

Evaluation of new classes of definite integrals in closed form and its applications to eddy current testing

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ABSTRACT

The new classes of definite integrals

$$A_n(\gamma) = \int_0^{\infty} \frac{\cos \gamma x dx}{(\sqrt{x^2 + a^2} + x)^{2n-1}}, \quad B_{n,\nu}(b) = \int_0^{\infty} \frac{x^{\nu+1} J_{\nu}(bx) dx}{(\sqrt{x^2 + a^2} + x)^{2n-1}}, \quad (1)$$

where $n = 1, 2, 3, \dots$, $\nu = 0, 1, 2, \dots$, $J_{\nu}(z)$ is the Bessel function of the first kind of order ν , are evaluated in closed form. Only one particular case of these integrals, $B_{1,0}(b)$, was evaluated in closed form (see [1], §69.5). The method used in [1] suitable only for calculation of $B_{1,0}(b)$. General formulae for $B_{n,\nu}(b)$ and even particular cases for $A_n(\gamma)$ seem to be absent from the literature. For evaluation of integrals (1) we use the divergent integrals which converge in the sense of Abel, [2]. The integrals (1) are expressed through finite sum of elementary functions and modified Bessel functions. The integrals $A_1(\gamma)$ and $B_{1,0}(b)$ are used in eddy current testing for calculation of impedance of a double conductor line or impedance of a conductor coil in the cases, when the double line or the coil are situated on the surface of a uniform conducting half-space, [3]. The impedance Z is expressed through $A_1(\gamma)$ or $B_{1,0}(b)$ also for nonuniform conducting half-space in the cases, when the size of the flaw more less then the distance between the wires of the double line or more less then the radius of the coil.

References

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