allocating a resource proportional to a requirement for it", but which differ in the method of determining this requirement at the decision point, i.e. in the method of determining the variable *m* (see Table 2). When modelling variant 1 the resource allocation strategy is based on the measured value: current length of queue S_i (see position of measuring points in the model, shown on Figure 7). When modelling variant 2 the total planned quantity of passengers in the flow s_i^* is known and another measured value is considered: the number of served passengers s_i at the current moment *t*.

Table 2. Calculation formulas for the two resource control strategies

Description of the variables	Strategy 1	Strategy 2
Remaining time till the end of the check-in process (*t* – current time)	$\tau_i^{rest} = t_i^{end} - t$	
Number of passengers which still have to be checked-in (s_i^* - total planned number of passengers in the flow; s_i – number of checked-in passengers at time *t*)		$S_i^{rest} = s_i^* - s_i$
Required average check-in rate (S_i – length of the queue at time *t*)	$\mu_i^* = \dfrac{S_i}{\tau_i^{rest}}$	$\mu_i^* = \dfrac{S_i^{rest}}{\tau_i^{rest}}$
Resource needs of a flow *i* (τ_i – average check-in time of a passenger)	$m_i^* = \tau_i \cdot \mu_i^*$	
Total calculated demand (number of check-in desks before rounding)	$m^* = \sum_{i=1}^{3} m_i^*$	
Real amount of resources (number of check-in desks after rounding)	$m = Int(m^* + 1)$	
Auxiliary sum	$Q = \sum_{i=1}^{3}\left(m_i^* \cdot \tau_i\right)$	
Real check-in rate (throughput of a funnel channel *i*)	$\mu_i = \dfrac{m_i^* \cdot m}{Q} = \mu_i^* \cdot \dfrac{m \cdot \tau_i}{Q}$	

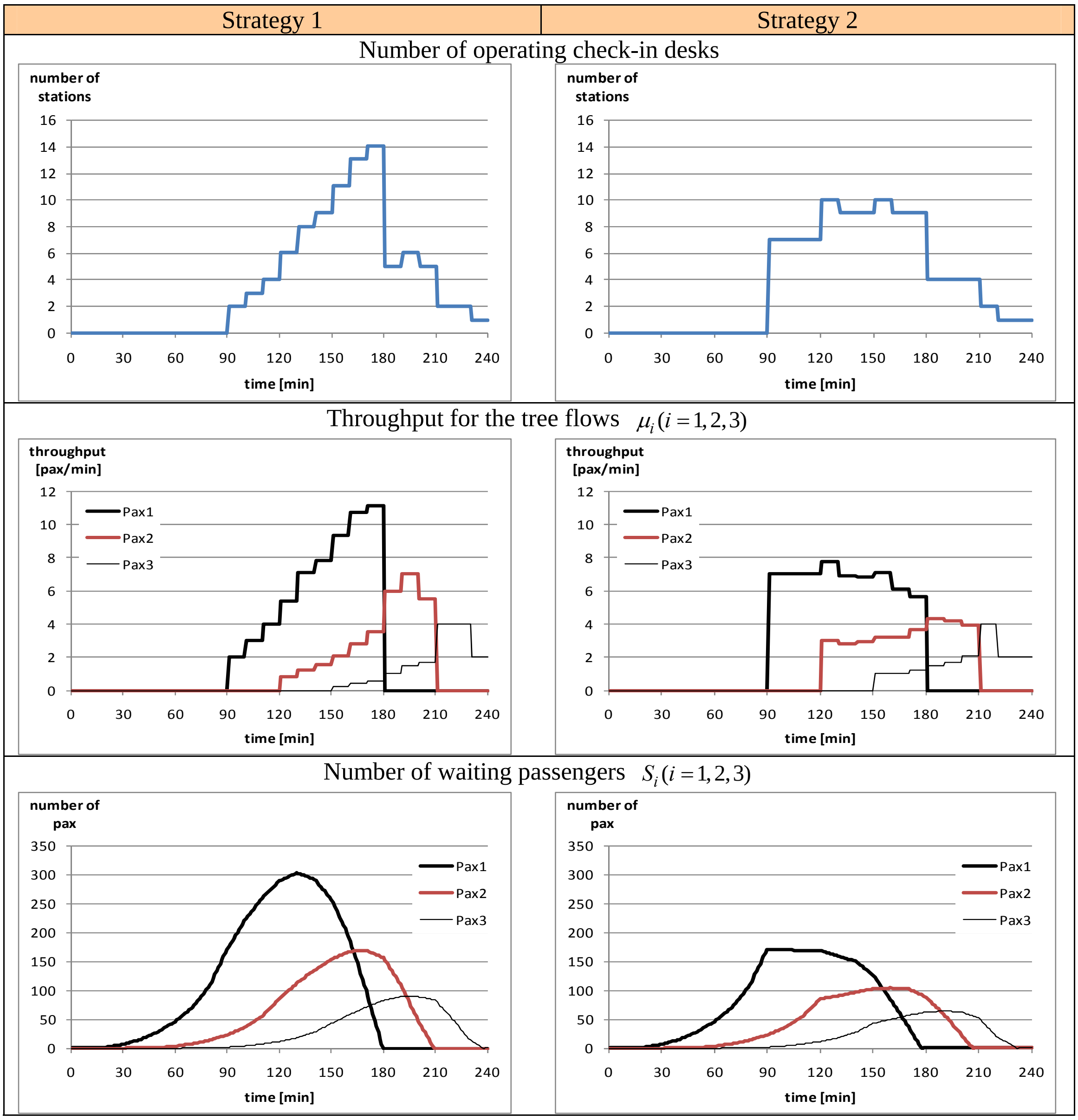

Figure 8. Mesoscopic modelling results

The results of modelling the two resource allocation strategies are very different from each other, nevertheless the basic condition, i.e. completing the check-in till t_i^{end}, is satisfied in both strategies (see Figure 8). Obviously, strategy 2 appears to be more efficient: the maximum length of every passenger queue is 40% lower than in strategy 1, while the maximum number of simultaneously operating check-in desks is reduced from 14 to 10.

6. Conclusions

The presented model has characteristics that are typical for the mesoscopic approach to modelling flow systems:

- the input flows of the system are not given with the help of distribution of time intervals between events, but numerically (graphically) in form of piecewise continuous intensity graphs;
- for the representation of heterogeneous flows the concept of “portions” is used (e.g. a group of passengers departing with the same flight), which is not defined in other modelling concepts;
- the service time of objects of the input flow is not given as a random variable, but as a constant equal to the average service time;

- the resource control (thus the flow control) is not done continuously, but only at the decision points;
- the resource allocation strategy is described not only with algorithms but to a great extend in form of analytical models (calculation formulas).

The time interval of 10 minutes was chosen only for the simplification of the model and results description. In mesoscopic models, any external and internal conditions for the occurrence of decision situations can be formulated. For example, a critical value for the length of one queue or for the total number of waiting passengers can be given. When reaching these thresholds immediate actions can be undertaken to increase the maximum throughput of the check-in process.

The advantages of a mesoscopic approach reveal themselves already at the stage of development of a conceptual model, since the developer has to operate with considerably more aggregated characteristics of resources and processed flows, compared to those of discrete event models. Mesoscopic models are not only more compact and clear than discrete event ones, they are also considerably faster processed on a computer, due to the fact that the number of modelled events can be reduced by several times. As a rule, they are quicker and more precise than continuous models, since they eliminate the known negative effects connected with the use of the fixed time interval Δt for the reproduction of processes in a model.

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