Impedance Change of a Conducting Plate with an Arbitrary Form Flaw

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Abstract: A new exact analytical formula for the impedance change used in non-destructive testing problems is derived. The derivation is based on the Green's formula in contrast with the previous studies that used Lorentz theorem for obtaining the formula known in literature. The new formula for the impedance change has the form of a triple integral of scalar product of two vector potentials: the vector potential in the flaw and the vector potential in the same region in the absence of the flaw over the region containing the flaw. The similar formula obtained earlier by previous authors has the form of a triple integral of scalar product of amplitude electric field vectors. It is proved that previous formula is not correct in the general case. The new formula is derived by considering the influence of a conducting plate with a flaw of an arbitrary form on a closed contour with current. The contour has an arbitrary form and is located in the horizontal plane parallel to the surface of the conducting plate.

Key-Words: Vector potential, Impedance change, Dirac delta function

1 Introduction

The formula for the change in impedance used in literature (see [1], [2]) has the form

$$Z^{ind} = -\frac{(\sigma_F - \sigma)}{I^2} \iiint_{V_F} \vec{E} \cdot \vec{E}_F \, dV, \qquad (1)$$

where V_F is the region of the flaw, σ_F and σ are the conductivities of the flawed and flawless regions, respectively, $\vec{E}_F$ is the amplitude electric field vector in the flawed region, $\vec{E}$ is the amplitude electric field vector in the same region in the absence of the flaw, I is the amplitude current vector density.

It can be easily shown that formula (1) is not correct in the general case even for a single-turn coil lying on the plane that is parallel to the interface of two media. An exact analytical formula has been obtained for the change in impedance for an emitter of an arbitrary closed shape located on the plane parallel to the interface of two media, and it has the form

$$Z^{ind} = \frac{\omega^2 (\sigma_F - \sigma)}{I^2} \iiint_{V_F} \vec{A} \cdot \vec{A}_F \, dV, \qquad (2)$$

where $\vec{A}_F$ is the amplitude vector potential in the flawed region, $\vec{A}$ is the amplitude vector potential in the same region in the absence of the flaw, ω is the frequency.

The relationship between vectors $\vec{E}$ and $\vec{A}$ in the case of harmonic oscillations of the external current with frequency ω is given by (see [3]):

$$\vec{E} = -j\omega\vec{A} + \frac{1}{\tilde{k}^2} grad \; div\vec{A}, \qquad (3)$$

where $\tilde{k}^2 = \mu_0\mu(\sigma + j\varepsilon_0\hat{\varepsilon}\omega)$, ε_0 and μ_0 are the electric and magnetic constants, respectively; $\hat{\varepsilon}$ and μ are the relative permittivity and relative magnetic permeability of the medium, respectively; $j = \sqrt{-1}$ is the imaginary unit. Thus, formulae (1) and (2) coincide only if $div\,\vec{A} = 0$.

2 New Formula for impedance change

Consider a conducting plate of finite thickness d situated in region $V_1 = \{-\infty < x, y < +\infty, \; -d < z < 0\}$ containing a flaw in the region $V_F \in V_1$ (see Fig.1). Regions V_0 and V_2 are free space. The source of the current is located in region V_0 on a closed line described by the equation:

$$z = h, \quad \rho = \rho(\varphi), \quad 0 \le \varphi \le 2\pi, \qquad (4)$$

where ρ, φ, z are the cylindrical polar coordinates. One can also use the Cartesian coordinates x, y, z.

The current in the contour obeys the law

$$\vec{I}^{\,e} = I\delta[\rho - \rho(\varphi)]\delta(z - h)\vec{e}_\tau, \; 0 \le \varphi \le 2\pi, \qquad (5)$$

where $\delta(x)$ is the Dirac delta function, $\vec{e}_\tau$ is a unit vector of a tangent to the line (4).

