NUMERICAL SCHEMES FOR THE SOLUTION TO DIFFERENTIAL EQUATIONS DRIVEN BY SEMI-MARKOV PROCESSES 1

M. ANNUNZIATO

Dipartimento di Matematica e Informatica, Università degli Studi di Salerno
Via Ponte Don Melillo, 84084 Fisciano (SA), Italia
E-mail: manunzi@unisa.it

We present numerical schemes for the Liouville Master Equation (LME) associated to a stochastic process that results from the action of a semi-Markov process on first order differential equations. The LME is a system of hyperbolic PDE with both local and non-local boundary conditions. We show that equation in differential and integral form and give a proof for the existence and uniqueness of the solution for the integral form. We use numerical schemes for solving the differential [1; 2] and integral form [3]. We study stability by Fourier analysis, and show some numerical experiments on cases of practical application that confirms the theoretical findings.

REFERENCES


1Work partially supported by Istituto Nazionale di Alta Matematica, Gruppo Nazionale per il Calcolo Scientifico.
AN ANALYSIS OF SMOOTHING–INTERPOLATING PROBLEMS

S. ASMUSS 1,3, N. BUDKINA 2,3 and J. BREIDAKS 1

1 University of Latvia
Zellu street 8, Riga, LV-1002, Latvia
2 Riga Technical University
Meza street 1/4, Riga, LV-1048, Latvia
E-mail: svetlana.asmuss@lu.lv, budkinanat@gmail.com, juris.breidaks@csb.gov.lv
3 Institute of Mathematics and Computer Science of University of Latvia
Rainis blvd. 29, Riga, LV-1459, Latvia

The talk deals with the smoothing – interpolating (smoothing for a part of data and interpolating for the rest) problems in the abstract setting of a Hilbert space. Let $X$, $Y$ be Hilbert spaces and assume that linear operators $T : X \rightarrow Y$, $A_1 : X \rightarrow \mathbb{R}^n$ and $A_2 : X \rightarrow \mathbb{R}^m$ are continuous. We consider the conditional minimization problem

$$||Tx|| \longrightarrow \min_{x \in H} ,$$

where restrictions given by $A_1$ (interpolating conditions) and $A_2$ (smoothing conditions) describe the set $H \subset X$.

We use known results separately for the problems of pure interpolation and for the problems of pure smoothing. For a given vector $u \in \mathbb{R}^n$ in the case $H = H_1 = \{ x \in X : A_1 x = u \}$ a solution of (1) is a spline from the space $S(T, A_1) = \{ s \in X: <Ts, Tx> = 0 \text{ for all } x \in \text{Ker}A_1 \}$ (called the interpolating spline). For a given vector $v \in \mathbb{R}^m$ and parameters $\delta, \varepsilon_i > 0$, $i = 1, \ldots, m$, in the case $H = H_2$ or $H = H_3$, where

$$H_2 = \{ x \in X : |(A_2 x)_i - v_i| \leq \varepsilon_i, \ i = 1, \ldots, m \}, \quad H_3 = \{ x \in X : \sum_{i=1}^m ((A_2 x)_i - v_i)^2 \leq \delta \},$$

a solution of problem (1) is a spline (called the smoothing spline) from the space $S(T, A_2)$. It should be noted that some results proved for smoothing splines in the case $H = H_2$ are true also when $\varepsilon_i = 0$ for some $i$, i.e. the corresponding interpolation conditions are fulfilled.

In this talk we consider problem (1) with mixed interpolating and smoothing conditions: $H = H_1 \cap H_2$ or $H = H_1 \cap H_3$. The solutions of such problems belong to the space of splines $S(T, A_1 \times A_2)$. We call these splines mixed interpolating – smoothing splines by analogy with the solution of the following conditional minimization problem:

$$||Tx||^2 + \frac{1}{\omega}||A_2 x - v||^2 \longrightarrow \min_{x \in H_1} ,$$

where the initial data $u$ are interpolated and the initial data $v$ are smoothed. Note that problem (2), which to a certain extent is connected with problem (1) considered here, was investigated by different authors (A.Y. Bezhaev, V.A. Vasilenko, C. Conti, S.N. Kersey and others).

1This work is partially supported by the project 2009/0223/1DP/1.1.1.2.0/09/APIA/VIAA/008 of the European Social Fund and by the grant 09.1570 of the Latvian Council of Science.
SEGMENTATION OF EYE FUNDUS IMAGES FOR IDENTIFYING THE BLOOD VESSELS

G. BALKYS and G. DZEMYDA
Institute of Mathematics and Informatics
Akademijos str. 4, LT-08663 Vilnius, Lithuania
E-mail: Gediminas.Balkys@ktl.mii.lt, Dzemyda@ktl.mii.lt

Retinal (eye fundus) images are widely used for diagnostic purposes by ophthalmologists. The normal features of eye fundus images include the optic disc, fovea and blood vessels. Algorithms for identifying blood vessels in the eye fundus image generally fall into two classes: extraction of vessel information and segmentation of vessel pixels. Algorithms of the first group start on known vessel point and trace the vasculature structure in the image. Algorithms of the second group perform a binary classification (vessel or non-vessel, i.e. background) in accordance of some threshold. We focus here on the binarization [1] methods that adapt the threshold value on each pixel to the global/local image characteristics. Global binarization methods [2] try to find a single threshold value for the whole image. Local binarization methods [3] compute thresholds individually for each pixel using information from the local neighborhood of the pixel.

In this paper, we modify and improve the local Sauvola method [3] by extending its abilities to be applied for eye fundus pictures analysis. In the original Sauvola algorithm, the threshold is defined as follows: 
\[ t(x, y) = \mu(x, y)[1 + k(\sigma(x, y)/R)] \]
where \( \mu(x, y) \) is the local mean and \( \sigma(x, y) \) is the standard deviation of pixel intensities in some window centered around the pixel \((x, y)\), \( R \) is the maximum value of the standard deviation. Parameter \( k \) controls the value of the threshold in the local window. The authors of [3] set the value of \( k \) to be the same for all pixels of the picture. In the improved Sauvola algorithm, we suggest to select \( k \) depending on pixel \((x, y)\). The experiments show the advantage of the modification when analyzing images having parts of different brightness. Example of eye fundus image analysis is presented in Fig. 1.

Fig. 1. Identifying the blood vessels: a) original image, b) extracted vasculature.

REFERENCES

EXPLICIT GENERAL LINEAR METHODS IN
NORDSIECK FORM WITH LARGE STABILITY
REGIONS

Z. BARTOSZEWSKI and Z. JACKIEWICZ

1Faculty of Applied Mathematics and Physics, Gdansk University of Technology, Poland,
2Department of Mathematics, Arizona State University, USA,
3AGH University of Science and Technology, Krakow, Poland

G. Narutowicza 11/12, 80-289, Gdansk, Poland,
Tempe, Arizona, 85287, USA,
al. Mickiewicza 30, 30-059 Krakow, Poland

E-mail: zbart@pg.gda.pl, jackiewi@math.la.asu.edu

We describe the construction of explicit Nordsieck methods for ordinary differential equations with large regions of absolute stability. These methods are obtained by minimizing the objective function for the negative area of the intersection of the region of absolute stability with a negative half plane. Examples of such methods of order $p$ and stage order $q = p$ will be presented for $p = 2, 3$ and 4 and their performance demonstrated on a number of numerical examples.
ON THE ESTIMATION OF THE STABLE LAWS

I. BELOVAS\textsuperscript{1,2} and V. STARIKOVIČIUS\textsuperscript{2}

\textsuperscript{1}Institute of Informatics and Mathematics
Akademijos 4, LT-08663, Vilnius, Lithuania
E-mail: igor\_belov@takas.lt

\textsuperscript{2}Vilnius Gediminas Technical University
Saulėtekio al. 11, LT-10223, Vilnius, Lithuania
E-mail: vs@vgtu.lt

Stable laws have a wide sphere of application: probability theory, physics, electronics, economics, sociology [6]. Particularly important role they play in financial mathematics, since the classical models of financial market, which are based on the hypothesis of the normality, often become inadequate [5].

Fitting stable distributions to empirical data is a nontrivial task [1]. Applying the maximum likelihood method we need fast and reasonably accurate procedures for obtaining initial estimates. Such procedures based on empirical characteristic functions were proposed [3; 4]. However their accuracy is heavily dependent on the location of values of characteristic functions [2].

The problem of optimal positioning of these values encounters difficulties when tackled analytically. So we found purposeful the numerical and Monte-Carlo approach. It requires significant computing resources and thus application of parallel computing. We present and discuss the results of our research.

REFERENCES


SPECIAL SOLUTIONS OF HUXLEY EQUATION

L. BIKULCIENE, Z. NAVICKAS and M.S RAGULSKIS

Kaunas University of Technology
Studentu 50, LT-51368, Kaunas, Lithuania
E-mail: liepa.bikulciene@ktu.lt, zenavi@ktu.lt, minvydas.ragulskis@ktu.lt

Huxley differential equation is a core mathematical framework for modern biophysically based neural modeling. It is often useful to obtain a generalized solitary solution for fully understanding its physical meanings. There are many methods to solve this equation: the traditional approaches to this task are variational iteration method [1], the homotopy perturbation method, Adomian’s decomposition method and the tanh method [2]; however, many methods may sometimes fail or the solution procedure becomes complicated as degree of nonlinearity increases. The Exp-function method, proposed by He and Wu [3] seemed to be most promising for that purpose. Zhou [4] obtained solutions of Huxley equation using this method.

In this paper the conditions when solutions of Huxley equation
\[ y'' + by' = 2a^2(y^3 + \alpha y^2 + \beta y + \gamma) \] (1)
where \(a, b, \alpha, \beta, \gamma \in R\) are fixed parameters can be expressed in special form
\[ y(x) = \frac{A_1 \exp(\lambda_1(x - \nu)) + A_2 \exp(\lambda_2(x - \nu))}{B_1 \exp(\lambda_1(x - \nu)) + B_2 \exp(\lambda_2(x - \nu))} \] (2)
where parameters \(A_1, A_2, B_1, B_2\) depends on Cauchy conditions and the procedure of finding exact solutions will be presented.

An analytical criterion determining if a solution of a differential equation can be expressed in an analytical form comprising exponential functions is developed in [5]. The employment of this criterion does not only give an answer to the above-stated question but gives the structure of the solution so that one does not have to guess what the form of the solution is. The load of symbolic calculations is brought before the structure of the solution is identified. This is in contrary to the Exp-function type methods where the structure of the solution is first guessed, and then symbolic calculations are exploited for the identification of parameters.

REFERENCES

A GRADIENT DESCENT METHOD FOR AN OPTIMAL CONTROL PROBLEM OF CONDUCTIVE-RADIATIVE HEAT TRANSFER

K. BIRGELIS

Faculty of Physics and Mathematics, University of Latvia
Zellu st. 8, Riga LV-1002, Latvia
E-mail: k.birgelis@gmail.com

We consider a pde-constrained optimal control problem arising in glass fabric industry to optimally control heating up and cooling of glass fabric sheets in special type furnaces (see [1], [2]). In order to adequately model heat transfer processes at high temperatures, various physical phenomena must be encountered in the model – heat conduction, medium convection, heat radiation propagation. Unfortunately, simultaneous modeling of all these processes leads us to a class of hard pde-constrained optimal control problems.

In order to construct a minimizing sequence and calculate an optimal control-state pair, we use gradient descent (steepest descent) method equipped with a simple line search algorithm. Each optimization iteration involves solving of the state equation as well as adjoint linearized equation. We use a Newton like iterative method combined with pseudo transient continuation as globalization procedure to solve these equations.

REFERENCES

COMPARING SOLUTIONS OF HYPERBOLIC AND PARABOLIC HEAT CONDUCTION EQUATIONS FOR L-SHAPE SAMPLES

T. BOBINSKA\textsuperscript{1}, M. BUIKE\textsuperscript{1,2} and A. BUIKIS\textsuperscript{1,2}

\textsuperscript{1}Faculty of Physics and Mathematics, University of Latvia
Zellu Street 8, LV-1002, Riga, Latvia
\textsuperscript{2}Institute of Mathematics and Computer Science, University of Latvia
Raina Blvd 29, LV-1459, Riga, Latvia
E-mail: tabita.magdalena@gmail.com, mbuike@lanet.lv, buikis@latnet.lv

As it is well known, heat conduction in a solid body can be described by the well-known Fourier equation
\begin{equation}
\frac{\partial V}{\partial t} = a^2 \left( \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \right) + \frac{1}{c\rho} f(x, y, z, t), \quad a^2 = \frac{k}{c\rho}, \quad (1)
\end{equation}
where $V(x, y, z, t)$ is the temperature of the body and $f(x, y, z, t)$ denotes the density of the heat sources. But there are several physical situations, for example, intensive steel quenching (IQ), when equation (1) should be upgraded to a hyperbolic form
\begin{equation}
\tau_r \frac{\partial^2 V}{\partial t^2} + \frac{\partial V}{\partial t} = a^2 \left( \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \right) + \Psi(x, y, z, t), \quad (2)
\end{equation}
where $\tau_r$ is a relaxation time and
\[
\Psi(x, y, z, t) = \frac{1}{c\rho} \left( \tau_r \frac{\partial f(x, y, z, t)}{\partial t} + f(x, y, z, t) \right).
\]
Here we state initial-boundary value problems for both of these equations describing IQ process for an L-shape sample. By means of Green’s function method (see [1] – [3]) we find solutions of the problems and compare the rate of change of the temperature in a small neighbourhood of the initial time $t = 0$.

REFERENCES


\textsuperscript{1}This work was partially supported by ESF research projects 2009/0138/1DP/1.1.2.1.2/09/PIIA/VIAA/004, 2009/0223/1DP/1.1.2.0/09/APIA/VIAA/008 and by the Latvian Council of Science research project 09.1572.
SOME CASES OF THE MATHEMATICAL MODELING APPLICATIONS IN ORTHODONTIC STOMATOLOGY

S. BOSIAKOV and K. YURKEVICH

Belarusian State University
Nezavisimosti avenue 4, 220030, Minsk, Belarus
E-mail: bosiakov@bsu.by

The research of mechanical influence on orthodontic appliance on displacement tooth and their metering influence are modern problems of biomechanics [1]. The prediction of value, the direction of displacement and the rotation of the root of tooth under concentrated forces and force moment are also well known nowadays problems [2]. This work is devoted to the influence of rod of orthodontic appliance on the roots of tooth. This appliance is used for widening of upper jaw.

The analysis of efforts are made for the case when boundary value problems of the roots of tooth correspond to clamped support. The relations between concentrated force, force moment and length of rod, diameter of cross-section, direction cosines (which define the rod position in space) are found. Obtained results are used for estimation of the displacement of the root of the tooth in bone stock, and also they are used for analysis of stress-deformed condition of periodontal ligament.

Mathematical modeling of displacement and rotations of tooth root is performed on the base of expression, which define fine movements of solids in elastic medium. The displacement of bone stock indefinitely decreases while moving away from the tooth roots. Geometry of fang is described by the equations of two different elliptic hyperboloid. The equilibrium equation are formulated with references to poles, which are the strength centers of fang. Expressions for rigidity of bone stock with translational displacement and rotations of fang are obtained.

Modified expressions for fine displacement of solids are applied during the analysis of normal and tangential stresses which appear at periodontal ligament. The displacements coincide at the area of contact between periodont and fang. The displacements are equal to zero at the boundary of periodont and bone stock.

Expressions for translational displacement, rotations of tooth roots could be applied by stomatologist for evaluation of stresses, which appear at bone stock and periodont.

REFERENCES


NORDSIECK METHODS WITH QUADRATIC STABILITY FUNCTION

M. BRAŚ

Department of Applied Mathematics, AGH University of Science and Technology
30059 Kraków, Poland
E-mail: brasmich@wms.mat.agh.edu.pl

We derive sufficient conditions which guarantee that the stability polynomial of Nordsieck method for ordinary differential equations has only two nonzero roots. Examples of such methods up to order four are presented which are A- and L-stable. These examples were obtained by computer search using the Schur criterion applied to the quadratic factor of the resulting stability polynomials.
ON THE SECOND-ORDER QUADRATIC RATIONAL DIFFERENCE EQUATIONS

I. BULA

University of Latvia, Department of Mathematics
Zelļu 8, Riga, LV 1002, Latvia
Institute of Mathematics and Computer Science of University of Latvia
Raina bulv. 29, Riga, LV 1459, Latvia
E-mail: ibula@lanet.lv

Recently there has been a great interest in studying nonlinear and rational difference equations. Nonlinear difference equations of order greater than one are of paramount importance in applications where the \((n + 1)\)st generation (or state) of the system depends on the previous \(n\) generations (or states). One of the reasons for this is a necessity for some techniques which can be used in investigating equations arising in mathematical models describing real-life situations in population biology, economics, genetics, psychology, sociology etc.

