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EQUILIBRIUM STABILITY OF DYNAMICAL SYSTEMS WITH FAST VARIABLES

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The stochastic averaging procedure is one of the most frequently used asymptotic methods for analysis of nonlinear oscillations (see, for example, [2],[3] and references there). To apply this method one should introduce a small positive parameter ε and split phase coordinates to two parts: fast motion $\{\varphi^\varepsilon(t)\}$ (called by "rotation") with inversely proportional to small parameter vector field and slow motion $\{r^\varepsilon(t)\}$ (called by "radial motion"). Both vector fields for radial motion and rotation are dependent not only on small parameter and phase coordinates but also are subjected to rapid switching by stochastic process $\xi(t/\varepsilon)$. Mostly in engineering application [2] the mathematical model of the above mentioned nonlinear oscillators has a form of system $\frac{dr^\varepsilon}{dt} = f(r^\varepsilon, \varphi^\varepsilon, \xi(t/\varepsilon), \varepsilon)$, $\frac{d\varphi^\varepsilon}{dt} = \frac{\omega}{\varepsilon} + g(r^\varepsilon, \varphi^\varepsilon, \xi(t/\varepsilon), \varepsilon)$, where f and g are continuous bounded functions with two continuous bounded derivatives according to phase variables $r \in \mathbf{R}^n$ and $\varphi \in \mathbf{\Phi}$, Φ is compact metric subset of the space $\mathbf{R}^m$ and $f(0, \varphi, \xi, \varepsilon) \equiv 0$. In this paper we assume that $\xi(t)$ is homogeneous ergodic Markov process with compact phase space $\mathbf{\Xi}$, invariant measure $\mu(d\xi)$ and weak infinitesimal operator Q. Let us remember [3] that stochastic averaging procedure for function $v(\xi, t)$ defines as an averaging by time and by measure $\mu(d\xi)$. We will denote this procedure by to lines over averaged function, that is, $\bar{\bar{v}} = \lim_{t\to\infty} \frac{1}{t} \int_0^t \int_{\mathbf{\Xi}} v(\xi, \omega t) d\xi dt$. As it has been proven in [1] the solutions of averaged equation for radial motion $\frac{dx}{dt} = \bar{\bar{f}}(x)$, where $\bar{\bar{f}}(r) = \overline{\overline{f(r, \cdot, \cdot, 0)}}$ are close approximations of the corresponding solutions of initial equations. That is, $\lim_{\varepsilon\to 0} \mathbb{P}_{\xi,y}\{\sup_{s\le t\le s+T} |x^\varepsilon(s+t) - x(t)| > c\} = 0$ for any $c > 0, s \in \mathbb{R}, T > 0$, $\xi \in \mathbf{\Xi}, y \in \mathbb{Y}$. Besides for sufficiently small ε stochastic equilibrium stability of initial radial motion follows equilibrium asymptotic stability of averaged equation. If averaged vector field for radial motion is identically equal to zero one can construct a diffusion approximation of radial motion as stochastic Ito equation $dx = a(x)dt + \sigma(x)dw(t)$ with shift $a(x)$ and diffusion $\sigma(x)$ defined by stochastic averaging procedure and potential of specially constructed compound Markov process on phase space $\mathbb{Y} \times \mathbf{\Xi}$ with weak infinitesimal operator $(\omega, \nabla_y) + Q$. The latter result efforts an opportunity to discuss a parametric resonance problem for linear stochastic oscillator [2] given by Mathieu type equation with diffusion parametric pump of frequency.

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