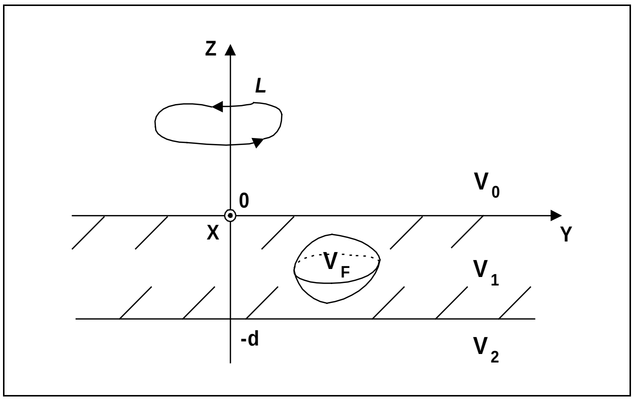

Fig.1. Contour L with current above a conducting plate V_1 containing an arbitrary form flaw in region V_F .

In this case, the complex-valued amplitude $\vec{A}(x,y,z)$ of the vector potential has three components $A_x(x,y,z)$, $A_y(x,y,z)$, $A_z(x,y,z)$ (see [3]):

$$\vec{A}(x,y,z)=\vec{A}_h(x,y,z)+A_z(x,y,z)\vec{e}_z\,, \qquad (6)$$

$$\vec{A}_h(x,y,z)=A_x(x,y,z)\vec{e}_x+A_y(x,y,z)\vec{e}_y\,. \qquad (7)$$

Since the source of current is situated in the horizontal plane, only the horizontal component of the vector potential, $\vec{A}_h(x,y,z)$, gives the contribution to the impedance. It is known that the problem for the horizontal component can be solved separately (see [3]). The solution of the problem for the vertical component is only needed in order to satisfy all the boundary conditions for the components A_x, A_y, A_z in the plane $z=0$. That is why it is not necessary to solve the problem for A_z .

Mathematical formulation of the problem for the horizontal component has the form:

$$\Delta\vec{A}_{0h}=-\mu_0\vec{I}^{\,e}\,, \quad -\infty<x,y<+\infty,\ 0<z<+\infty, \qquad (8)$$

$$\Delta\vec{A}_{1h}+k_1^2\vec{A}_{1h}=0,\ \ -\infty<x,y<+\infty,\ -d<z<0,\quad (x,y,z)\notin V_F\,, \qquad (9)$$

$$\Delta\vec{A}_{Fh}+k_F^2\vec{A}_{Fh}=0,\ \ -\infty<x,y<+\infty,\ -d<z<0,\quad (x,y,z)\in V_F\,, \qquad (10)$$

$$\Delta\vec{A}_{2h}=0, \qquad -\infty<x,y<+\infty,\ -\infty<z<-d, \qquad (11)$$

$$\Delta=\partial^2/\partial x^2+\partial^2/\partial y^2+\partial^2/\partial z^2\,,$$

with the corresponding boundary conditions on boundaries of ~~the~~ regions V_0, V_1, V_F and V_2. The problem for a non-uniform medium can be transformed into the problem for a uniform medium with a non-uniform right-hand side so that system (8)-(11) can be rewritten in the form

$$\Delta\vec{A}_{0h}=-\mu_0\vec{I}^{\,e}\,,\ -\infty<x,y<+\infty,\ 0<z<+\infty, \qquad (12)$$

$$\Delta\vec{A}_{1h}+k_1^2\vec{A}_{1h}= \begin{cases} 0, & (x,y,z)\notin V_F,\ -d<z<0,\\ (k_1^2-k_F^2)\vec{A}_{Fh}, & (x,y,z)\in V_F,\ -d<z<0, \end{cases} \qquad (13)$$

$$\Delta\vec{A}_{2h}=0\,,\ -\infty<x,y<+\infty,\ -\infty<z<-d, \qquad (14)$$

with the boundary conditions

$$z=0:\ \vec{A}_{0h}=\vec{A}_{1h},\quad \frac{\partial\vec{A}_{0h}}{\partial z}=\frac{\partial\vec{A}_{1h}}{\partial z}; \qquad (15)$$

$$z=-d:\ \vec{A}_{1h}=\vec{A}_{2h},\quad \frac{\partial\vec{A}_{1h}}{\partial z}=\frac{\partial\vec{A}_{2h}}{\partial z}; \qquad (16)$$

$$\begin{aligned} &x^2+y^2+z^2\to\infty\ (z>0):\quad \vec{A}_{0h}\to 0;\\ &x^2+y^2\to\infty\ (-d<z<0):\quad \vec{A}_{1h}\to 0;\\ &x^2+y^2+z^2\to\infty\ (z<-d):\quad \vec{A}_{2h}\to 0. \end{aligned} \qquad (17)$$