The problems of second-order rational (but not quadratic) difference equations are described in [2]. In our talk we discuss about second-order quadratic rational difference equations in form

\[
x_{n+1} = \frac{\alpha + \beta x_n x_{n-1} + \gamma x_{n-1}}{A + B x_n x_{n-1} + C x_{n-1}}, \quad n = 0, 1, 2, \ldots,
\]

which are specially research in [1]. For example, we investigate the rational difference equation

\[
x_{n+1} = \frac{\alpha}{(1 + x_n) x_{n-1}}, \quad n = 0, 1, 2, \ldots
\]

and give some answers of Open Problems whose are formulated in [1]. We consider the boundedness nature of solutions, the stability of the equilibrium points, the periodic character of the equation, and the convergence to periodic solutions. We use some ideas from [3].

REFERENCES


SOLVABILITY OF THE BOUNDARY VALUE PROBLEMS FOR CERTAIN SELF-SIMILAR EQUATIONS

J. CEPĪTIS

University of Latvia
Zel̂ļu street 8, LV-1002, Rīga, Latvia
E-mail: janis.cepitis@lu.lv

Introducing the stream function in governing partial differential equations of mathematical models of various physical processes and choosing the self-similar variables frequently one obtains the boundary value problem for the self-similar ordinary differential equation or their system. As a rule these self-similar equations have quadratic non-linearity and depend on parameters. Wide range of investigations, among them qualitative investigations, during the century paid attention to such problems, initially from the classical works along the latest ones. It must be mentioned that the considering of non-Newtonian processes notably extends the area of exploration.

For example, using the boundary layer approximation for the investigations of laminar boundary layers on slender bodies of revolution in axial flow or free convection about a flat heated plate embedded in a porous medium, or etc. (see, [1]), we can obtain the boundary value problems on the half-axis for the equation

\[ f'''' + a_1(\alpha)ff'' + a_2(\alpha)f'f'' = 0, \]

where \( a_1 \) and \( a_2 \) are rational functions of parameter \( \alpha \). This form of equation includes the classical Blasius and Falkner-Skan equations. As the equation has the fourth order the most artless boundary value problems in this case is the de la Vallee-Poussin problems. Equations of such type arising in hydrodynamics explored also in works of Latvian physicists (for example, [2], [3]).

Some existence and uniqueness results with respect to values of the parameter for the aforesaid self-similar problems will be considered.

REFERENCES


---

1 This work was partially supported by ESF research project 2009/0223/1DP/1.1.1.2.0/09/APIA/VIAA/008 and by the Latvian Council of Sciences research project 09.1220
2D MICRO- AND MACROSCOPIC MODELS FOR SIMULATION OF VEHICULAR TRAFFIC

N. CHURBANOVA, B. CHETVERUSHKIN, I. FURMANOV and M. TRAPEZNIKOVA

Institute for Mathematical Modeling RAS
4 Merisskaya Square, Moscow 125047, Russia
E-mail: nata@imamod.ru

Mathematical modeling is the most economical tool for planning and designing new motorways, estimating their capacity and optimizing traffic control. There are two basic types of the traffic flow models [1]. The macroscopic models use the continuum approach and derive equations similar to the gas dynamics equations. The microscopic models treat vehicles as separate particles, which interact according to certain laws. Most of the present models are one-dimensional and do not account for parameter distribution across the road. The proposed paper reflects the progress of investigations [2] and is devoted to creation of original 2D macro- and microscopic models of multilane traffic to predict flows for the real road geometry.

The macroscopic model is constructed in a similar way as the Quasi-Gas-Dynamic System of Equations [3]. Additional mass fluxes guarantee solution smoothness on the reference distances. The following driving strategy is implemented: drivers aim at moving with the velocity providing traffic safety; they try to move to the lane either with a lower density or a higher velocity; drivers aim at the planned destination such as a road exit. Two last intentions are included in the model with the help of the transverse velocity equation.

The proposed microscopic model is based on the cellular automata theory [4]. In this case the computational domain is the two-dimensional grid. The number of cells across the road is equal to the number of lanes. Each cell may or may not contain one and only one particle, representing a car. Particles can skip from one cell to another, which in this case has to be vacant. Time and space are quantified, therefore the velocity and the acceleration are quantified too. The particle movement is performed under special laws incorporating stochastic observations.

Both macro- and microscopic models have been improved to simulate heterogeneous flows. A large number of test predictions have been performed: traffic jam evolution on the road, the traffic flow on multilane roads with entries and exits, the flow on the road with a local widening, multiphase flows. The results obtained demonstrated good agreement of the models in both qualitative and quantitative senses. Moreover the microscopic model was used for simulating traffic flows on a crossroad to optimize the traffic lights regime.

REFERENCES

PARALLEL PREDICTOR CORRECTOR ALGORITHMS FOR SYSTEMS OF PARABOLIC EQUATIONS ON GRAPHS

R. ČIEGIS and N. TUMANOVA

Vilnius Gediminas Technical University
Saulėtekio al. 11, LT-10223 Vilnius, Lithuania
E-mail: rc@vgtu.lt

We consider the system of Hodgkin–Huxley reaction–diffusion equations formulated on the edges of the graph:

$$\frac{\partial v^k}{\partial t} = \mu v^k_{xx} - g_1(m^k)^3 h^k(v^k - E_1) - g_2(n^k)^4(v^k - E_2) - g_3(v^k - E_3), \quad 0 < x < l_k, \quad k = 1, \ldots, K,$$

$$m^k_t = (1 - m^k)\alpha_m(v^k) - m^k\beta_m(v^k),$$

$$h^k_t = (1 - h^k)\alpha_h(v^k) - h^k\beta_h(v^k),$$

$$n^k_t = (1 - n^k)\alpha_n(v^k) - n^k\beta_n(v^k),$$

The equations for the transmembrane potential $v^k$ are coupled due to some conjugation conditions at branch points. Here the gating variables $m, h$ control the sodium current, $n$ is the controlling gate of potassium, $\mu$ is the diffusion coefficient, and the equilibrium potentials of the sodium, potassium and leakage currents are denoted by $E_j, j = 1, 2, 3$.

We construct and investigate two types of efficient parallel numerical algorithms for the solution of this problem. For both algorithms the basic template is obtained by using the implicit backward Euler scheme to approximate the system of differential equations. The system of nonlinear equations is linearized by decoupling the PDE and three systems of ODE with the Picard type iterations (we restrict to 2 iterations). Data distribution paradigm is used to distribute the discrete problem among processors. Metis tool is applied for a load balanced distribution of local tasks.

The first algorithm is based on a parallel direct solver of the obtained large systems of linear equations. The modified factorization algorithm is implemented locally on each processor and the reduced system of equations, the size of which is equal to the number of vertexes of the graph, is solved on the master processor. Then solutions on the branch points are distributed to all processors and the global solution is computed in parallel on all processors.

In the second algorithm, the predictor-corrector algorithm is implemented to decouple computations at each edge of the graph. In the predictor step the values of the solution at branch points are computed by using the explicit approximation of the conservation equations. Then equations on edges are solved in parallel by all processors, since these equations are independent. During the corrector step, the values of the solution at the branch points are recomputed by the implicit algorithm.

Results of computational experiments are presented and the efficiency of the proposed parallel algorithms is investigated.
ON APPROXIMATION OF VALUE FUNCTIONS FOR CONTROLLED DISCONTINUOUS RANDOM PROCESSES

S. DANILENKO and H. PRAGARAUSKAS

Vilnius Gediminas Technical University, Institute of Mathematics and Informatics
Sauletekio al. 11, LT-10223, Vilnius, Lithuania, Akademijos 4, LT-08 663, Vilnius, Lithuania
E-mail: svetlana.danilenko@vgtu.lt, pragarauskash@yahoo.com

We consider the rate of approximation of value functions for controlled discontinuous random processes by using piece-wise constant control policies. Such policies are important in numerical computations since on each time interval where a policy is constant we are dealing with a process without control what simplifies the calculation of value function and suboptimal policy.

The $d$-dimensional controlled process $X_t = X^{\alpha,s,x}_t$, $t \geq 0$, is a solution of Itô’s stochastic equation

$$X_t = x + \int_0^t b(\alpha_r, s + r, X_r) dr + \int_0^t \int c(\alpha_r, s + r, X_r - z, z) q(dr, dz)$$

defined on a stochastic basis $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})$ satisfying the usual conditions. Here, $q$ is a Poisson $(\mathbb{F}, \mathbb{P})$-martingale measure on Borel subsets of $[0; \infty) \times \mathbb{R}^d$ and $\alpha = \alpha_t$, $t \geq 0$, is a control policy, i.e. an $\mathbb{F}$-predictable random process with values in a separable metric space $A$. We assume that the coefficients $b : A \times [0, \infty) \times \mathbb{R}^d \to \mathbb{R}^d$ and $c : A \times [0, \infty) \times \mathbb{R}^d \times \mathbb{R}^d \to \mathbb{R}^d$ are bounded measurable functions continuous in $\alpha$ and Lipschitz-continuous $(t, x)$.

The value function is defined by

$$v(s, x) = \sup_{\alpha \in \mathfrak{A}} \mathbb{E}_\mathbb{P}(X^{\alpha,s,x}_T),$$

where $\mathfrak{A}$ is the class of all control policies and $g$ is a bounded real-valued Lipschitz-continuous function.

Let $\mathfrak{A}_h$ be the class of all control policies $\alpha \in \mathfrak{A}$ which are constant on each time interval $[kh, kh + h)$, $k = 0, 1, \ldots$. The value function for piece-wise constant control policies is given by

$$v_h(s, x) = \sup_{\alpha \in \mathfrak{A}_h} \mathbb{E}_\mathbb{P}(X^{\alpha,s,x}_T).$$

We give sufficient conditions under which

$$|v(s, x) - v_h(s, x)| \leq const h^\kappa$$

for some $\kappa \in (0, 1)$ and all $(s, x) \in [0, T] \times \mathbb{R}^d$.

This problem for controlled diffusion processes was considered in [1].

REFERENCES

MATHEMATICAL MODELS AND NUMERICAL ANALYSIS OF LASER DIODE PUMPED ACTIVELY AND PASSIVELY Q-SWITCHED SOLID-STATE LASERS

A. DEMENTJEV

Institute of Physics
Savanoriu pr. 231, LT-02300, Vilnius, Lithuania
E-mail: aldement@ktl.mii.lt

Exactly 50 years ago (on May 16, 1960) Theodore Maiman was the first to present a working laser. It was a solid-state (synthetic ruby crystal) laser (SSL) pumped by spring-shaped high-power flash lamp. The first gas Ne-Ne laser was also invented in 1960 and the first laser diode was built in 1962. The invention of lasers strongly changed our lives. They are working in CD and DVD players, printers, bar code readers, etc. Therefore, laser jubilee is widely celebrated in many countries this year.

SSLs have undergone a very impressive progress during these years and they are now most widely used lasers. For modeling of the generation dynamics of Q-switched lasers the so called point laser model (PLM) is usually used. The traveling wave model (TWM) was rarely used previously for mathematical modeling of generation dynamics due to significant difficulties of very long numerical simulations of giant pulse generation from the spontaneous noise level. Therefore, TWM was sometimes used for setting the more detailed background of PLM only [1].

In the paper, a hierarchy of theoretical semiclassical models of generation dynamics will be presented for actively and/or passively Q-switched SSL. The slowly varying amplitude (SVA) approximation provides a considerable simplification of an electromagnetic wave equation. Such approximation allows the detailed analysis of pulse amplification spatio-temporal dynamics in solid-state laser amplifiers taking into account the main parameters of active media. For lasers with a Fabry-Perot resonator an approximation of counter-propagating plane waves is often good enough. The plane wave approximation with its disregard of the transverse dependence considerably simplifies the system of coupled nonlinear partial differential equations governing the laser generation dynamics. The final two simplification steps are the usage of intensity plane waves [2] and the rate equation for the time-dependent only photons and population densities [1].

Results of numerical simulation of generation dynamics for the laser diode end-pumped actively and passively Q-switched SSL using point and traveling wave laser models will be reported. It is shown that widely used PLM cannot give accurate results when the generated waves are subjected to the strong amplitude modulations inside the resonator by active or passive elements [2].

REFERENCES


TRANSLATIONALLY INVARIANT TWO-PARTICLE DENSITY MATRICES

A. DEVEIKIS

Vytautas Magnus University
Vileikos 8, LT-44404, Kaunas, Lithuania
E-mail: a.deveikis@if.vdu.lt

The ab initio no-core nuclear shell-model approach based on using of wave functions for many particle system description is not the simplest one. The only quantities really needed for calculation of identical particle systems are translationally invariant density matrices [1].

The presented two-particle translationally invariant density matrices are defined as two-particle density matrices integrated over centre-of-mass position vector of two last particles and complemented with isospin variables [2]. In order to simplify shell-model calculations, the set of the quantum numbers responsible for unambiguous enumeration of the antisymmetric states contains the number of oscillator quanta, momentum, isospin and only one additional integer quantum number instead of quantum numbers of higher order groups, such as seniority.

The two-particle translationally invariant basis for construction of considered density matrices is obtained by projecting out the unexcited centre-of-mass subspace of the centre-of-mass Hamiltonian matrix eigenvectors. An arbitrary number of oscillator quanta can be involved. The two-particle translationally invariant density matrix may be expressed in terms of this considerable more simple basis instead of very sophisticated translationally invariant coefficients of fractional parentage:

\[
W_{E\Gamma J\Pi T, E'\Gamma' J\Pi T}^{elj\pi t, el'j'\pi t} = \sum_{(E \Delta \Pi T)} \langle (E \Delta \Pi T); ((el)_{-1}, elj\pi t)J''T''| | E\Gamma J\Pi T \rangle \times \langle (E \Delta \Pi T); ((el)_{-1}, el'j'\pi t)J''T''| | E'\Gamma' J\Pi T \rangle
\]  

The procedures for calculation of two-particle translationally invariant density matrices were implemented in computer code. The theoretical formulation have been illustrated by calculation of density matrices for higher excitations in the case of \( A = 6 \) nuclei. The new computer code proves to be quick, efficient, numerically stable, and produces results possessing only small numerical uncertainties.

REFERENCES


A MODEL FOR MULTI-PHASE 2D COMPOSITE MATERIALS OF ELLIPTIC SHAPE

M. DUBATOVSKAYA and S. ROGOSIN

Belarusian State University
Nezavisimosti ave 4, BY-220030, Minsk, Belarus
E-mail: marina.dubatovskaya@gmail.com; rogosin@gmail.com

For multi-phase (multi-component, heterogeneous) media it is characteristic that there are surfaces of discontinuity of the considered media properties (see, e.g. [1]). As a particular case of the multi-phase media one can consider composite materials with different characteristics of their components (see [2], [3]). Analytical methods applied at the study of a steady heat conduction in various types of composites are described in [4]. Several analytical formulas for the effective (macroscopic) conductivity tensor which are deduced by using different approaches based on the recent results in the theory of partial differential equations and complex analysis (see also [5] and references therein). We can mention here Clausius-Mossotti's formula (for small concentration of inclusions), Dykhne's formula (for symmetric composite with three components), generalized Keller-Dykhne formula (for self-dual two-phase system with arbitrary concentration of compact inclusions of one phase into another), Mityushev's formula (for circular inclusions in 2D composites).

Recently, attempts were made to get formulas for effective conductivity of 2D heterogeneous media and composite materials with components separated by the second order lines, namely hyperbolas and/or ellipses (see, e.g. [6], [7]).

In this report we consider a steady heat conduction in the multi-component heterogeneous media geometrically represented by different elliptical cylinders, embedded into each other. The method of integral equations in the form, developed in [7], is applied. Formulas for effective conductivity are obtained.

The work is partially supported by the Belarusian Fund for Fundamental Scientific Research.

REFERENCES

ON THE DIOPHANTINE EXPONENT OF A BINARY WORD

A. DUBICKAS

Vilnius University

Department of Mathematics and Informatics, Naugarduko 24, LT-03225, Vilnius, Lithuania

E-mail: arturas.dubickas@mif.vu.lt

Let \( \omega \) be an infinite binary word, i.e., an infinite sequence of two elements, say, 0 and 1. Then any (finite or infinite) string of consecutive symbols of \( \omega \) is called a factor of \( \omega \), and any factor of \( \omega \) starting from the first symbol of \( \omega \) is called a prefix of \( \omega \). The Diophantine exponent of \( \omega \) is defined as

\[
\text{Dio}(\omega) := \limsup_{n \to \infty} \frac{|u_n| + \tau_n |v_n|}{|u_n| + |v_n|},
\]

where the limit superior is taken over every prefix of \( \omega \) of the form \( u_n v_n^\tau \), where \( u_n \) (possibly empty) and \( v_n \) are the words over the alphabet \( A = \{0, 1\} \) for each \( n \geq 1 \) and \((\tau_n)_{n=1}^\infty \) is a sequence of real numbers satisfying \( \tau_n \geq 1 \) for \( n \geq 1 \) and \( |v_n^\tau| \to \infty \) as \( n \to \infty \). Note that \( \lim_{n \to \infty} |v_n^\tau| = \infty \) if say \( \lim_{n \to \infty} \tau_n = \infty \) or \( \lim_{n \to \infty} |v_n| = \infty \).