Note that $\vec{A}_{Fh}$ on the right-hand side of (13) is taken from the solution of problem (8) - (11) with the corresponding boundary conditions. Besides, the right-hand side of (13) is chosen so that by substituting $\vec{A}_{1h}=\vec{A}_{Fh}$ into (13), equation (13) is transformed into equation (10).

As the problem is linear, it is convenient to rewrite functions $\vec{A}_{0h}$, $\vec{A}_{1h}$ and $\vec{A}_{2h}$ in the form

$$\vec{A}_{0h}(x,y,z)=\vec{A}_{0h}^{absnt}(x,y,z)+\vec{A}'_{0h,ind}(x,y,z)+ +\vec{A}''_{0h,ind}(x,y,z), \qquad (18)$$

$$\vec{A}_{1h}(x,y,z)=\vec{A}'_{1h}(x,y,z)+\vec{A}''_{1h}(x,y,z), \qquad (19)$$

$$\vec{A}_{2h}(x,y,z)=\vec{A}'_{2h}(x,y,z)+\vec{A}''_{2h}(x,y,z), \qquad (20)$$

in order to obtain a formula for Z^{ind} . In the above,

$\vec{A}_{0h}^{absnt}$ is the solution of equation (12) in the absence of the conducting plate in region $-d<z<0$;

$\vec{A}'_{0h,ind}$ is the reaction of the conducting medium under the condition that the medium is uniform, i.e. $k_F=k_1$ (i.e. the right-hand side of (13) is equal to zero) , but $I\neq 0$;

$\vec{A}''_{0h,ind}$ is a contribution to the reaction of the conducting medium under the condition that the medium is non-uniform , i.e. $k_F\neq k_1$, but $I=0$ (in other words, $\vec{A}''_{0h,ind}$ is the solution of problem (12) - (17) for $I=0$, but $k_F\neq k_1$).

Similarly,

$\vec{A}'_{1h}$ is the solution of equation (13) for $k_F = k_1$, but $I \neq 0$;

$\vec{A}''_{1h}$ is the solution of equation (13) for $k_F \neq k_1$, but $I = 0$;

aAnd

$\vec{A}'_{2h}$ is the solution of equation (14) for $k_F = k_1$, but $I \neq 0$;

$\vec{A}''_{2h}$ is the solution of equation (14) for $k_F \neq k_1$, but $I = 0$.

It follows from the boundary conditions (15), (16) and formulae (18) - (20) that the following conditions are to be satisfied:

$$z = 0: \begin{cases} \vec{A}'_{1h} = \vec{A}^{absnt}_{0h} + \vec{A}'_{0h,ind}; & \vec{A}''_{1h} = \vec{A}''_{0h,ind}; \\ \dfrac{\partial \vec{A}'_{1h}}{\partial z} = \dfrac{\partial \vec{A}^{absnt}_{0h}}{\partial z} + \dfrac{\partial \vec{A}'_{0h,ind}}{\partial z}; & \dfrac{\partial \vec{A}''_{1h}}{\partial z} = \dfrac{\partial \vec{A}''_{0h,ind}}{\partial z}. \end{cases} \quad (21)$$

$$z = -d: \begin{cases} \vec{A}'_{1h} = \vec{A}'_{2h}; & \vec{A}''_{1h} = \vec{A}''_{2h}; \\ \dfrac{\partial \vec{A}'_{1h}}{\partial z} = \dfrac{\partial \vec{A}'_{2h}}{\partial z}; & \dfrac{\partial \vec{A}''_{1h}}{\partial z} = \dfrac{\partial \vec{A}''_{2h}}{\partial z}. \end{cases} \quad (22)$$