This quantity was recently introduced by Adamczewski and Bugeaud in [2]. It also appears in [1], [3], where it is explained (although it is not introduced explicitly) how it can be used to derive irrationality measures.

Clearly, for an ultimately periodic sequence \( \omega \), we have \( \omega = uv^\infty \), hence \( \text{Dio}(\omega) = \infty \). On the other hand, \( \text{Dio}(\omega) \geq 1 \) for every binary word \( \omega \). In [5] we prove the every real number \( \xi \geq 1 \) is the Diophantine exponent of some infinite binary word \( \omega \):

**Theorem 1.** Every real number \( \xi \geq 1 \) is the Diophantine exponent of some infinite word \( \omega \) over the alphabet \( \{0, 1\} \).

Earlier, it was proved that every real number \( \xi > 1 \) is a critical exponent of some infinite word [6] and that every real number \( \xi > 2 \) is a critical exponent of some binary infinite word [4].

**REFERENCES**


[4] J.D. Currie and N. Rampersad. For each \( \alpha > 2 \) there is an infinite binary word with critical exponent \( \alpha \). *Electronic J. Comb.*, **15** #N34, 5 p., 2008.


WEB SERVICE-BASED DATA MINING: LARGE MULTIDIMENSIONAL DATA VISUALIZATION

G. DZEMYDA, V. MARCINKEVIČIUS and V. MEDVEDEV

Institute of Informatics and Mathematics
Akademijos 4, LT-08663, Vilnius, Lithuania
E-mail: Dzemyda@ktl.mii.lt, VirgisM@ktl.mii.lt, Viktor.M@ktl.mii.lt

Interaction between humans and machines is one of the areas in computer science that has evolved a lot in the last years. Progresses and innovations are mainly due to increases in computer power and technology of interactive software. It is also the result of new ways of considering the interaction with computers and the role of computers in everyday life. Real data of natural and social sciences are often high-dimensional. So, it is very difficult to understand these data and extract patterns. One way for such understanding is to make visual insight into the analyzed data set using various approaches [1]. In this paper, we present an approach and architecture of the Web service-based data mining oriented to the multidimensional data visualization. The World Wide Web is the ideal platform to implement a service for visualization and to make this service available to customers. This paper focuses on visualization methods as a tool for the visual pattern recognition in large-scale multidimensional datasets. The proposed service simplifies the usage of visualization methods that are often very sophisticated and include a lot of the know-how of their developers.

Web services refer to a new breed of Web applications as self-contained, self-describing modular applications that can be published, located, and invoked across the Web [2], [3]. The proposed service simplifies the usage of three visualization methods and makes them wide-accessible [4], [5]: multidimensional scaling (MDS), relative MDS, diagonal majorization algorithm. The proposed Web service architecture for the multidimensional data visualization is a three-layer model. The client Interface and Data Visualization Components layers are the main parts of the system. Client responsibility is sending a data, which must be accepted, processed and returned from the visualization service.

Proposed a realization of the service receives a multidimensional dataset and as a result produces a visualization of the dataset. It also supports different configuration parameters of the used data mining methods. These parameters allow the user to control the visualization process and to extract knowledge from their data set much more comprehensively. The proposed service for the visual data mining covers a cluster for parallel computation.

REFERENCES

ALGEBRAIC OPERATIONS WITH RELATIONS ON A SET AND CORRESPONDING ROUGH APPROXIMATION OPERATORS

A. ELKINS

Department of Mathematics
Zellu iela 8, Riga LV-1002, Latvia
E-mail: acerfromriga@gmail.com

Given an arbitrary binary relation $\rho \subseteq X \times X$ on a set $X$, we consider operators of upper and lower approximation $u_{\rho} : 2^X \to 2^X$ and $l_{\rho} : 2^X \to 2^X$, defined respectively by $u_{\rho}(A) = \{x \in X \mid \rho(x) \cap A \neq \emptyset\}$ and $l_{\rho}(A) = \{x \in X \mid \rho(x) \subseteq A\}$ where $A \subseteq X$ and $\rho(x) = \{x' \in X \mid (x, x') \in \rho\}$. Note that such operators in case when $\rho$ is an equivalence relation were first introduced by Z. Pawlak [1] and further under various assumptions on $\rho$ were studied by different authors, see e.g. [2], [3], et. al.

We study the following problem. Given a family of relations $F = \{\rho_i \subseteq X \times X \mid i \in I\}$ we investigate how do the set theoretical operations on this family influence the resulting relation. In particular we show that the following interrelations are valid:

1. If $\rho = \bigcap_{i \in I} \rho_i$, then $\bigcap_{i \in I} l_{\rho_i}(A) \subseteq l_{\rho}(A)$ and $\bigcap_{i \in I} u_{\rho_i}(A) \supseteq u_{\rho}(A)$;
2. If $\rho = \bigcup_{i \in I} \rho_i$, then $\bigcap_{i \in I} l_{\rho_i}(A) = l_{\rho}(A)$ and $\bigcup_{i \in I} u_{\rho_i}(A) = u_{\rho}(A)$;
3. If $\rho = \rho_1 \setminus \rho_2$, then $l_{\rho}(A) \supseteq l_{\rho_1}(A) \setminus l_{\rho_2}(A)$ and $u_{\rho}(A) \supseteq u_{\rho_1}(A) \setminus u_{\rho_2}(A)$;
4. If $\rho = \rho_1 \circ \rho_2$, then $l_{\rho}(A) = l_{\rho_1}(l_{\rho_2}(A))$ and $u_{\rho}(A) = u_{\rho_1}(u_{\rho_2}(A))$

REFERENCES

PARALLEL COMPUTING AND MATLAB TO SOLVE MULTIPLE CRITERIA OPTIMIZATION PROBLEMS

E. FILATOVAS and O. KURASOVA

Institute of Informatics and Mathematics
Akademijos 4, LT-08663, Vilnius, Lithuania
E-mail: ernest.filatov@gmail.com, kurasova@ktl.mii.lt

Matlab Parallel Computing Toolbox lets us solve computationally and data-intensive problems on multicore computers and computer clusters. In this investigation, we solve a time-consuming multiple criteria optimization problem (MOP): the optimal selection of feed ingredients in cattle-breeding [1], [2]. An user-friendly decision support system has been designed for interactive multiple criteria optimization in Matlab. In order to parallelise a solving process of the optimization problem, Parallel Computing Toolbox has been used [3], [4].

In our problem, the cost price $Φ(x_1, ..., x_n)$ as well as the violations of requirements of the nutritive characteristics $Ψ_j(x_1, ..., x_n)$, $j = 1, m$ values must be minimized (where $x_i$ is a constituent part of the $i$th ingredient in feed; $n$ is the number of ingredients; $m$ is the number of nutritive characteristics in feed). The criteria $Φ$ and $Ψ$ are contradictory, - with an increase in violation of the permissible amount of nutritive characteristics the price of feed is falling. The most widely spread method, weighted-sum approach, is used to solve the MOP: different coefficients values of the criteria are selected and single criteria optimization problems are solved by the processors of the computer cluster.

The goal of this research is to investigate the efficiency of the computer cluster, solving the multiple criteria optimization problem interactively, and to assess convenience of the designed decision support system for decision maker.

REFERENCES

EXACT SOLUTIONS OF EINSTEIN’S EQUATION FOR SOLITARY ELECTROMAGNETIC WAVES

D. FUNARO

Dipartimento di Matematica - Università di Modena
Via Campi 213/B, 41125 Modena, Italy
E-mail: funaro@unimo.it

We show exact solutions of the full Einstein’s equation when the right-hand side is the electromagnetic stress tensor. These were firstly presented in [1] and then generalized in [2]. We begin with defining the fields representing a solitary vector wave with compact support. Successively, using the expression of these fields, we build the corresponding energy tensor with respect to a certain unknown metric tensor. Such a metric tensor is assumed to have a simple form (only six entries different from zero). This is sufficient to recover an exact solution without complicating the computations too much. By imposing the Ricci curvature tensor to be equal to the right-hand side, we end up with a rather complex system that we are able to solve, through symbolic manipulation, in order to find out the metric tensor.

In this way we can understand how the space-time is deformed by the passage of the wave and draw interesting conclusions.

REFERENCES

ON A SIXTH ORDER MAXIMUM PRINCIPLE

T. GARBUZA

Daugavpils University
Parades str. 1, Daugavpils, LV-5400, Latvia
E-mail: garbuza@inbox.lv

Our motivation goes from the paper by Dunninger [1] where maximum principle was stated for the 4-th order differential inequalities. The result states that if the inequalities

\[ u^{(4)}(t) \geq 0, \quad u \in C^{(4)}[a, b], \]
\[ u(a) \geq 0, \quad u'(a) \geq 0, \quad u(b) \geq 0, \quad u'(b) \leq 0 \]  

(1)

hold then \( u(t) \) cannot have a negative minimum.

Similarly, if

\[ u^{(4)}(t) \leq 0, \quad u \in C^{(4)}[a, b], \]
\[ u(a) \leq 0, \quad u'(a) \leq 0, \quad u(b) \leq 0, \quad u'(b) \geq 0 \]  

(2)

hold then \( u(t) \) cannot have a positive maximum.

The results were applied then for the fourth order BVP

\[ x^{(4)}(t) = f(t, x), \quad t \in [a, b], \]
\[ x(a) = 0, \quad x'(a) = 0, \quad x(b) = 0, \quad x'(b) = 0. \]  

(3)

Functions \( u(t) \) were identified with \( x - \beta \) and \( x - \alpha \), where \( \alpha \) and \( \beta \) are the lower and upper functions, and the respective estimates \( x \leq \beta \) and \( x \geq \alpha \) were obtained.

We consider the sixth order ordinary differential inequalities and prove a sixth order maximum principle. We use it to prove a priori estimates for the sixth order nonlinear boundary value problems.

REFERENCES

CLASSIFICATIONS OF NONTRIVIAL ZEROS OF THE Riemann ZETA-FUNCTION

R. GARUNKŠTIS

Vilnius university
Naugarduko 24, Vilnius, Lithuania
E-mail: ramunas.garunkstis@mif.vu.lt

Let $0 < \alpha \leq 1$. The Hurwitz zeta-function is defined by the following Dirichlet series

$$
\zeta(s, \alpha) = \sum_{m=0}^{\infty} \frac{1}{(m + \alpha)^s}.
$$

For $\alpha = 1$ we get the famous Riemann zeta-function $\zeta(s) = \zeta(s, 1)$. Nontrivial zeros of $\zeta(s)$ lie in the strip $0 < \Re s < 1$. The Riemann hypothesis claims that all these zeros are on the critical line $\Re s = 1/2$. In [1] we introduced the classification of nontrivial zeros of $\zeta(s)$ based on trajectories of zeros of $\zeta(s, \alpha)$ in the respect of the parameter $\alpha$. In this talk we consider other classifications. One example is the classification of nontrivial zeros of the Riemann zeta-function based on trajectories connecting zeros of the $\zeta(s)$ with the zeros of the derivative of $\zeta(s)$.

REFERENCES

IN VolVING FUZZY ORDER IN THE DEFINITION OF MONOTONICITY FOR THE AGGREGATION PROCESS\textsuperscript{1}

O. GRIGORENKO

University of Latvia
Raina bulvāris 19, Rīga LV-1586, Latvia
E-mail: ol.grigorenko@gmail.com

Since the introduction of the concept of a fuzzy set by L. A. Zadeh \cite{4} and its generalization by J. A. Goguen \cite{2}, fuzzy analogues of basic concepts of classical mathematics were introduced and investigated, fuzzy relations \cite{5} among them. In the last years theoretical results obtained in the theory of fuzzy relations were involved for solving problems of practical nature (see eg.\cite{1}). The aim of this work is to involve fuzzy order relation in the study of aggregation process (see eg.\cite{3}). Namely, we use the fuzzy order relation instead of the crisp order relation in the definition of monotonicity. Recall that aggregation function is a mapping satisfying boundary conditions and the condition of monotonicity. In this work we focus only on the condition of monotonicity and define the degree of monotonicity in the following way:

**Definition 1.** Let $f : [0, 1]^n \rightarrow [0, 1]$ be a function (aggregation function), $P : [0, 1]^2 \rightarrow [0, 1]$ be a fuzzy order relation and $\rightarrow_T$ the residuum corresponding to a t-norm $T : [0, 1]^2 \rightarrow [0, 1]$.

We define the degree of monotonicity for a function (aggregation function) $f$ w.r.t fuzzy relation $P$ and residuum $\rightarrow_T$ in the following way:

$$M_{P, \rightarrow_T}(f) = \inf_{x,y} (\land_i P(x_i, y_i) \rightarrow_T P(f(x), f(y))).$$

After giving main definitions we illustrate the introduced notions by examples and study the properties of aggregation functions which have a certain degree of monotonicity.

REFERENCES

\begin{itemize}
  \item \cite{1} U. Bodenhofer, J. Küng, S. Saminger. Flexible query answering using distance-based fuzzy relations. Lecture Notes in Artificial Intelligence 4342, TARSKI, 207–228, 2006.
\end{itemize}

\textsuperscript{1}This work was partially supported by ESF research project 2009/0223/1DP/1.1.1.2.0/09/APIA/VIAA/008.
NONLINEAR ASYMMETRIC OSCILLATIONS

A. GRITSANS and F. SADYRBAEV

Daugavpils University
Parades 1, LV-5400, Daugavpils, Latvia
E-mail: armands.gricans@du.lv, felix@latnet.lv

We consider a nonlinear asymmetric oscillator governed by the equation

\[ x'' = -\lambda f(x^+) + \mu g(x^-), \quad (i) \]

where \( x^+ = \max\{x, 0\} \), \( x^- = \max\{-x, 0\} \). Generally nonlinear functions \( f, g : [0, +\infty) \rightarrow [0, +\infty) \) are continuous and such that \( f(0) = g(0) = 0 \). The boundary conditions

\[ x'(a) = x'(b) = 0, \quad (ii) \]

are imposed. Spectrum is a set of \((\lambda, \mu)\) such that there exists a nontrivial solution of \((i), (ii)\). We consider the problem under the normalization conditions \(|x(a)| = \alpha\). Description of the spectrum is given in terms of the time-map functions associated with functions \( f \) and \( g \). Interesting and unusual properties of the spectrum are pointed out and discussed provided that different normalization conditions are applied.

REFERENCES


THE REPRESENTATION OF SOLUTIONS OF HAMILTON-JACOBI EQUATION WHEN THE HAMILTONIAN IS THE MAXIMUM OF LINEAR FUNCTIONS

G. GUDYNAS

Klaipeda University
Manto str. 84, Klaipeda, Lithuania
E-mail: gvgintaut@balticum-tv.lt

We define the fundamental solution and prove the representation formula for solutions of

\[ u_t + H(u_x) = 0 \]  \hspace{1cm} (1)

\[ u(0, x) = u_0(x) \]  \hspace{1cm} (2)

where \((t, x) \in S_T = \{(t, x) : t \in (0, T], x \in \mathbb{R}^n\}\). When hamiltonian has the form

\[ H(p) = \max_{i=1}^{m-1} ((a^i, p) + b_i), \]  \hspace{1cm} (3)

where \(a^i, p \in \mathbb{R}^n, b_i \in \mathbb{R}\).

The solution of (1),(2) in this case can be represented by formula

\[ u(t, x) = \min_{\xi \in \mathbb{R}^n} [u_0(\xi) + F(t, x, \xi)] \]

were

\[ F(t, x, \xi) = \begin{cases} \frac{b_i - b_{i+1}}{a_{i+1} - a_i} (x - \xi - a_i t) - b_i t, & \xi \in [x - a_{i+1} t, x - a_i t], \; i = 1 \div m - 1, \\ +\infty, & \xi < x - a_m t, \; \xi > x - a_1 t. \end{cases} \]

The similar structure of the solution (1),(2) for the hamiltonians (3) may be realized in \(\mathbb{R}^n\).

REFERENCES


LINEAR/LINEAR RATIONAL SPLINE INTERPOLATION

E. IDEON and P. OJA

Institute of Mathematics, University of Tartu
J.Liivi 2, 50409 Tartu, Estonia
E-mail: erge5@ut.ee, Peeter.Oja@ut.ee

For a strictly monotone function $f$ on $[a,b]$ we describe the construction of finding an interpolating linear/linear rational spline $S$ of $C^1$ smoothness and on each subinterval of the grid $a = x_0 < x_1 < \ldots < x_n = b$ of the form

$$S(x) = a_i + \frac{c_i(x - \xi_i)}{1 + d_i(x - \xi_i)}, \quad x \in [x_{i-1}, x_i], \quad 1 + d_i(x - \xi_i) > 0, \quad i = 1, \ldots, n,$$

using interpolation points $\xi_i = (x_{i-1} + x_i)/2, i = 1 \ldots, n$. In addition, the spline should satisfy two consistent boundary conditions $S(a) = \alpha, S(b) = \beta$ or $S'(a) = \alpha, S'(b) = \beta$. This leads to a nonlinear system for which we get the uniqueness of a solution.