Since $\vec{A}^{absnt}_{0h}$ is the solution of the non-homogeneous equation (12), then functions $\vec{A}'_{0h,ind}$ and $\vec{A}''_{0h,ind}$ must be the solutions of the corresponding homogeneous equations

$$\Delta \vec{A}'_{0h,ind} = 0, \quad \Delta \vec{A}''_{0h,ind} = 0. \quad (23), (24)$$

Since $\vec{A}'_{1h}$ is the solution of (13) for $k_F = k_1$, then function $\vec{A}'_{1h}$ satisfies the equation

$$\Delta \vec{A}'_{1h} + k_1^2 \vec{A}'_{1h} = 0. \quad (25)$$

Since $\vec{A}'_{2h}$ is the solution of the homogeneous equation (14), then function $\vec{A}''_{2h}$ satisfies the equation

$$\Delta \vec{A}''_{2h,ind} = 0. \quad (26)$$

In the case of problem (8) - (11), (12) – (17) for the non-uniform medium, the change in impedance due to a flaw in the conducting medium has the form (see [3]):

$$Z^{ind} = \frac{j\omega}{I} \oint_L \vec{A}''_{0h,ind}(x,y,z)\, d\vec{l}. \quad (27)$$

Consider the Green's formula

$$\iiint_V (u\,\Delta v - v\,\Delta u)\, dV = \oiint_S \left(u \frac{\partial v}{\partial n} - v \frac{\partial u}{\partial n}\right) dS, \quad (28)$$

where S is the closed surface bounding region V, $\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$, $\vec{n}$ is the outer normal to the surface S, and the functions $u, v, \Delta u, \Delta v$ are continuous in the closed region. It is easy to prove that Green's formula (28) is valid for the two vector functions

$$\vec{A} = A_x(M)\vec{e}_x + A_y(M)\vec{e}_y + A_z(M)\vec{e}_z$$

$$\vec{B} = B_x(M)\vec{e}_x + B_y(M)\vec{e}_y + B_z(M)\vec{e}_z$$

in the Cartesian coordinates, and it has the form

$$\iiint_V (\vec{A}\Delta\vec{B} - \vec{B}\Delta\vec{A})\, dV = \oiint_S \left(\vec{A}\frac{\partial \vec{B}}{\partial n} - \vec{B}\frac{\partial \vec{A}}{\partial n}\right) dS. \quad (29)$$

In order to prove formula (29), it is sufficient to write formula (28) for three pairs of projections (A_x, B_x), (A_y, B_y), (A_z, B_z) and to sum the results.

I. Consider region $-d < z < 0$. By taking the scalar product of (13) with $\vec{A}'_{1h}$ and (25) with $\vec{A}_{1h}$, and by subtracting the first product from the second one, one obtains

$$\vec{A}_{1h}\Delta\vec{A}'_{1h} - \vec{A}'_{1h}\Delta\vec{A}_{1h} = -(k_1^2 - k_F^2)\vec{A}'_{1h}\vec{A}_{Fh}. \quad (30)$$

Integrating (30) over the region $-d < z < 0$ one obtains

$$\iiint_{-d<z<0} (\vec{A}_{1h}\Delta\vec{A}'_{1h} - \vec{A}'_{1h}\Delta\vec{A}_{1h})\, dV =$$

$$= -(k_1^2 - k_F^2)\iiint_{V_F} \vec{A}'_{1h}\vec{A}_{Fh}\, dV. \quad (31)$$

Using Green's formula (29) yields

$$\oiint_S \left(\vec{A}'_{1h}\frac{\partial \vec{A}_{1h}}{\partial n} - \vec{A}_{1h}\frac{\partial \vec{A}'_{1h}}{\partial n}\right) dS =$$

$$= (k_1^2 - k_F^2)\iiint_{V_F} \vec{A}_{Fh}\vec{A}'_{1h}\, dV, \quad (32)$$

where S is the closed surface of the integration (see Fig.2.).

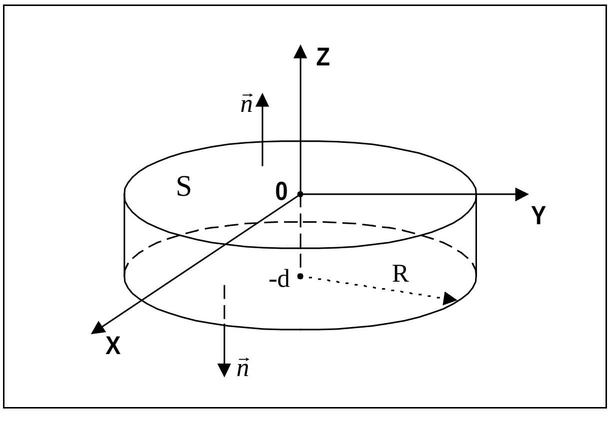

Fig.2. Closed surface S of integration ($-d < z < 0$).

Since $\vec{A}_{1h}$, $\vec{A}'_{1h} \to 0$ as $R^2 = x^2 + y^2 \to \infty$ ($-d < z < 0$), then instead of the surface integral over the closed surface S, only double integral over planes $z = 0$ and $z = -d$ remains on the left-hand side of (32):

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left(\vec{A}'_{1h}\frac{\partial \vec{A}_{1h}}{\partial z} - \vec{A}_{1h}\frac{\partial \vec{A}'_{1h}}{\partial z}\right)\Bigg|_{z=0} dxdy \; - \; \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left(\vec{A}'_{1h}\frac{\partial \vec{A}_{1h}}{\partial z} - \vec{A}_{1h}\frac{\partial \vec{A}'_{1h}}{\partial z}\right)\Bigg|_{z=-d} dxdy = (k_1^2 - k_F^2)\iiint_{V_F} \vec{A}_{Fh}\vec{A}'_{1h}\, dV \,. \quad (33)$$

The minus sign at the second term on the left-hand side of (33) is used since the outer normal $\vec{n}$ to the surface $z = -d$ is in the negative *z*-direction. By using the decomposition (19) for $\vec{A}_{1h}$ and the boundary conditions (15) at $z = 0$, one can perform the transformations

$$z = 0: \; \vec{A}'_{1h}\frac{\partial \vec{A}_{1h}}{\partial z} - \vec{A}_{1h}\frac{\partial \vec{A}'_{1h}}{\partial z} = \vec{A}'_{1h}\frac{\partial \vec{A}''_{1h}}{\partial z} - \vec{A}''_{1h}\frac{\partial \vec{A}'_{1h}}{\partial z} = \vec{A}'_{0h}\frac{\partial \vec{A}''_{0h,ind}}{\partial z} - \vec{A}''_{0h,ind}\frac{\partial \vec{A}'_{0h}}{\partial z}, \quad (34)$$

where $\vec{A}'_{0h} \equiv \vec{A}^{absnt}_{0h} + \vec{A}'_{0h,ind}$. Similarly, by using the decomposition (19) and the boundary conditions (16) at $z = -d$, one can obtain

$$z = -d: \; \vec{A}'_{1h}\frac{\partial \vec{A}_{1h}}{\partial z} - \vec{A}_{1h}\frac{\partial \vec{A}'_{1h}}{\partial z} = \vec{A}'_{1h}\frac{\partial \vec{A}''_{1h}}{\partial z} - \vec{A}''_{1h}\frac{\partial \vec{A}'_{1h}}{\partial z} = \vec{A}'_{2h}\frac{\partial \vec{A}''_{2h}}{\partial z} - \vec{A}''_{2h}\frac{\partial \vec{A}'_{2h}}{\partial z}\,. \quad (35)$$

Substituting (34) and (35) into (33) yields

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left(\vec{A}'_{0h}\frac{\partial \vec{A}''_{0h,ind}}{\partial z} - \vec{A}''_{0h}\frac{\partial \vec{A}'_{0h}}{\partial z}\right)\Bigg|_{z=0} dxdy \; - \; \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left(\vec{A}'_{2h}\frac{\partial \vec{A}''_{2h}}{\partial z} - \vec{A}''_{2h}\frac{\partial \vec{A}'_{2h}}{\partial z}\right)\Bigg|_{z=-d} dxdy = (k_1^2 - k_F^2)\iiint_{V_F} \vec{A}_{Fh}\vec{A}'_{1h}\, dV \,. \quad (36)$$