It is known that $\|S - f\|_{\infty} = O(h^3)$, with uniform mesh $x_i = a + ih, i = 0, \ldots, n$, for the proof see, e.g., [1]. For the quadratic splines, the superconvergence results are known. We show that for the linear/linear rational splines we obtain $\|S(x_i) - f(x_i)\|_{\infty} = O(h^4)$. For $S_i = S(x_i), f_i = f(x_i), i = 0, \ldots, n$, using, e.g., boundary conditions

$$S(a) = f(a) - \frac{h^4}{128}\psi_0, \quad S(b) = f(b) - \frac{h^4}{128}\psi_n,$$

we get

$$S_i = f_i - \frac{h^4}{128}\psi_i + R \quad \text{with} \quad \psi_i = f_i\left(\frac{f'''}{f'}\right)_i - 3f_i\left(\frac{f'''}{f'}\right)_i, \quad i = 1, \ldots, n - 1,$$

$R = o(h^4)$ for $f \in C^4[a,b]$ and $R = O(h^{4+\alpha})$ for $f^{IV} \in \text{Lip } \alpha, \alpha \in (0,1]$. We show also superconvergence of order $h^3$ for the first derivative and of order $h^2$ for the second derivative of $S$ in certain points.

In our study we used the spline representation by spline values $S_i = S(x_i), i = 0, \ldots, n, S_i = S(\xi_i), i = 1, \ldots, n,$

$$S(x) = \bar{S}_i + \frac{4(S_i - \bar{S}_i)(\bar{S}_i - S_{i-1})(x - \xi_i)}{h(S_i - S_{i-1}) + 2(2S_i - S_{i-1} - \bar{S}_i)(x - \xi_i)}, \quad x \in [x_{i-1}, x_i].$$

Numerical examples support the obtained theoretical results.

REFERENCES

POWER CONSUMPTION IN PARALLEL COMPUTATIONS OPTIMIZING TOPOLOGY OF TRUSS STRUCTURES USING BRANCH AND BOUND

A. IGUMENOV and J. ŽILINSKAS

Institute of Mathematics and Informatics
Akademijos 4, LT-08663 Vilnius, Lithuania
E-mail: igumenov@gmail.com, julius.zilinskas@mii.lt

Topology optimization of truss structures [1] is considered. The problem is formulated as a combinatorial optimization problem [2], with the goal to minimize the mass of construction according to constraints. Branch and bound [3] technique is used to develop an algorithm for optimization of truss structures guaranteeing accuracy of solution. Such problems are computationally expensive, therefore distributed parallel computing is used to solve them.

In this talk results of computational experiments are presented. Problems of topology optimization are solved using distributed computations in a cluster of computers with quad-core processors - "grid.mii.lt. Power consumption as well as computational requirements are evaluated. Additional experiments have been performed to evaluate power consumption when computers are not loaded and when they are loaded with different number of processes of computations.

REFERENCES

CALCULATION OF HEAT EXCHANGE COEFFICIENT AS PER TEMPERATURE MEASUREMENTS INSIDE A SOLID

M. ILTINA and I. ILTINS

Riga Technical University
Meza Street 1, Riga, LV-1048, Latvia
E-mail: marijai@inbox.lv

To model heat exchange processes mathematically, one should know temperature distribution in a fixed moment of time and rules of heat exchange on boundary of a body. These rules often are definable by technical means, for instance, with measuring temperature of surface, or calculable, with knowing amount of supplied heat. If it is impossible, then boundary conditions must be calculated, by measuring temperature at an inner point of a body. One-dimensional temperature field is considered as follows:

\[
\frac{\partial t}{\partial \tau} = a^2 \left( \frac{\partial^2 t}{\partial x^2} + \frac{k-1}{x} \cdot \frac{\partial t}{\partial x} \right), \quad x \in [0; b],
\]

(1)

\[t(x, 0) = t_0,
\]

(2)

Boundary conditions are unexplored whereas a change of temperature at an inner point of a body is known.

\[x_1 \in (0, b), \quad t(x_1, \tau) = t_1(\tau).
\]

Thus, \(t(b, \tau)\) and heat exchange coefficient \(a\) should be calculated following boundary conditions of the third type:

\[
\lambda \frac{\partial t(b, \tau)}{\partial x} = \alpha (t_e(\tau) - t(b, \tau)),
\]

where \(t_e\) is temperature of environment. The task is solved, by using projection of temperature field in series among measured function \(t_1\) derivatives:

\[t(x, \tau) = \sum_{n=0}^{\infty} t_1^{(n)}(\tau) P_n(x).
\]

REFERENCES


ANALYSIS OF DELAY DIFFERENTIAL EQUATIONS USING THE LAMBERT FUNCTION

I. IVANOVIENĖ and J. RIMAS

Kaunas University of Technology
Studentų g. 50, LT-51368 Kaunas
E-mail: irma.paleviciute@stud.ktu.lt, jonas.rimas@ktu.lt

The method of finding of solutions of the scalar and matrix differential equations with the delayed argument, based on application of the Lambert function, is analyzed. Results obtained for the scalar equation are compared to the results obtained by the method of consecutive integration with application of Laplace transformation. Results received for the matrix equation are compared to the results received by the numerical algorithm of finding of solution of the differential equations with delayed argument (this algorithm (dde23) is presented in the Matlab package).
LOCAL ONE DIMENSIONAL DIFFERENCE SCHEME FOR PSEUDOPARABOLIC EQUATION WITH NONLOCAL CONDITIONS

J. JACHIMAVIČIENĖ and M. SAPAGOVAS

Institute of Informatics and Mathematics
Akademijos 4, LT-08663, Vilnius, Lithuania

E-mail: justina_jachimaviciene@yahoo.com, m.sapagovas@ktl.mii.lt

In this work we consider the two-dimensional nonlinear pseudoparabolic equation with nonlocal conditions

\[
\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \eta \left( \frac{\partial}{\partial t} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \right) + f(x, y, t), \quad t \geq 0, \quad 0 < x < 1, \quad 0 < y < 1,
\]

\[
\begin{align*}
\frac{u(t, 0, y) - u(t, 1, y)}{\tau} &= \gamma_1 \int_0^1 u(t, x, y) \, dx + \mu_1(y), \\
\frac{u(t, 1, y) - u(t, 0, y)}{\tau} &= \gamma_2 \int_0^1 u(t, x, y) \, dx + \mu_2(y), \\
\frac{u(t, x, 0) - u(t, x, 1)}{\tau} &= \mu_3(t), \quad u(t, x, 1) = \mu_4(t), \\
\frac{u(0, x, y) - \varphi(x, y)}{\tau} &= 0.
\end{align*}
\]

We investigate LOD difference scheme for the solution of this problem

\[
\begin{align}
\frac{u^{n+1/2} - u^n}{\tau} &= \Lambda_1 u^{n+1/2} + \frac{\eta}{2} \frac{u^{n+1/2} - \Lambda_1 u^n}{\tau/2} + \frac{1}{2} f_n, \\
\frac{u^{n+1} - u^{n+1/2}}{\tau} &= \Lambda_2 u^{n+1} + \frac{\eta}{2} \frac{u^{n+1} - \Lambda_2 u^{n+1/2}}{\tau/2} + \frac{1}{2} f_n.
\end{align}
\]

We rewrite the system (6)-(7) in the form

\[
u^{n+1} = Su^n + F,
\]

where matrix S is nonsymmetric matrix.

The main method for investigating the stability of difference scheme is analysis of spectrum structure for the transition matrix S.

REFERENCES


HIGHLY STABLE GENERAL LINEAR METHODS FOR
ORDINARY DIFFERENTIAL EQUATIONS

Z. JACKIEWICZ

Department of Mathematics, Arizona State University
Tempe, Arizona, 85287, USA
E-mail: jackiewi@math.la.asu.edu

We describe the construction of highly stable general linear methods (GLMs) for the numerical solution of ordinary differential equations (ODEs). We describe the construction of some classes of GLMs which are $A$-stable and $L$-stable using the Schur criterion, and algebraically stable methods using criteria proposed recently by Hill, Nonlinear stability of general linear methods, Numer. Math. 103(2006), 611-629, and Hewitt and Hill, Algebraically stable general linear methods and the $G$-matrix, BIT 49(2009), 93-111. We illustrate the results for the class of two-step Runge-Kutta methods with inherent Runge-Kutta stability for which one of the coefficient matrices is assumed to have a one-point spectrum. We also describe our search for algebraically stable methods in this class without imposing any restrictions on the coefficient matrices, and in the class of diagonally implicit Nordsieck methods. Part of these results was obtained in collaboration with M. Braš from AGH University of Science and Technology, Kraków, Poland, G. Izzo from University of Naples and R. D’Ambrosio from University of Salerno.
TWO–DIMENSIONAL DIFFUSION EQUATION WITH NONLOCAL BOUNDARY CONDITION

K. JAKUBĖLIENĖ and M. SAPAGOVAS

Institute of Informatics and Mathematics
Akademijos 4, LT-08663, Vilnius, Lithuania
E-mail: m.sapagovas@ktl.mii.lt, gibaite@gmail.com

We consider the two–dimensional parabolic equation
\[
\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + f(x, y, t), \quad 0 \leq x, y \leq 1, \quad 0 < t < T
\]
with boundary conditions
\[
\begin{align*}
    u(0, y, t) &= \mu_1(y, t), \\
    u(1, y, t) &= \mu_2(y, t), \\
    u(x, 1, t) &= \mu_3(x, t),
\end{align*}
\]
the nonlocal integral condition instead boundary condition
\[
u(x, 0, t) = \gamma(x) \int_0^1 \int_0^1 u(x, y, t) dx dy + \mu_4(x, t)
\]
and initial condition
\[
u(x, y, 0) = \varphi(x, y).
\]
We construct and investigate for this differential problem the implicit alternating direction method.

REFERENCES


NUMERICAL SIMULATION OF HEAT CONDUCTION IN COMPOSITE MATERIALS

G. JANKEVIČIUTĖ, R. ČIEGIS and O. SUBOČ

Vilnius Gediminas Technical University
Saulėtekio al. 11, LT-10223 Vilnius, Lithuania
E-mail: (gerda;rc;os)@vgtu.lt

In two dimensional rectangular region $D = (0, L_1) \times (0, L_2)$ we solve the nonlinear stationary problem, which describes a distribution of the temperature $T(X)$ in domain $D$. The main purpose is to study a finite volume approximation to the solution of the steady-state heat radiation with nonlinear Stefan–Boltzmann boundary condition in given domain. The domain is made up of three layers of different materials with different thermal diffusion coefficients. We note, that one material is metal, the other two are isolators (the thickness of metal, compared to thickness of isolator, is very thin), therefore the diffusion coefficients have very different values.

The mathematical model consists of the elliptic differential equation [1]:

$$-2 \sum_{i=1}^{2} \frac{\partial}{\partial x_i} \left( k(X) \frac{\partial T}{\partial x_i} \right) = \left( \frac{I(X)}{A} \right)^2 \rho_0 (1 + c(T - 20)), \quad X \in D, \quad (1)$$

subject to the symmetry condition on $\partial D_1 = \{ (0, x_2), x_2 \in [0, L_2] \}$

$$k(X) \frac{\partial T}{\partial x_1} = 0, \quad X \in \partial D_1 \quad (2)$$

and the nonlinear boundary condition on the remaining part of the boundary:

$$k(X) \frac{\partial T}{\partial \eta} + \alpha_K(T)(T(X) - T_a) + \varepsilon \sigma (\tilde{T}^4 - \tilde{T}_a^4) = 0, \quad X \in \partial D \setminus \partial D_1, \quad (3)$$

where $\tilde{T} = T + 273.15$ is a temperature in the Kelvin scale. The convection heat transfer coefficient $\alpha_K(T) \geq \alpha_0 > 0$ for $T \geq T_a$. $I$ is a given current, $A$ is an area of the metal, $\rho_0$ is the specific resistivity of the conductor, $c > 0$ is a constant, $T_a$ is the temperature of the environment, $k$ is diffusion coefficient. The heat source is generated by a current in the metal region and it depends linearly on the temperature of the body.

We also investigate the stability and convergence of the 1D differential problem in composite material. A discrete problem is constructed using finite volume method, a special attention is given to the approximation of the nonlinear Robin boundary conditions [2]. The Picard iterative method is used to linearize the nonlinear discrete problem, the convergence of iterations is also proved.

REFERENCES

IDENTIFICATION OF NONLINEAR MICROSTRUCTURED MATERIALS

J. JANNO

Institute of Cybernetics
Akadeemia tee 21, EE-12618, Tallinn, Estonia
E-mail: janno@ioc.ee

Microstructured materials like alloys, crystallites, ceramics, functionally graded materials, etc. have gained wide application. Determination of parameters of these materials is a problem of great practical importance. For this purpose wave processes going on in macro-level could be used.

We consider a mathematical model of the microstructure, which was derived according to the Mindlin ideas by means of the hierarchical approach due to Engelbrecht and Pastrone [1]. The governing system of this model contains two coupled nonlinear equations related to the macro- and microscale. The system involves 7 coefficients that are related to the material properties. It is possible to simplify the model eliminating the micro-deformation by means of the slaving principle and reach a single hierarchical equation with nonlinearities in lower and higher order terms. The hierarchical equation contains 5 coefficients.

We will establish conditions for the existence of solitary waves in both models and study inverse problems to recover the coefficients of the models by means of the measurement of solitary waves.

In particular, we will prove that in the case of the hierarchical equation the 5 unknown coefficients can be uniquely recovered by the amplitudes and half-lengths of two independent solitary waves in fixed levels [2]. However, in the case of the original coupled system the uniqueness issue is more complicated. We pose some over-determined inverse problems using solitary waves in this system and present new uniqueness results for them.

In addition, we discuss stability issues in some simpler subcases. To this end we deduce local Lipschitz-estimates. Analysis of the Lipschitz-coefficients enables to optimize the measurement levels of the waves.

REFERENCES


ROLES OF WEIGHT FUNCTIONS FOR STABILITY OF DIFFERENCE SCHEMES OF NONLOCAL PARABOLIC PROBLEM

Ž. Jesevičiūtė, R. Čiupaila and M. Sapagovas

1 Kaunas University of Medicine, Vytautas Magnus University,  
2 Vilnius Gediminas Technical University,  
3 Institute of Mathematics and Informatics

1 Eiveniu 4, LT-50009, Kaunas, Lithuania,  
2 Saulėtekio al. 11, LT-10223, Vilnius, Lithuania,  
3 Akademijos 4, LT-08663, Vilnius, Lithuania

E-mail: zivile.js@gmail.com, reciup@lrs.lt, m.sapagovas@ktl.mii.lt

The stability of an implicit difference scheme for a parabolic equation subject to nonlocal integral conditions is considered. We investigate the parabolic differential equation subject to nonlocal integral conditions with variable weight functions

\[
\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + f(x, t), \quad 0 < x < 1, \ 0 \leq t \leq T, \\
u(0, t) = \int_0^1 \alpha(x)u(x, t)dx + \mu_1(t), \\
u(1, t) = \int_0^1 \beta(x)u(x, t)dx + \mu_2(t), \\
u(x, 0) = \varphi(x).
\]

The stability analysis is based on the spectral structure of a matrix of the difference scheme. We consider the structure of the spectrum for the nonlocal parabolic problem. We investigate how eigenvalues depend on the parameters \(\alpha(x)\) and \(\beta(x)\), occurring in the nonlocal boundary conditions. Some properties of the spectrum for this differential problem are proved. We consider the cases in which zero, negative and positive eigenvalues appear. If we would like to investigate zero eigenvalue, we should to analyse the equation, which represents a hyperbola in the coordinate system \((\gamma_1, \gamma_2)\). The hyperbola’s branches can change position or degenerate to a straight line depending on the expression of \(\alpha(x)\) and \(\beta(x)\).

The corresponding difference problem is investigated also. The results of the numerical experiment are presented. The area, where all the eigenvalues are real and positive, is found.

REFERENCES


POLYLOGARITHMS: A PROBABILISTIC VIEW

P. JODRÁ

Universidad de Zaragoza

María de Luna 3, 50018 Zaragoza, Spain

E-mail: pjodra@unizar.es

The polylogarithm function has a long history and was already studied by Leibniz in 1696 and later by Euler (1768) and Landen (1760, 1780), whom introduced the simplest forms called dilogarithm and trilogarithm. For complex numbers s and z, the polylogarithm function of order s and argument z is defined as \( \text{Li}_s(z) := \sum_{n=1}^{\infty} \frac{z^n}{n^s}, \) |z| < 1, and it can be extended by analytic continuation to the whole complex z-plane with the exception of [1, \infty) (cf. Lewin [3] for full details). Perhaps, the best-known integral representation is \( \text{Li}_s(z) = (z/F(s)) \int_0^\infty \frac{u^{s-1}}{(e^u - z)} \, du, \) \( \Re (s) > 0, \ z \notin (1, \infty). \)

The polylogarithms of integer order are of great importance, for instance, in statistical mechanics (cf. Lee [2] for a survey). However, these functions are little known in standard reference textbooks and, in probability and mathematical statistics. In the current work, we see that the polylogarithms of integer order play an essential role to describe basic properties of some probabilistic models, and, at the same time, can be reinterpreted as moments corresponding to certain random variables.