II. Consider region $z < -d$. One can perform the same transformations with equation (14) for $\vec{A}_{2h}$ and with $\Delta\vec{A}''_{2h} = 0$ in the region $z < -d$. As a result, one can obtain

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left(\vec{A}'_{2h}\frac{\partial \vec{A}''_{2h}}{\partial z} - \vec{A}''_{2h}\frac{\partial \vec{A}'_{2h}}{\partial z}\right)\Bigg|_{z=-d} dxdy = 0\,. \quad (37)$$

It follows from (37) that the second term on the left-hand side of (36) is equal to zero, and expression (36) takes the form

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left(\vec{A}'_{0h}\frac{\partial \vec{A}''_{0h,ind}}{\partial z} - \vec{A}''_{0h}\frac{\partial \vec{A}'_{0h}}{\partial z}\right)\Bigg|_{z=0} dxdy = (k_1^2 - k_F^2)\iiint_{V_F} \vec{A}_{Fh}\vec{A}'_{1h}\, dV \,. \quad (38)$$

III. Consider region $z > 0$. By performing the same transformations with equation (12) for $\vec{A}_{0h}$ and with $\Delta\vec{A}''_{0h,ind} = 0$ in the region $z > 0$, one can obtain

$$-\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left(\vec{A}_{0h}\frac{\partial \vec{A}''_{0h,ind}}{\partial z} - \vec{A}''_{0h,ind}\frac{\partial \vec{A}_{0h}}{\partial z}\right)\Bigg|_{z=0} dxdy = \mu_0 \iiint_{z>0} \vec{I}^{\,e}\vec{A}''_{0h,ind}\, dV \,. \quad (39)$$

The minus sign on the left-hand side of (39) is used because the outer normal to the surface $z = 0$ in region $z > 0$ is in the negative *z*-direction. By using decomposition $\vec{A}_{0h} = \vec{A}^{absnt}_{0h} + \vec{A}'_{0h,ind} + \vec{A}''_{0h,ind}$ and the boundary conditions (15) at $z = 0$, and by substituting (5) for $\vec{I}^{\,e}$, it follows from (39) that

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left(\vec{A}'_{0h}\frac{\partial \vec{A}''_{0h,ind}}{\partial z} - \vec{A}''_{0h}\frac{\partial \vec{A}'_{0h}}{\partial z}\right)\Bigg|_{z=0} dxdy = -\mu_0 I \iiint_{z>0} \delta[\rho - \rho(\varphi)]\delta(z-h)\vec{e}_\tau \vec{A}''_{0h,ind}\, dV \,, \quad (40)$$

where $\vec{A}'_{0h} = \vec{A}^{absnt}_{0h} + \vec{A}'_{0h,ind}$.

One can transform the right-hand side of (40) by using the main property of the delta function:

$$\iiint_{z>0} \delta[\rho - \rho(\varphi)]\delta(z-h)\vec{e}_\tau \vec{A}''_{0h,ind}\, dxdydz = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \delta[\rho - \rho(\varphi)]\vec{e}_\tau \vec{A}''_{0h,ind}\Bigg|_{z=h} dxdy \,. \quad (41)$$

By introducing the cylindrical polar coordinates $x = \rho\cos\varphi$, $y = \rho\sin\varphi$, $dxdy = \rho\, d\rho d\varphi$, it follows from (41) that

$$F \equiv \int_0^{2\pi} d\varphi \int_0^{\infty} \delta[\rho - \rho(\varphi)]\vec{e}_\tau \vec{A}''_{0h,ind}\Bigg|_{z=h} \rho\, d\rho = \int_0^{2\pi} \vec{A}''_{0h,ind}(\rho(\varphi), h)\, \vec{e}_\tau\, \rho(\varphi) d\varphi \,. \quad (42)$$