Clearly, the polylogarithms of negative integer order are connected with the non-central moments of the discrete geometric distribution, which has probability function \( P(X = n) := p (1 - p)^n, \) where \( 0 < p < 1 \) and \( n = 0, 1, 2, \ldots \) More precisely, we have \( E[X^k] = p \text{Li}_{-k}(1 - p), \) for \( k = 1, 2, \ldots \) As a consequence, the non-central moments of the binomial negative distribution can also be expressed in terms of polylogarithms.

On the other hand, let \( Y \) be a continuous random variable having density function \( f(y) := (1 + \beta)e^{-y}/(1 + \beta e^{-y})^2, \) where \( y > 0 \) and \( \beta > -1, \) which is widely used in marketing science. In particular, for any \( \beta > -1 (\beta \neq 0) \) we have \( E[Y^k] = -(1 + \beta)k! \text{Li}_k(-\beta)/\beta, \) for \( k = 0, 1, 2, \ldots \) which leads to the integral representation \( \text{Li}_k(z) = (z/k!) \int_0^1 \log^k(1/u)/(1 - zu)^2 \, du, \ z < 1 \) (see Jodrá [1]). This result can be extended as follows.

Let \( Y_{1:n} \) be the minimum order statistic of a random sample of size \( n \) from \( Y \) and denote by \( \mu_{1:n}^{(k)} \) the \( k \)th non-central moment of \( Y_{1:n}, \ n = 2, 3, \ldots \) Then, for any \( \beta > -1 \) we have

\[
\mu_{1:n}^{(k)} = \frac{1 + \beta}{\beta} \left( \mu_{1:n-1}^{(k)} - \frac{k}{(n - 1)} \mu_{1:n-1}^{(k-1)} \right), \quad k = 1, 2, \ldots, \ n = 2, 3, \ldots, \ (\beta \neq 0),
\]

where \( \mu_{1:n}^{(k)} := E[Y^k] \) for \( k = 0, 1, \ldots \) and \( \mu_{1:n}^{(0)} := 1 \) for \( n = 2, 3, \ldots \) The above recursive relation implies that for any \( n = 1, 2, \ldots \) the following integrals

\[
\int_0^1 \frac{u^{n-r}(1-u)^{r-1} \log^k(1/u)}{(1-zu)^{n+1}} \, du, \quad z < 1, \quad r = 1, \ldots, n, \quad k = 1, 2, 3, \ldots,
\]

can be expressed in terms of polylogarithms of non-negative integer order.

REFERENCES


POTENTIAL REPRESENTATION METHOD FOR SCHRÖDINGER EQUATION

D. JURGAITIS, A.J. JANAVIČIUS and S. TURSKIENĖ

Šiauliai University
P. Višinskio 19, LT-77156, Šiauliai, Lithuania
E-mail: pletra@cr.su.lt, AYanavy@gmail.com, sigita@fm.su.lt

The potential representation method [1] is based on following postulate: new wave function, when additional potential is included, can be expressed multiplying previous wave function on multiplier depending on this potential. Using this method was obtained a general solutions of the Schrödinger equation for the negative discrete [1] proper energies. The integral equations were obtained by modified method of undetermined coefficients. The general solution

\[ U_\alpha = \varphi U_nL = (c_1(r) + c_2(r)F_nL/U_nL)U_nL \]

of Schrödinger equation

\[ \frac{d^2}{dr^2} U_\alpha - \frac{L(L+1)}{r^2} U_\alpha + C(E_\alpha - V_\delta - V_0(r))U_\alpha = 0, \quad V_\delta = V(r) - V_0(r), \quad C = \frac{2m}{\hbar^2} \]

for potential \( V(r) \) can be expressed by combination of physical \( U_nL \) and linearly independent non-physical \( F_nL \) solutions for model potential \( V_0(r) \). Taking for derivatives additional requirement \( c_1 + c_2 F_nL/U_nL = 0 \), we can find \( c_1 \) and \( C_2 \) for standard boundary conditions \( U_\alpha(0) = U_\alpha(\infty) = 0 \) for bound states, and obtain following integral equations

\[ \varphi U_nL = U_nL + \frac{U_nL}{W_0} \int_0^r F_nL C \Delta V \varphi U_nL dr_1 - \frac{F_nL}{W_0} \int_0^r U_nL C \Delta V \varphi U_nL dr_1, \quad \Delta E_nL = \int_0^\infty U_nL V_\delta \varphi U_nL dr - \int_0^\infty U_nL \varphi U_nL dr \]

where \( \Delta V = V_\delta - \Delta E_nL, \Delta E_nL = E_\alpha - E_nL \). \( W_0 \) is Wronskian of linearly independent solutions \( U_nL \) and \( F_nL \) for model potential. For convergence of iteration processes (for first iteration calculating \( \Delta E_nL \) was taken \( \varphi = 1 \)) we must require that after \( N \) iterations the perturbations \( \Delta E_nL \) practically become constant. Then the proper energy values \( E_\alpha \) are defined. The FORTRAN program was prepared for exponentially decreasing at large distances Woods - Saxon potential, where model harmonic oscillator potential \( mw^2r^2 \) was used [1]. The proper values of neutron energies -39.97, -29.56, -15.38, -1.425, expressed in mega electron volts, for states \( n = 1, 2, 3, 4 \) and \( L = 0 \) calculated by the discretization method [2], we can compare with -39.91, -29.44, -15.55, -1.855 obtained by solution of integral equations. Differences depends on the different asymptotic of model potential.

REFERENCES


In 1975, S.M. Voronin [1] proved, that any non-vanishing analytic function can be approximated uniformly by certain shifts of the Riemann zeta-function \( \zeta(s) \) (now it is called as an universality). In the proof, the essential role play two facts: the linearly independence of the set \( \{ \log p : p \text{ is prime} \} \) over the field of rational numbers \( \mathbb{Q} \) and the existence of Euler product representation for \( \zeta(s) \).

A more complicated problem is the investigation of joint universality property for a collection of zeta- and \( L \)-functions, especially, the case for different functions. The main aim of the report is to present the joint discrete universality for the Dirichlet \( L \)-functions \( L(s, \chi) \) and a periodic Hurwitz zeta-function \( \zeta(s, \alpha; b) \) with a transcendental parameter \( \alpha \) [2]. Note that in this case, the set \( \{ \log p : p \text{ is prime} \} \cup \{ \log(m + \alpha) : m \in \mathbb{N} \cup \{0\} \} \) is linearly independent over \( \mathbb{Q} \), also.

REFERENCES


METHOD OF LINES AND FINITE DIFFERENCE SCHEMES OF EXACT SPECTRUM FOR SOLUTION THE HYPERBOLIC HEAT CONDUCTION EQUATION

H. KALIS and A. BUIKIS

Institute of Mathematics and Informatics, Faculty of Physics and Mathematics University of Latvia
Raina bulvāris 29, Rīga LV-1459, Zellu ielā 8, Rīga LV-1002, Latvija
E-mail: buikis@latnet.lv, kalis@lanet.lv

The solutions of corresponding 1-D initial-boundary value problem and inverse problem for hyperbolic heat conduction equation are obtained numerically, using for approach differential equations the discretization in space applying the finite difference scheme (FDS) and the difference scheme with exact spectrum (DSES) [1]. The solution in the time is obtained analytically and numerically with continuous and discrete Fourier methods.

Using the spectral method are obtained news transcendental equation and algorithms for obtaining the last eigenvalue and eigenvector of finite difference scheme.

We define the DSES, where the finite difference matrix $A$ is represented in the form

$$A = PDP^T$$

($P, D$ is the matrixes of finite difference eigenvectors and eigenvalues correspondently ) and the elements of diagonal matrix $D$ are replaced with the first eigenvalues from the differential operator.

For finite difference approximation with central differences strong numerical oscillations are presented, when the initial and boundary conditions are discontinuous [2].

For the inverse problem the function $V_0(x)$ is unknown and then we can used the aditional condition $T(x,t_f) = T_f(x)$, where $T_f$ is given final temperature.

For finite difference approximation with central differences strong numerical oscillations are presented, when the initial and boundary conditions are discontinuous [2]. The method of DSES is without oscillations and this is effective for numerical solutions

REFERENCES


$^{1}$Authors wish to thank for partial support the ESF project Nr. 2009/0223/1DP/1.1.2.0/09/APIA/VIAA/008 and Latvian Science Foundation grant Nr. 09.1572
CRYPTOGRAPHIC ALGORITHM "SAFER" MODERNIZATION

S. KAMENCHENKO

Transport and Telecommunication Institute, Lomonosova 1, LV-1019, Riga, Latvia
E-mail: freeon@inbox.lv

Nowadays, the problem of using cryptographic methods in information systems has become particularly relevant, because of the routine introduction and extensive use of global computer networks, which carry large amounts of information, this fact not allowing the access of unauthorized persons to it. The rapid development of computing and neural technologies constantly reduces the time for breaking ciphers, which until now had the high cryptographic strength [1; 2].

Such algorithms, as SAFER and its modification [3] have not found a wide application in practical cryptography because of their significant shortcomings: weak cryptographic resistance, low encryption/decryption speed and keys expansion [3].

The article is devoted to modification of SAFER algorithm using two variants. Both modification versions allow us to build matrix classes of codes, where:

- Matrices have the property $A = A^{-1}$ and thus, unlike the pseudo-Hadamard matrices, the problem of handling to ill-conditioned matrix is self-eliminated;
- Elements of the matrix consists only with integers $\pm 1, \pm 2$ or $\pm 1, \pm 2, \pm 4$;
- power of constructed matrix classes of codes increases with the exponential rate with matrix dimension increasing. If the matrix has the dimension $2^n \times 2^n$ ($2 < n \in N$), then the power of a matrix class with property $A = A^{-1}$ is equal to $c \cdot 2^{2n+1}$, where $1 < c \equiv const$. For example, the matrix with dimension $4 \times 4$ allows us to construct a matrix class of codes with $A = A^{-1}$ property, whose power is equal to $4.5 \cdot 2^8$.

Obviously, the use of pseudo-Hadamard matrices for data encryption/decryption contributes to slowing the algorithm, because their specific structure consist of a large numbers $2^n$, where $n \in N$.

To improve the reliability (ie, resistance to cryptanalysis) of the proposed encoding algorithm is proposed to use not one pair of matrices, but several pairs, appropriate transformation of which greatly increases the resistance of encoding.

Based on the above mentioned facts of specialized encryption/decryption matrix class, we can assume that the proposed method allows to form a new cryptosystem that satisfies the requirements of the cryptographic resistance.

REFERENCES


The article is written with the financial assistance of European Social Fund. Project Nr. 2009/0159/1DP/1.1.2.1.2/09/IPIA/VIAA/006 (The Support in Realisation of the Doctoral Programme “Telematics and Logistics” of the Transport and Telecommunication Institute).
STABILITY AND BLOW-UP OF ONE DIFFERENCE SCHEME FOR PARABOLIC EQUATIONS WITH NONLINEAR SOURCE

A. KANDRATSIUK

Belarusian State University, Minsk, Nezavisimosti, 4
Kurchatova 8, V124B, BY-220064, Minsk, Belarus
E-mail: unreal_universal@mail.ru

In the rectangle $Q_T = \{(x,t) : 0 \leq x \leq l, 0 \leq t \leq T_{cr}\}$ we consider the initial boundary value problem for the semilinear parabolic equation

$$
\frac{\partial u}{\partial t} = \frac{\partial}{\partial x} \left( k(x) \frac{\partial u}{\partial x} \right) + \lambda u^2,
$$

(1)

$$
u(x,0) = u_0(x), \quad u(0,t) = 0, \quad u(l,t) = 0,
$$

(2)

$$
0 < k_1 \leq k(x) \leq k_2, \quad |u_0'(x)| \leq c_0,
$$

(3)

where $k_1, k_2, \lambda$ are positive constants.

The main purpose of this talk is to prove the stability of the linearized difference schemes approximating initial-boundary problem (1)-(3). As is known [1], [2], stability analysis in the nonlinear case is based on preliminary estimates of the difference solution in strong norms. Expression $\frac{\partial}{\partial x} \left( k(x) \frac{\partial u}{\partial x} \right)$ in (1) is approximated by the operator $Ay = - (ay_x)_x$. It is shown that the condition

$$
1 - \frac{\lambda l^2}{16k_1^2} \|u_0\|_A \geq 0,
$$

(4)

is sufficient for the global (as $t \to \infty$) boundedness and stability of the difference solution[3]. Moreover, if inequality (4) holds strictly, then the solution decays in the uniform norm exponentially.

If the condition on the input data (4) are not fulfilled, then the solution may become infinite in finite time. In this case, the a priori estimates obtained by the use of the grid analog of Lemma Bihari will be valid only up to a certain point in time.

We also study the monotonicity of difference schemes in the nonlinear case. Conditions for automatic selection of time step in numerical calculation are obtained by the monotonicity analysis.

REFERENCES

ON THE RANGES OF SOME FUNCTIONALS ON THE CLASS OF UNIVALENT IN THE HALF-PLANE FUNCTIONS

J. KIRJACKIS

Vilnius Gediminas Technical University
Sauletekio al. 11, LT-10223, Vilnius, Lithuania
E-mail: jk@fm.vgtu.lt

Let $U$ be a set of univalent (that is analytic and one-to-one) in the right half-plane $\Pi = \{ \Re z > 0 \}$ functions, normalized by conditions $F''(1) - 1 = F(1) = 0$. For the class $S$ of univalent and normalized in the unit disc $E = \{ |w| < 1 \}$ functions $f(\omega)$, $f'(0) - 1 = f(0) = 0$ it is known that the ranges of functionals $J_{n}(F) = f^{(n)}(0)$ are circles $|J_{n}(F)| \leq n! n$. With respect to the class $U$ it is established that range of functional $I_{2}(F) = F''(1)$ is a circle too, but the range of functional $I_{3}(F) = F'''(1)$ is not a circle. The shapes of the ranges of functionals $I_{n}(F) = F^{(n)}(1)$, $F \in U$ are considered.
ON A CLASS OF TWO-DIMENSIONAL HYPERBOLIC EQUATIONS WITH NONLOCAL BOUNDARY CONDITIONS

E. KLIMOVA and L. PULKINA

Faculty of Mechanics and Mathematics, Samara State University; Technical University Dresden
Ak. Pavlova 1, 443011, Samara, Russia
E-mail: elenaklimova25@gmail.com, louise@samdiff.ru

In this talk, we prove the existence and uniqueness on a solution of a class of hyperbolic partial differential equations with nonlocal boundary conditions. Results are obtained by using a functional analysis method based on an a priori estimate and on the density of the range of the operator corresponding to the abstract formulation of the considered problem. The precise statement of the problem is of the form:

Let \( a, b > 0 \) and \( Q := \{(x, y) \in \mathbb{R}^2 : 0 < x < a, 0 < y < b \} \). Find a function \( (x, y) \rightarrow u(x, y) \), where \((x, y) \in Q\), satisfying the equation

\[
uxy + (Ax)_x + (By)_y + Cu = f(x, y)
\]

and the integral conditions

\[
\begin{align*}
    u(x, 0) + \int_{0}^{b} u(x, y)dy &= g_1(x), \\
    u(0, y) + \int_{0}^{a} u(x, y)dx &= g_2(y),
\end{align*}
\]

where \( g_1 \) and \( g_2 \) are given functions:

\[
g_1(0) + \int_{0}^{a} g_1(x)dx = g_2(0) + \int_{0}^{b} g_2(y)dy.
\]

Problem with mixed integral conditions for hyperbolic equations are studied by different method in [1; 2] and others.

REFERENCES


DOUBLE CONDUCTOR LINE ABOVE A TWO-LAYER MEDIUM WITH VARYING ELECTRIC CONDUCTIVITY AND MAGNETIC PERMEABILITY

V. KOLISKINA and I. VOLODKO

Riga Technical University
Meza 1/4, LV-1048, Riga, Latvia
E-mail: v.koliskina@gmail.com, inta.volodko@rtu.lv

Theoretical models for eddy current testing problems are well developed for the case where the parameters of a conducting medium (such as electric conductivity and magnetic permeability) are constants [1]. In engineering applications (examples include surface hardening, decarbonization, and determination of thickness of metal coatings) the properties of conducting materials can change with depth [2]. Two basic approaches are widely used in such cases: (a) conducting medium with depth-varying properties is replaced by a multilayer medium with piecewise constant properties (the number of layers can be quite large, up to 50 layers were used in the literature); (b) relatively simple model profiles that take into account electric conductivity and magnetic permeability variation in the vertical direction are used with the hope to obtain analytical solution of the problem in terms of known special functions. The second approach is followed in the present paper. Consider a double conductor line formed by two infinitely long parallel wires carrying alternating current. The double line is located above a two-layer medium where the properties of the upper layer (the electric conductivity and magnetic permeability) are exponentially varying functions of the vertical coordinate. Closed-form solution for the vector potential in terms of improper integrals containing Bessel functions of complex argument is obtained in the paper. The change in impedance per unit length of the double conductor line is calculated for different values of the parameters of the problem. The results are compared with the calculations for the case of a conducting medium with constant properties.

REFERENCES

Linear and weakly nonlinear stability analyses of shallow water flows are usually performed under the assumption that the base flow profile is symmetric with respect to the transverse coordinate (see [1]). Recent experimental data [2] showed that in some cases the base flow velocity distribution is not symmetric. A typical example of such a case is a flow in an open channel with vegetation in the floodplains. The analysis of experiments in [2] confirmed the presence of the two-layer structure of the base flow: a boundary-layer type of the velocity distribution is observed in the main channel while a hyperbolic tangent profile can be used to approximate the velocity distribution in the floodplain.

In the present paper a spatial linear stability problem for an asymmetric base flow with the two-layer structure described above is analyzed. Linear stability characteristics (marginal stability curves and growth rates for the most unstable mode) are calculated using collocation method based on Chebyshev polynomials. Growth rates and critical values of the bed-friction number are compared with linear stability characteristics for the corresponding symmetric base flow profile.

REFERENCES

DEPENDENCE OF THE EFFECTIVE CONDUCTIVITY OF 2D COMPOSITE MATERIALS ON DOMAIN PERTURBATIONS

A. KOROLEVA, S. ROGOSIN and A. SHATSKO

Belarusian State University
Nezavisimosti ave 4, BY-220030, Minsk, Belarus
E-mail: ankakoroleva@gmail.com; rogosin@gmail.com

Analytical methods applied at the study of a steady heat conduction in various types of composites are described in [1]. Several analytical formulas for the effective (macroscopic) conductivity tensor which are deduced by using different approaches based on the recent results in the theory of partial differential equations and complex analysis (see also [2] and references therein). We can mention here Clausius-Mossotti formula (for small concentration of inclusions), Dykhne formula (for symmetric composite with three components), generalized Keller-Dykhne formula (for self-dual two-phase system with arbitrary concentration of compact inclusions of one phase into another), Mityushev’s formula (for circular inclusions in 2D composites).

The main goal of the work is to study the dependence of the effective (macroscopic) conductivity tensor on the local perturbation of a finite number of initial circular inclusions. For sake of simplicity we consider only potential steady heat flow under ideal contact condition. This problem leads to \( R \)-linear conjugation problem for analytic functions in multiply connected domain (see [1])

\[
\phi(t) = \phi_k(t) - \rho \phi_k'(t) - t, \quad t \in \Gamma_k, \quad k = 1, \ldots, n,
\]

where \( \rho \) is Bergman parameter (see, e.g. [1]).

In the case when \( \Gamma_k \) are circles the system of relations (1) are solved (see [2]) by using the method of the functional equations and the method of successive approximation. On the base of this solution an exact formula for effective conductivity is found (see [3]).

The changes in this formula caused by the local perturbation of the inclusion are studied. The machinery of this work is based on the approach of M. Lanza de Cristoforis developed in [4], [5].

The work is partially supported by the Belarusian Fund for Fundamental Scientific Research.

REFERENCES


WEAK SOLUTIONS OF BOUNDARY VALUE PROBLEMS IN NONTUBE DOMAINS FOR FOURTH-ORDER EQUATIONS OF COMPOSITE TYPE

V.I. KORZYUK and O.A. KONOPELKO

Belarusian State University
4, Nezavisimosti av., Minsk, Belarus, 220030
E-mail: Korzyuk@bsu.by, KonopelkoOA@bsu.by

In a domain Q of (n + 1)-dimensional Euclidean space $\mathbb{R}^{n+1}$ of independent variables $x = (x_0, \ldots, x_n)$ with respect to function $u: \mathbb{R}^{n+1} \supset Q \ni x \mapsto u(x) \in \mathbb{R}$ we consider a linear partial differential fourth-order equation of composite type

$$Lu = \frac{\partial^4 u}{\partial x_0^4} + (b^2 - a^2) \frac{\partial^2}{\partial x_0^2} \Delta u - a^2 b^2 \Delta^2 u + A^{(2)} u = f(x),$$

where $a, b \in \mathbb{R}$ and $a^2 > b^2$, $\Delta = \sum_{i=1}^{n} \frac{\partial^2}{\partial x_i^2}$ is the Laplacian with respect to the variables $x' = (x_1, \ldots, x_n)$, $A^{(2)} = \sum_{|\alpha| \leq 2} a^{(\alpha)}(x) D^\alpha$, $D^\alpha = \frac{\partial^{\alpha}}{\partial x_0^{\alpha_0} \ldots \partial x_n^{\alpha_n}}$, $\alpha = (\alpha_0, \ldots, \alpha_n)$, $\alpha_i, i = 0, \ldots, n$, are nonnegative integers, $|\alpha| = \alpha_0 + \ldots + \alpha_n$, $a^{(\alpha)}(x) \in C^2(\overline{Q})$ and $f \in H^{-1}$ are given functions, $H^{-1}$ is space with negative norm $[1]$

Boundary $\partial Q$ of the domain $Q$ consists of three parts: $S_1 = \{x \in \partial Q \mid \nu_0^2 - a^2 |\nu'|^2 \geq \delta, \nu_0 \leq -\delta\}$, $S_2 = \{x \in \partial Q \mid \nu_0^2 - a^2 |\nu'|^2 \leq -\delta\}$ and $S_3 = \{x \in \partial Q \mid \nu_0^2 - a^2 |\nu'|^2 \geq \delta, \nu_0 \geq \delta\}$, $\partial Q = S_1 \cup S_2 \cup S_3$, where $\delta$ is positive number, $\nu(x) = (\nu_0(x), \ldots, \nu_n(x))$ is the outward with respect to the domain $Q$ unit normal on the hypersurface $\partial Q$ at a point $x \in \partial Q$, $|\nu'|^2 = \nu_1^2 + \ldots + \nu_n^2$.

We join to equation (1) the following boundary conditions:

$$u|_{S_1} = \frac{\partial u}{\partial p_1}|_{S_1} = 0, \quad u|_{S_2} = \frac{\partial u}{\partial p_2}|_{S_2} = 0$$

where $\frac{\partial u}{\partial p_i}$ is derivative in the direction of $p_i$ from vector field $P$ of $C^1$, which is not tangent to $S_i$, $i = 1, 2$. Boundary conditions (2) can be nonhomogeneous.

Under some restrictions to $\partial Q$, extending the operator $L$ in coarse topology we prove the existence of unique weak solution of problem (1)–(2).

REFERENCES


TWO-POINT BOUNDARY VALUE PROBLEM FOR ONE-DIMENSIONAL WAVE EQUATION

V.I. KORZYUK and I.S. KOZLOVSKAYA

Institute of Mathematics of National Academy of Science, Belarussian State University
11 Surganova St., 220072; 4 Nezavisimosti Ave., 220030, Minsk, Belarus
E-mail: Korzyuk@bsu.by; Kozlovskaja@bsu.by

With respect to unknown function $u : \mathbb{R}^2 \ni Q \ni x \rightarrow u(x) \in \mathbb{R}$ we consider one-dimensional wave equation

$$\frac{\partial^2 u}{\partial t^2} - a^2 \frac{\partial^2 u}{\partial x^2} = f(t, x), \quad (1)$$

where $Q = [0, \infty) \times (0, l)$.

We join to it an initial condition

$$u|_{t=0} = \varphi(x), \quad x \in (0, l), \quad (2)$$

and boundary conditions

$$u|_{x=0} = \mu^{(1)}(t), \quad u|_{x=l} = \mu^{(2)}(t), \quad t \in (0, \infty). \quad (3)$$

In addition at $t = \frac{kl + \tau}{a}$ the value of derivative

$$\frac{\partial u}{\partial t}|_{t=\frac{kl + \tau}{a}} = \tilde{\psi}(x), \quad x \in (0, l), \quad (4)$$

is given, where $k \in \{0, 1, 2, \ldots\}$, $\tau \in [0, l]$.

The problem is to determine $k$ and $\tau$, for which equation (1) has classical solution satisfying conditions (2)–(4).

Problem (1)–(4) is reduced to a system of equations with respect to the arguments with displacement. We show that for functions $\varphi \in C^2[0, l]$, $\tilde{\psi} \in C^2[0, l]$, $\mu^{(j)} \in C^2[0, \infty)$, $j = 1, 2$, $f \in C^0, 1(Q)$ if fitting conditions

$$\varphi(0) = \mu^{(1)}(0), \quad a^2 \varphi''(0) = \mu^{(1)}''(0) - f(0, 0),$$

$$\varphi(l) = \mu^{(2)}(0), \quad a^2 \varphi''(l) = \mu^{(2)}''(0) - f(0, l),$$

$$\tilde{\psi}(0) = \mu^{(1)}'\left(\frac{kl + \tau}{a}\right), \quad \tilde{\psi}(l) = \mu^{(2)}'\left(\frac{kl + \tau}{a}\right)$$

are satisfied, then in the class of functions $C^2(Q)$ for any $k = 0, 1, 2, \ldots$ and $\tau = 0, l/3, 2l/3, l$ a unique classical solution of problem (1)–(4) exists.
ASYMPTOTICAL ANALYSIS OF SOME COUPLED NONLINEAR WAVE EQUATIONS

R. KRIAUIZENĖ\textsuperscript{1} and A. KRYLOVAS\textsuperscript{2}

\textsuperscript{1,2}Mykolas Romeris University
Ateities st. 20, LT-08303 Vilnius, Lithuania
\textsuperscript{2}Vilnius Gediminas Technical University
Saultekio al. 11, LT-10223 Vilnius, Lithuania
E-mail: krylovas@mruni.eu, akr@fm.vgtu.lt

We consider coupled nonlinear equations modelling a family of travelling wave solutions. The propose of the our work is to show that the method of internal averaging along characteristics [1], [2] can be used for wide classes of coupled non-linear wave equations, such as Klein-Gordon, Hirota-Satsuma, etc. The asymptotical analysis reduces a system of coupled non-linear equations to a system of integro-differential averaged equations. For the periodical initial conditions the averaged system disintegrates into independent equations which describe simple weakly non-linear travelling waves in the non-resonance case. In the resonance case the integro-differential averaged systems describe interaction of waves and give a good asymptotical approximation for exact solutions [3].

REFERENCES


UNANNOUNCED INTERIM INSPECTIONS:
DO FALSE ALARMS MATTER?

TH. KRIEGER and R. AVENHAUS

Universität der Bundeswehr München
ITIS
Werner-Heisenberg-Weg 39, 85577 Neubiberg, Germany
E-mail: thomas.krieger@unibw.de, rudolf.avenhaus@unibw.de

Unannounced Interim Inspections (UII) in nuclear plants of the European Union have recently attracted major attention by the International Atomic Energy Agency (IAEA) and by European Atomic Energy Community (EURATOM) in the context of the IAEA/EURATOM Partnership Approach. Therefore, a research project had been organized by the Joint Research Centre in Ispra in collaboration with the Universität der Bundeswehr München in the framework of which the assumptions have been classified which are necessary for a quantitative analysis and a few variants have been studied in detail.

In that project only so-called Attribute Sampling procedures were considered which means that only errors of the second kind, but not those of the first kind, where taken into account, see [2].

In this paper now the impact of errors of the first kind on UII’s is investigated. Two kinds of planning UII’s are considered: In the fully sequential one both players, the inspector and the plant operator, decide step by step to inspect resp. to start an illegal activity, see [1]. In the hybrid-sequential one the inspector decides at the beginning of the reference time interval where to place his UII’s, whereas the plant operator acts again sequentially.

It turns out that in both cases the equilibrium strategies of the inspector and the equilibrium payoffs to both players are the same, but not the equilibrium strategies of the plant operator. We try to present a plausible explanation for this surprising result.

REFERENCES


CLASSIFICATION OF INDIVIDUALS BASED ON PLANAR HISTOGRAM OF A HUMAN POSTURE

R. KRUSINSKIENE, R. JASINEVICIUS and V. PETRAUSKAS

Kaunas University of Technology
Studentu g. 50-407, LT-51368 Kaunas
E-mail: radvile.krusinskiene@ktu.lt

Human posture classification and disorder recognition in early stages becomes more and more important as society gets older and ability to maintain bipedal posture vastly influences the quality of life of an individual [2]. The time law of small vibrations of the center of pressure (COP) of humans in standing position (human posture) may provide useful information about physical condition of an individual [1]. This paper presents the ability to distinguish two classes of individuals healthy (HL) ones and those suffering from multiple sclerosis disease (MSD) based on information extracted from the COP signal (stabilogram) and generalized as planar histogram.

The pattern recognition method is based on solving linear programming problem [3]. The solution for the class \( p \) is based on the information of the description of objects (\( N \) dimensional vectors) \( \alpha_k^p \), \( \forall k \) from \( p \) and consists of the obtained value of the dependence function \( \Phi_{p_{\text{max}}} \) together with corresponding generalized pattern \( K_p \): \( \forall p \):

\[
\begin{align*}
\max \Phi_p(K_p, \alpha_k^p) &= \Phi_{p_{\text{max}}}, \\
K_p &= (K_{p1}^1, K_{p2}^1, \ldots, K_{pi}^1, \ldots, K_{pN}^1).
\end{align*}
\]

The results provided by the method allowed to construct the generalized pattern \( K_p \), which emphasizes main distinctive features of each class. The obtained values of \( \Phi_{p_{\text{max}}} \) and \( K_p \) were used for classification of individuals into HL and MSD classes.

Thirty female individuals (fifteen from HL class and fifteen from MSD class) 32±2 (average ± standard deviation) years old took part in clinical experiments. Numerical experiments show that it is possible to find a hyperplane, which distinguishes HL and MSD classes of individuals using information from planar histograms of stabilograms. The main differences between planar histograms of HL and MSD classes are situated on both sides of secondary diagonal.

The generalized patterns constructed using only several randomly selected planar histograms (six from HL class and two from MSD class) were informative enough to assign 70% of individuals from "unknown class" correctly.

REFERENCES


ON RECENT DEVELOPMENTS FOR FILTRATION RELATED MULTISCALE PROBLEMS

Z. LAKDAWALA and O. ILIEV

Fraunhofer ITWM
Fraunhofer-Platz 1, D-67663 Kaiserslautern, Germany
E-mail: lakdawala, iliev@itwm.fhg.de

Filtration is typically a multiscale problem. Nowadays, Computational Fluid Dynamics (CFD) simulations are often used in studying various filtration processes and design of filter elements at different independent scales. The efficient filtration of suspended particles is an important topic for many technical applications. Examples are oil filtration, air filtration etc. To develop optimal filter designs, reliable computational flow and transport models and algorithms are essential. Although the separate macro or microscale simulations contribute essentially to the design of filters with improved performance, they have certain clear limitations. Algorithms and software for CFD simulations of filtration processes were earlier presented by Fraunhofer ITWM [1; 2]. Some algorithmic aspects of the software tool, SuFiS, are discussed in [3]. The current talk discusses advanced recent developments on multiscale simulation of flow in plain and porous media.

Our aim is to discuss the complex nature of filtration, models employed at different scales and the numerical algorithms for solving a coupled system at different scales. However, due to a multitude of relevant length scales (particles, pores, fibres, device), a naïve approach is computationally infeasible and therefore accurate upscaling, homogenization and multiscale methods are required. The process entails derivation of averaged particle concentration equations, upscaling from pore to Darcy scale, and multiscale methods to treat heterogeneous domains. In typical filtration problems, some of the flow domain is governed by the Navier Stokes equations, whereas the Darcy law is applied to the other areas. The integration of both systems into an efficient and general multiscale framework is a major challenge. Special attention is paid to requirements of computer resources and computational bottlenecks while simulating such problems. A macroscopic framework for such a problem is formulated. The numerical algorithm is discussed in details, followed by some numerical results that are validated against measurements.

REFERENCES

ON THE FINITE PART CONCEPT OF DIVERGING INTEGRALS

K. LÄTT

Faculty of Mathematics and Computer Sciences, University of Tartu
J. Liivi 2, 50409, Tartu, Estonia
E-mail: klatt@ut.ee

Diverging integrals have been used in the solution of applied problems for a long time, see [1], [2] and [3].

We consider some approaches to finding the finite part (f.p.) of diverging integrals which have the form

\[ \int_a^b a(r) v(r, \lambda) \, dr, \]

i.e., diverging integrals with integrand represented as a product of two functions, one with a parameter-dependent non-integrable singularity at one point of the integration interval and the other absolutely integrable. We examine methods which are based on the expansion of the absolutely integrable function in a Taylor series with centre at the singular point (f.p.) and on the analytic continuation with respect to the parameter of the singularity (a.f.p) (c.f. [4]).