As $\vec{e}_\tau \rho(\varphi) d\varphi = \vec{e}_\tau dl = d\vec{l}$, (42) has the form

$$F = \oint_L \vec{A}''_{0h,ind}(\rho(\varphi), h)\, d\vec{l} = \frac{I}{j\omega} Z^{ind} . \qquad (43)$$

Thus,

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \left(\vec{A}'_{0h} \frac{\partial \vec{A}''_{0h,ind}}{\partial z} - \vec{A}''_{0h} \frac{\partial \vec{A}'_{0h}}{\partial z} \right)\Bigg|_{z=0} dxdy =$$

$$= \frac{I}{j\omega} Z^{ind} . \qquad (44)$$

Since the left-hand sides of (38) and (44) are equal, then the right-hand sides must be equal too, i.e.

$$\mu_0 I^2 Z^{ind} (j\omega)^{-1} = (k_1^2 - k_F^2) \iiint_{V_F} \vec{A}_{Fh} \vec{A}'_{1h}\, dV . \qquad (45)$$

Since $k_1^2 = j\omega\sigma_1\mu_{0,}\ k_F^2 = j\omega\sigma_F\mu_0$, then it follows from (45) that

$$I^2 \mu_0 Z^{ind} (j\omega)^{-1} =$$

$$= j\omega\mu_0 (\sigma_1 - \sigma_F) \iiint_{V_F} \vec{A}_{Fh} \vec{A}'_{1h}\, dV , \qquad (46)$$

or

$$Z^{ind} = \frac{\omega^2 (\sigma_F - \sigma_1)}{I^2} \iiint_{V_F} \vec{A}_{Fh} \cdot \vec{A}'_{1h}\, dV . \qquad (47)$$

i.e. the obtained formula is similar to formula (2).

Besides, it can be easily proved that formula (47) can be generalized to the case of *n* arbitrary form flaws present in the conducting plate.

Note that in the previous studies trying to prove the formula for impedance change, it is assumed that $div\,\vec{A} = 0$ in formula (3). Besides,

1) In [1] it is assumed that the scalar potential gives change in the static field only. That statement is not true;

2) In [2] it is suggested to use the Coulomb's gauge $div\,\vec{A} = 0$. At the same time, the authors use the following equation for the vector potential $\vec{A}$:

$$\Delta\vec{A} + k^2 \vec{A} = \mu_0 \vec{I}^{\,ext} . \qquad (48)$$

That is not correct, because in the case of Coulomb gauge the equation for the vector potential is more complicated (see [3], p.10), and it has the form

$$\Delta\vec{A} = \mu_0\mu\sigma\left(\nabla\varphi + \frac{\partial\vec{A}}{\partial t}\right) + \mu_0\varepsilon_0\mu\varepsilon\frac{\partial}{\partial t}\left(\nabla\varphi + \frac{\partial\vec{A}}{\partial t}\right) -$$

$$- \mu_0\mu\vec{I}^{\,e} . \qquad (49)$$

3 Conclusions

A new formula for the change in impedance is obtained for the case of a closed emitter of arbitrary form located in a horizontal plane above a conducting plate with an arbitrary form flaw. The formula has the form of a triple integral over the region of the flaw of scalar product of two vector potentials: the vector potential in the flaw and the vector potential in the same region in the absence of the flaw. It is noted that the similar formula obtained in the previous papers is not correct in the general case.

References:

[1] Zaman A.J.M., Gardner C.G. and Long S.A., Change in impedance of a single turn coil due to a flaw in a conducting half-space, *J. Nondestr. Eval.*, Vol.3**,** No.1, 1982, pp. 37- 43.

[2] Satveli R., Moulder J.C., Wang B. and Rose J.H., Impedance of a coil near an imperfectly layered metal structure: The layer approximation, *J. Applied Phys*., Vol.79, No. 6, 1996, pp. 2811-2821.

[3] Antimirov M.Ya., Kolyshkin A.A., Vaillancourt R, *Mathematical Models for Eddy Current Testing*, Montreal: Les Publication CRM, 1997.