REFERENCES

NUMERICAL ALGORITHMS FOR SCHRÖDINGER EQUATION WITH ARTIFICIAL BOUNDARY CONDITIONS

I. LAUKAITYTĖ, R. ČIEGIS and M. RADZIUNAS

1 Vilnius Gediminas Technical University
Sauletekio al. 11, LT-10223, Vilnius, Lithuania
E-mail: rc@vgtu.lt
2 Weierstrass Institute for Applied Analysis and Stochastics
Mohrenstrasse 39, 10117 Berlin, Germany
E-mail: radziunas@wias-berlin.de

Schrödinger type mathematical models are used in many areas of physical and technological interest, for example, in electromagnetic wave propagation, in seismic migration and in semiconductor devices [1]. Depending on the considered real life application we frequently need to solve Schrödinger type equations in infinite or, at least, quite large spatial domains using rather fine numerical mesh [2; 3]. In these cases due to computational restrictions (CPU time and memory resources of the computer), one has to restrict the computational domain and to solve the problem only in the region of interest or a slightly larger domain.

Then the main challenge is to introduce special artificial boundary conditions which enable us to simulate accurately the asymptotical behavior of the solution and do not induce numerical reflections at the boundaries. These boundary conditions must give a well posed problem and discrete approximations of the new boundary value problem should be constructed, which are stable under non-restrictive conditions on space and time steps of the discrete grids.

We investigate the performance of numerical schemes for one-dimensional linear Schrödinger equations with different artificial boundary conditions. By comparing the numerical solutions with the initially known solution of the corresponding problem in infinite domain we draw several conclusions about effectiveness of the considered boundary conditions.

REFERENCES

APPLICATION OF PROBABILISTIC MODEL IN APPROXIMATION OF ANALYTIC FUNCTIONS

A. LAURINČIKAS

Vilnius University
Naugarduko 24, LT-03225, Vilnius, Lithuania

Siauliai University
P. Višinskio 19, LT-77156, Siauliai, Lithuania

E-mail: antanas.laurincikas@maf.vu.lt

By the Mergelian theorem, analytic functions are uniformly approximated on some compact sets by polynomials. This means that, for each given analytic function \( f \), there exists a polynomial depending on \( f \) and approximating it with desired accuracy. However, it turns out that there exist analytic functions \( g(s), s = \sigma + it \), whose shifts \( g(s + i\tau) \) uniformly approximate every analytic function, i.e., these functions in some sense are universal. The mentioned universality property is due to the Riemann zeta-function \( \zeta(s) \), and other zeta and \( L \)-functions. For example, the following modification of the Voronin theorem is true [1].

Suppose that \( K \) is a compact subset of the strip \( \{ s \in \mathbb{C} : \frac{1}{2} < \sigma < 1 \} \) with connected complement, and \( f(s) \) is a continuous non-vanishing function on \( K \) and analytic in the interior of \( K \). Then, for every \( \varepsilon > 0 \),

\[
\liminf_{T \to \infty} \frac{1}{T} \text{meas} \left\{ \tau \in [0, T] : \sup_{s \in K} |\zeta(s + i\tau) - f(s)| < \varepsilon \right\} > 0.
\]

There exists a direct analytic method for the proof of universality theorems on approximation of analytic functions. However, a probabilistic model gives the best and general results in the field. This model is based on limit theorems in the sense of weak convergence of probability measures in the space of analytic functions. Our report is devoted to the above probabilistic approach in approximation of analytic functions. Both the cases of zeta-functions having the Euler product and without Euler product will be discussed. Also, the effectivization problem of universality theorems will be touched.

REFERENCES

ON TWO CONSTRUCTIONS OF A GENERALIZED AGGREGATION OPERATOR

J. LEBEDINSKA

Department of Mathematics, University of Latvia
Zellu iela 8, LV - 1002, Riga, Latvia
E-mail: julija.lebedinska@e-apollo.lv

Aggregation operators [1] are an indispensable tool for different communities dealing with procession of the information coming from different sources. When information is presented in the form of fuzzy sets [3] the appropriate tool to use is generalized aggregation operator [2] or shortly gagop. Let \( \prec \) be some order relation on \( F(\mathbb{R}) \) (the class of all fuzzy subsets defined on the real line \( \mathbb{R} \) and taking values from the unit interval) with the least element \( \tilde{0} \in F(\mathbb{R}) \) and the greatest element \( \tilde{1} \in F(\mathbb{R}) \) then:

**Definition 1.** [2] A mapping \( \tilde{A} : \bigcup_{n \in \mathbb{N}} F(\mathbb{R})^n \rightarrow F(\mathbb{R}) \) is called a generalized aggregation operator w.r.t. the order relation \( \prec \), if for every \( n \in \mathbb{N} \) the following conditions hold:

1. \( \tilde{A}(\tilde{0}, \ldots, \tilde{0}) = \tilde{0} \)
2. \( \tilde{A}(\tilde{1}, \ldots, \tilde{1}) = \tilde{1} \)
3. \( \forall i = 1, \ldots, n \) \( (P_1 \prec Q_1) \Rightarrow \tilde{A}(P_1, \ldots, P_n) \prec \tilde{A}(Q_1, \ldots, Q_n) \),

where \( P_1, \ldots, P_n, Q_1, \ldots, Q_n \in F(\mathbb{R}) \).

Different methods of construction of a gagop are proposed. \( \tilde{A} \) is a pointwise extension of an agop \( A \) provided that:

\[ \forall x \in \mathbb{R} \quad \tilde{A}(P_1, \ldots, P_n)(x) = A(P_1(x), \ldots, P_n(x)). \]

\( \tilde{A} \) is a \( T \)-extension of an agop \( A \) provided that:

\[ \tilde{A}(P_1, \ldots, P_n)(x) = \sup \{ T(P_1(x_1), \ldots, P_n(x_n)) | (x_1, \ldots, x_n) \in [0, 1]^n : A(x_1, \ldots, x_n) = x \}, \]

where \( T \) is an arbitrary lower semicontinuous t-norm.

In the paper we compare these construction methods from the perspective to preserve boundary conditions \((A1), (A2)\) and the monotonicity \((A3)\) w.r.t. different order relations; and also we compare properties of pointwise extension and \( T \)-extension.

**REFERENCES**


EXISTENCE OF THE SOLUTION OF CLASSICAL SMOOTHING PROBLEMS WITH OBSTACLES AND WEIGHTS

E. LEETMA and P. OJA

Institute of Mathematics, Tartu University
J. Liivi 2, 50409 Tartu, Estonia
E-mail: Evely.Leetma@ut.ee, Peeter.Oja@ut.ee

We consider the classical smoothing problem with obstacles

\[ \min_{f \in \Omega_{\alpha\beta}} \|Tf\|^2, \]  

where \( \Omega_{\alpha\beta} = \{ f \in L^2_2(\mathbb{R}^n) \mid f(X_i) = z_i, i \in I_0, \alpha_i \leq f(X_i) \leq \beta_i, i \in I_1 \} \) with given index sets \( I_0, I_1 \), pairwise distinct points \( X_i \in \mathbb{R}^n, i \in I = I_0 \cup I_1 \), interpolation values \( z_i \in \mathbb{R}, i \in I_0 \), obstacle values \( \alpha_i, \beta_i, \alpha_i < \beta_i, i \in I_1 \), the Beppo Levi space \( L^2_2(\mathbb{R}^n) = \{ f \mid D^\alpha f \in L^2_2(\mathbb{R}^n), |\alpha| = r \} \) and \( T : L^2_2(\mathbb{R}^n) \to L^2_2(\mathbb{R}^n) \times \ldots \times L^2_2(\mathbb{R}^n) \) as \( Tf = \left\{ \sqrt{|\alpha|} D^\alpha f \right\} \). The classical smoothing problem with weights is

\[ \min_{f \in \Omega_0} \left( \|Tf\|^2 + \sum_{i \in I_1} w_i |f(X_i) - z_i|^2 \right), \]  

where \( \Omega_0 = \{ f \in L^2_2(\mathbb{R}^n) \mid f(X_i) = z_i, i \in I_0 \} \) with given weights \( w_i \geq 0 \) and values \( z_i \in \mathbb{R}, i \in I_1 \).

It is quite well known that the solutions of problems (1) and (2) (if they exist) are natural splines. The existence of solutions is usually established, see, e.g. [1], on some restrictions about corresponding interpolation problems. On the other hand, it is known that the interpolation problem with natural splines is always solvable [2]. We show that this yields the existence of solutions for problems (1) and (2). The uniqueness of the solution needs the unique solvability of special interpolation problems with polynomials.

REFERENCES

ANALYTICAL SOLUTION OF A TWO-DIMENSIONAL DOUBLE-FIN ASSEMBLY

M. LENCMANE\textsuperscript{1} and A. BUIKIS\textsuperscript{1,2}

\textsuperscript{1}University of Latvia, Department of Mathematics
Zelzlu Street 8, Riga LV-1002, Latvia
\textsuperscript{2}Institute of Mathematics and Computer Science
Raina boul., Riga LV-1459, Latvia
E-mail: marija.lencmane@lu.lv, buikis@latnet.lv

Extended surface is used specially to enhance the heat transfer between a solid and surrounding medium. Such an extended surface is termed a fin. The rate of heat transfer is directly proportional to the extent of the wall surface, the heat transfer coefficient and to the temperature difference between solid and the surrounding medium. Finned surfaces are widely used in many applications such as air conditioners, aircrafts, chemical processing plants, etc. Finned surfaces are also used in cooling electronic components. In [1] is considered performance of a heat - exchanger consisting of rectangular fins attached to both sides of plane wall. In [1,2] works one-dimensional steady-state double-fin assembly problem is compared with the single-fin assembly. In paper [3] mathematical three-dimensional formulation of transient problem for one element with one rectangular fin is examined, reduce it by conservative averaging method to the system of three heat equations with linear sink terms. In [4] was considered exact analytical solution for two-dimensional steady-state process for system with one rectangular fin by the method of Green function [5].

This paper deals with mathematical three-dimensional formulation of steady-state problem and transient problem for heat-exchanger consisting of rectangular fins attached to both sides of a plane wall(double-fin assembly) and reduce it by conservative averaging method to the two-dimension system of the three Laplace equations (for steady-state system). Analytical solution based on Green function approach is proposed. This solution is obtained in the form of 2\textsuperscript{nd} kind Fredholm integral equations.

REFERENCES

SURVIVAL ANALYSIS METHODS IN INSURANCE: ANALYSIS AND MODELLING

T. LEONAVICIENE and K. JUCIKAITE

Vilnius Gediminas Technical University
Saulėtekio av. 11, Vilnius, Lithuania
E-mail: terese.brazauskaite@gmail.com, KJucikaite@gmail.com

In this paper, non-life insurance invoked rarely encountered in the field of survival analysis model. Survival analysis model described in [2], [3], [4], [5] was adapted to the insurance company analysis. Analysis was created using SAS program [1].

The housing and household property insurance contract details: the object of insurance, the events of policyholders types and characteristics signed an insurance company in Lithuania for 2007 – 2008 were investigated using the Survival analysis methods. 8608 insurance policies (contracts), which covered 17420 objects (events) were selected for the research. Survival analysis methods were applied to this data. It allows us to asses the moments of the greatest risk and distinguish its decisive factors. The main factors are: the location of the object (10 Lithuanian counties), object type (building, room, house and property values, civil liability), the object structure (frame / log (wood), wooden/stone, stone), and of course the insurance amount (small amount of insurance is less than 250000 Lt, the average amount of insurance – 250000 – 500000 Lt and big – over 500000 Lt).

During the process of the research life tables were created, the hazard and the risk functions calculated, the survival function evalutions were calculated using Kaplan – Meier method. Then the statistical hypothesses were formulated and checked and risk decisive factors were determined.

The object location and object type has the greatest influence the survival of objects. Objects in the city of Vilnius have 2 – 2.5 times higher level of risk in relation to objects in other areas of Lithuania. Rooms have 2.5 – 3 times more risk than the other objects. Survival functions in the groups distinguished according to the Lithuanian countys are significantly different. Survival functions also depends on the object type and amount of insurance.

We presume that survival analysis results would allow the insurance company to organize the work in more efficient way, to forsee the motion of capital. These results also were used for modelling the events.

REFERENCES


62
In today’s automobile’s electrical systems, high temperature loads stem from a multitude of current-carrying components (wirings, fuses, etc). In order to keep the heat generation as low as possible, it is necessary to optimize the geometrical composition and the employed material of all components as well as their interaction. Before this optimization can be performed, the single components have to be modelled and simulated by mathematical methods.

In this work, we investigate the multi cable – a cable consisting of several single cables. To describe its heat generation, we use the two dimensional heat equation with discontinuous coefficients for the heat conductivity on the interfaces of different materials and suitable boundary conditions.

The increase of the electrical resistance resulting from the rise of temperature is approximated linearly:

\[
c \rho_{MET} \frac{\partial T}{\partial t} - \nabla \cdot ( \lambda_{MET} \nabla T ) = \left( \frac{I}{A_{MET}} \right)^2 \frac{1}{\kappa_{MET}} \left( 1 + \alpha_{\rho_{MET}} (T - T_{ref}) \right) \quad \text{in } \Omega_{MET}. \quad (1)
\]

The heat transmission from solid material to air is described by a Robin type boundary condition

\[
\lambda_{ISO} \frac{\partial T}{\partial n} + \alpha(T) T = \alpha(T) T_U \quad \text{on } \Gamma_{INT}, \quad (2)
\]

where \( \alpha(T) \) denotes the temperature-dependent heat transmission coefficient. It contains convective and radiative effects and causes the system to be nonlinear.

To solve the problem numerically, we use the finite element method. The nonlinear equation system is solved iteratively by means of the Newton-Raphson Method. Numerical results and comparisons to other works concerning this subject \([1; 2]\) are presented.

REFERENCES


DISCRETE APPROXIMATION OF ANALYTIC FUNCTIONS BY SHIFTS OF PERIODIC HURWITZ ZETA-FUNCTIONS

R. MACAITIENĖ

Šiauliai University, Šiauliai College
P. Višinskio 19, LT-77156, Šiauliai, Lithuania
E-mail: renata.macaitiene@mi.su.lt

One remarkable property of zeta-functions is that every analytic function can be approximated uniformly on compact sets by shifts of these functions. In 1975, for the Riemann zeta-function this property was discovered by S. M. Voronin. He proved that any non-vanishing continuous function \( g(s) \) on the disc \( \{ s \in \mathbb{C} : |s - 3/4| \leq r \}, \ r < 1/4, \) and analytic in the interior, can be approximated with any degree of accuracy by shifts \( \zeta(s + i\tau) \). Later, it was proved that the set of shifts of \( \zeta(s) \) which approximate a given function \( g(s) \) has a positive lower density. Now the remarkable Voronin theorem has a more general statement, which was obtained by A. Laurinčikas [1]. The concept of discrete approximation of analytic functions is due to A. Reich: here the shifts are taken from certain arithmetic progressions.

In the report, we discuss the approximation of analytic functions by discrete shifts of periodic Hurwitz zeta-functions.

Let \( \mathcal{A} = \{ a_m : m \in \mathbb{N}_0 \} \) be a periodic sequence of complex numbers with minimal period \( k \in \mathbb{N}. \) Then the periodic Hurwitz zeta-function \( \zeta(s, \alpha; \mathcal{A}), s = \sigma + it, \) with parameter \( \alpha, \ 0 < \alpha \leq 1, \) is defined, for \( \sigma > 1, \) by

\[
\zeta(s, \alpha; \mathcal{A}) = \sum_{m=0}^{\infty} \frac{a_m}{(m+\alpha)^s},
\]

and is meromorphically contiinuable to the whole complex plane with possible simple pole \( s = 1, \) and \( \Re s = 1 \). In [2] and [3], the continuous approximation theorems for \( \zeta(s, \alpha; \mathcal{A}) \) and \( (\zeta(s, \alpha_1; \mathcal{A}_1), ..., \zeta(s, \alpha_n; \mathcal{A}_n)) \) were investigated. The aim of this report is to present the discrete version of these theorems. One of the results is the following theorem.

**Theorem 1.** Suppose that \( \alpha \) is transcendental number and \( h > 0 \) is a fixed number such that \( \exp \left( \frac{2\pi h}{\alpha} \right) \) is rational. Let \( K \) be a compact subset of the strip \( D \) with connected complement, and let \( f(s) \) be a continuous function on \( K \) which is analytic in the interior of \( K. \) Then, for every \( \epsilon > 0, \)

\[
\liminf_{N \to \infty} \frac{1}{N+1} \# \left\{ 0 \leq m \leq N : \sup_{s \in K} |\zeta(s + imh, \alpha; \mathcal{A}) - f(s)| < \epsilon \right\} > 0.
\]

**REFERENCES**


ESTIMATION OF DIFFUSION AND REACTION RATES OF THE YTTRIUM ALUMINIUM GARNET

M. MACKEVIČIUS\(^1\), F. IVANAUSKAS\(^2\) and A. KAREIVA\(^3\)
\(^1\) Institute of Mathematics and Informatics
Akademijos 4, Vilnius LT-08663, Lithuania

\(^2\) Vilnius University, Department of Mathematics and Informatics
Naugarduko 24, Vilnius LT-03225, Lithuania

\(^3\) Vilnius University, Department of Chemistry
Naugarduko 24, Vilnius LT-03225, Lithuania

We consider the diffusion and reaction rates at high temperatures (1000–1600 °C) for heterogeneous reaction of the yttrium aluminium garnet (YAG) [1]. The reaction is described by the formula

\[3\text{Y}_2\text{O}_3 + 5\text{Al}_2\text{O}_3 \rightarrow 2\text{Y}_3\text{Al}_5\text{O}_{12},\]

which we consider as a bimolecular one \((A + B \rightarrow C)\). In our model, the YAG synthesis is described by the following reaction–diffusion system in the region \((x, t) \in V \times (0, \infty)\):

\[
\begin{align*}
\frac{\partial c_1}{\partial t} &= 2 \sum_{j=1}^{2} D_1 \frac{\partial^2 c_1}{\partial x_j^2} - \frac{1}{5} kc_1 c_2, \\
\frac{\partial c_2}{\partial t} &= 2 \sum_{j=1}^{2} D_2 \frac{\partial^2 c_2}{\partial x_j^2} - \frac{1}{3} kc_1 c_2, \\
\frac{\partial c_3}{\partial t} &= 2 \sum_{j=1}^{2} D_3 \frac{\partial^2 c_3}{\partial x_j^2} + \frac{2}{15} kc_1 c_2,
\end{align*}
\]

with initial conditions \(c_i(x, 0) = c_i^0, x \in \bar{V} = V \cup \partial V, i = 1, 2, 3\), and boundary conditions \(\frac{\partial c_i}{\partial x_j} = 0, x \in \partial V, t \geq 0, i = 1, 2, 3\). Here \(D_i\) is the diffusion coefficient, \(c_i\) is the concentration of the \(i\)th reactant, and \(k\) is the reaction rate.

By analyzing the relations between parameters \(k\) and \(D_i\), we find their values that are in agreement with true chemical reactions in the two-dimensional model (where \(V = [0, a] \times [0, a]\)) for temperatures 1000 °C, 1200 °C, and 1600 °C. We use the synthesis time, the dimensions of reactants, and the synthesis type as data, describing reaction–diffusion processes.

REFERENCES

CONSTRUCTION OF LOW DIMENSION REPRESENTATIONS OF 2-D AND 3-D REGIONS IN THE IMAGE PROCESSING

M. MEILŪNAS¹, M. PAULINAS², J. ROKICKI² and L. LAUKAITYTĖ¹

¹ Vilnius Gediminas Technical University
Sauletekio al. 11, LT-10223, Vilnius, Lithuania
E-mail: mm@vgtu.lt

² Vilnius Gediminas Technical University
Naugarduko 41, LT-03227, Vilnius, Lithuania

Thinning and skeletonization are widely used for processing of thresholded images, data reduction, pattern recognition etc.

Skeletonization is an iterative process frequently used in image processing to reduce the pictorial content of an object to a graph representing only the general shape of the object. Among many other areas, skeletonization techniques are finding an increasing number of applications in automated medical diagnostics and analysis.

We propose and discuss algorithms for skeletonization and building of low dimension representations of some biological structures. These algorithms are based on step-wise approximations of medial axes and centerlines.

REFERENCES


ON INTEGRAL REPRESENTATION OF THE TRANSLATION OPERATOR

P. MIŠKINIS

Vilnius Gediminas Technical University
Sauletekio Ave.11, LT-10223, Vilnius, Lithuania
E-mail: paulius.miskinis@vgtu.lt

The translation operator ˆT_a is an operator in which

\[ ˆT_a f(x) = f(x + a), \]  

where a is a parameter of the translation. For the smooth functions \( f(x) \), the corresponding generator is the exponential map of an ordinary derivative [1]:

\[ ˆT_a = e^{a \frac{d}{dx}}. \]  

In the case of non-smooth functions when the ordinary derivative does not exist, we have to generalize the corresponding representation (2), substituting the ordinary derivative by the fractional one and turning to the generalized exponential function. The idea of this generalization was first mentioned in the monograph [2].

The proof of the fractional representation of the translation operator and the corresponding integral representation is considered. Some aspects of the translations in graduate spaces and their integral representation, as well as applications in physics are discussed.

REFERENCES


SOME ASPECTS OF MODELING INDUSTRIAL PROBLEMS

H. NEUNZERT

Fraunhofer-Institut fuer Techno- und Wirtschaftsmathematik (ITWM)
Fraunhofer-Platz 1, D-67663 Kaiserslautern, Germany
E-mail: neunzert@itwm.fraunhofer.de

Solving an industrial problem means modeling, computing and analyzing the results. Computing often poses time problems and models have to be simplified in order to get results in a time given by the industrial partner. In that way, hierarchies of models are sometimes generated, where complex models are used for parameter identification of simpler models. The lecture shows examples from the work of our Fraunhofer Institute for Industrial Mathematics at Kaiserslautern, one from textile industry and one from polishing jewels.
ADJOINT PROBLEMS FOR STATIONARY PROBLEMS WITH NONLOCAL CONDITIONS

J. NORKŪNAITĖ and A. ŠTIKONAS

1Faculty of Mathematics and Informatics, Vilnius University
2Institute of Mathematics and Informatics, Vilnius University

We are investigating adjoint problems in differential and discrete cases for stationary problems with NBCs. We define a class of problems with differential operators with NCs and propose a summarized theoretical results for nonlocal problems. We consider the second order discontinuous differential problem with NBCs in general form:

\[ L(u) := - \left( (p(x)u'(x))' + q(x)u(x) + \kappa_0(x)l_0^*(u) + \kappa_1(x)l_1^*(u) \right) = f(x), \quad x \in (0, 1) \]
\[ l_i(u) = \gamma_i(k_i, u) + g_i, \quad g_i \in \mathbb{R}, \gamma_i \geq 0, \quad i = 0, 1, \]

where pair \((l_i(u), l_i^*(u))\) is equal to \((u(i), (-1)^i p(i)u'(i))\) or \((-1)^i p(i)u'(i) + \eta_i u(i), u(i)\), \(\eta_i \in \mathbb{R}\). Functions \(f(x), \kappa_i(x)\) and linear functionals \(k_i\) can be introduced in the following forms:

\[ f(x) = \sum_{j=1}^{L} \left( \mu_j^0 \delta(x - \xi_j) + \mu_j^1 \delta'(x - \xi_j) \right) + \mu(x), \quad \mu_j^0, \mu_j^1 \in \mathbb{R}, \quad \mu \in L_1(0, 1), \]
\[ \kappa(x) = \sum_{j=1}^{M} \left( \beta_j^0 \delta(x - \xi_j) + \beta_j^1 \delta'(x - \xi_j) \right) + \beta(x), \quad \beta_j^0, \beta_j^1 \in \mathbb{R}, \quad \beta \in L_1(0, 1), \]
\[ \langle k, u \rangle = \sum_{j=1}^{N} \left( \alpha_j^0 u(\xi_j \pm 0) + \alpha_j^1 u'(\xi_j \pm 0) \right) + \int_0^1 \alpha(x)u(x) \, dx, \quad \alpha_j^0, \alpha_j^1 \in \mathbb{R}, \quad \alpha \in L_1(0, 1), \]

where \(\xi_j \in (0, 1)\), \(\xi_j \pm 0\) denotes \(\xi_j + 0\) or \(\xi_j - 0\). We search the solution of this nonlocal problem in the form \(u(x) = \varphi(x) + y_0 \varphi_0(x) + y_1 \varphi_1(x)\), where functions \(\varphi, \varphi_0, \varphi_1\) are the solution of the auxiliary problems with classical BCs. The existence and uniqueness of the solution satisfies the same conditions as in the article [1].

REFERENCES

SPLINE COLLOCATION FOR VOLTERRA INTEGRAL EQUATIONS

P. OJA

Institute of Mathematics, Tartu University
J. Liivi 2, 50409 Tartu, Estonia
E-mail: Peeter.Oja@ut.ee

We consider the spline collocation method for the Volterra integral equation of the second kind

\[ y(t) = \int_0^t K(t, s, y(s))ds + f(t), \quad t \in [0, T]. \]

For given integers \( m \geq 1 \) and \( d \geq -1 \) we use the space of splines

\[ S_{m+d}^d = \left\{ u \in C^d[0, T] : u|_{\sigma_n} \in P_{m+d}, \ n = 1, \ldots, N \right\} \]

with a family of grids \( 0 = t_0 < t_1 < \ldots < t_N = T, \ N \to \infty, \) intervals \( \sigma_n = (t_{n-1}, t_n), \ n = 1, \ldots, N, \) and \( P_k \) denoting the set of all polynomials of degree not exceeding \( k. \) The collocation method uses fixed (independent of \( N \)) parameters \( 0 < c_1 < \ldots < c_m \leq 1 \) and determines the approximate solution \( u \in S_{m+d}^d \) by collocation conditions

\[ u(t) = \int_0^t K(t, s, u(s))ds + f(t), \quad t = t_{n-1} + c_i(t_n - t_{n-1}), \ i = 1, \ldots, m, \ n = 1, \ldots, N. \]

In addition, some initial or near-boundary conditions are imposed.

The study of convergence in given situation is our main objective. The results are basing on general convergence theorems for operator equations in Banach spaces. We analyse in details some particular cases of \( m \) and \( d. \) Most part of our results are established for linear equations.
AGGREGATION OF L-FUZZY REAL NUMBERS: PROPERTIES AND SOME APPLICATIONS

P. ORLOVS

University of Latvia, Department of Mathematics
Zellu street 8, Riga, LV-1002, Latvia
E-mail: pavels.orlovs@gmail.com

Aggregation of some values (e.g. arithmetic mean, weighted mean, minimum or maximum) is an essential tool in many fields of mathematics, physics, computer science. Our talk deals with aggregation of L-fuzzy numbers which are analogue of real numbers in the case when \( L = (L, \wedge, \lor) \) is a completely distributive lattice. We use the notion of an L-fuzzy real number originating from B. Hutton’s paper (see [1]) and later developed by several authors.

We investigate a \( t \)-norm extension of an ordinary aggregation operator \( A \) defined on \( \mathbb{R} \) to \( \tilde{A} \) which is defined on the L-fuzzy real line \( \mathbb{R}(L) \) by the following formula (see [2]):

\[
\tilde{A}(z_1, \ldots, z_n)(x) = \bigvee_{x = A(x_1, \ldots, x_n)} T(z_1(x_1), \ldots, z_n(x_n)),
\]

where \( z_1, \ldots, z_n \in \mathbb{R}(L), x_1, \ldots, x_n \in \mathbb{R} \) and \( T \) is a continuous \( t \)-norm.

The aim of our research is to analyze properties of the extended aggregation operator \( \tilde{A} \) on \( \mathbb{R}(L) \) depending on properties of the ordinary aggregation operator and the \( t \)-norm. In particular we consider such properties as associativity, symmetry, idempotence, existence of a neutral and an absorbent element, stability for a linear transformation. By using the extended aggregation operator we define \( t \)-norm based operations with L-fuzzy real numbers and investigate their properties.

This approach in the case when \( L = [0, 1] \) provides us a possibility to aggregate the survival functions of random variables. This kind of functions plays important role in actuarial science and some applications of statistical analysis.

We discuss also the connection (noted by E. Pap and J. Stajner [3]) between the extended aggregation and the optimization problem of finding the extremum of a function on the domain \( D = \{(x_1, x_2, \ldots, x_n) \mid x_1 + x_2 + \ldots + x_n = x, x_i \geq 0, i = 1, \ldots, n\} \). This type of problem often occurs in the mathematical economy and operation research.

This work is a development of the results presented in [4], [5].

REFERENCES

SIMPLICIAL AND RECTANGULAR BRANCH AND BOUND WITH IMPROVED COMPUTATIONALLY CHEAP BOUNDS

R. PAULAVIČIUS and J. ŽILINSKAS

Institute of Mathematics and Informatics
Akademijos 4, LT-08663 Vilnius, Lithuania
E-mail: r.paulavicius@vpu.lt, julius.zilinskas@mii.lt

Lipschitz optimization solves global optimization problems

\[ f^* = \max_{x \in \mathcal{D}} f(x), \]

in which the multivariate objective functions \( f : \mathcal{D} \rightarrow \mathbb{R}, \mathcal{D} \subset \mathbb{R}^n \) satisfy the Lipschitz condition

\[ |f(x) - f(y)| \leq L \|x - y\|, \quad \forall x, y \in \mathcal{D}, \]

where \( L > 0 \) is a constant called Lipschitz constant, the domain \( \mathcal{D} \) is compact and \( \| \cdot \| \) denotes the norm. Apart from the global optimum \( f^* \), one or all global optimizers \( x^* : f(x^*) = f^* \) should be found.

The most studied case of Lipschitz optimization is the unconstrained univariate one \((n = 1)\), for which numerous algorithms have been proposed, compared, and theoretically investigated. The most studied of these methods is due to Piyavskii [2]. For \((n > 2)\), however, finding a Piyavskii type bound and a new sample point constitutes a difficult optimization problem.

Most of methods for Lipschitz optimization fall into three main classes. First, multivariate Lipschitz optimization can be reduced to the univariate case. The second class contains direct extensions of Piyavskii’s method to the multivariate case. Various modifications using Euclidean norm or other norms or close approximations have been proposed. The third class contains many simplicial [3] and rectangular branch and bound techniques, but, in general, considerably weaker bounds. These algorithms differ by the selection, branching and bounding rules.

New bounds over hyper-rectangles and simplices are proposed in this talk. These improved bounds are stronger than trivial bounds, but they are usually still computationally cheap. Branch and bound algorithms with these bounds have been implemented using a branch and bound template [1]. The efficiency of the proposed algorithms is evaluated experimentally and compared with the results of other well-known algorithms. The proposed algorithms often outperform the comparable branch and bound algorithms.

REFERENCES

PIECEWISE POLYNOMIAL COLLOCATION METHODS BASED ON PRODUCT INTEGRATION TECHNIQUES FOR SOLVING WEAKLY SINGULAR INTEGRO-DIFFERENTIAL EQUATIONS

A. PEDAS and E. TAMME

Institute of Mathematics, University of Tartu
J. Liivi 2, 50409, Tartu, Estonia
E-mail: arvet.pedas@ut.ee; enn.tamme@ut.ee

In [1] we studied a piecewise polynomial collocation method for solving boundary value problems of Fredholm integro-differential equations with weakly singular kernels. In [1] the convergence results were established under the assumption that the integrals occurring in the collocation equation can be computed exactly. Unfortunately, this is not a case, often. In the present contribution we propose a discrete version of the collocation method which is based on product integration techniques: we approximate smooth parts of the integrals by piecewise polynomial interpolation and integrate exactly the remaining (more singular) parts of those integrals. In the talk the attainable order of local and global convergence of proposed algorithms is discussed and some results of numerical experiments are presented.

REFERENCES

ADI APPROACH TO THE PARTICLE DIFFUSION PROBLEM IN MAGNETIC FLUID

V. POLEVIKOV and L. TOBISKA

1Belarusian State University, 2Otto-von-Guericke University of Magdeburg
14 Independence Ave., 220030 Minsk, Belarus, 2PF4120, D-39106, Magdeburg, Germany
E-mail: polevikov@bsu.by, tobiska@mathematik.uni-magdeburg.de

We consider a two-dimensional problem of the time-dependent process of diffusion of ferromagnetic particles in a magnetic fluid under the action of a nonuniform magnetic field. The process in a closed magnetic-fluid domain \( \Omega \) is described by the dimensionless equation

\[
\frac{\partial C}{\partial t} = \nabla \cdot (\nabla C - CL(\xi) \nabla \xi)
\]

with the Robin-type boundary condition

\[
\frac{\partial C}{\partial n} - L(\xi)(\frac{\partial \xi}{\partial n})C = 0,
\]

and the uniform initial condition

\( C = 1 \) at \( t = 0 \) where \( C = C(x, t) \) is the relative volume concentration of particles, \( L(\xi) = \coth \xi - 1/\xi \) the Langevin function, \( \xi = \xi(x) \) the dimensionless magnetic-field intensity, \( x = (x_1, x_2) \in \Omega \).

Notice that the exact solution of the steady-state particle concentration problem is given in [1] and is of the form

\[
C = b|\Omega|/\int_{\Omega} b \, dx_1 dx_2,
\]

where \( b = b(x) = \sinh \xi/\xi > 0 \).

To simplify the mathematical statement, we change the variables by

\[
C(x, t) = b(x)u(x, t).
\]

With this substitution the problem becomes

\[
\frac{b \, \partial u}{\partial t} = \nabla \cdot (b \nabla u), \quad x \in \Omega; \quad \frac{\partial u}{\partial n} = 0, \quad x \in \partial \Omega; \quad u = \frac{1}{b}, \quad t = 0.
\]  

(1)

For the numerical solution of problem (1) in a rectangular domain \( \Omega \), the ADI-type scheme

\[
b(x) \frac{y^{j+1/2} - y^j}{\tau/2} = \Lambda_1 y^{j+1/2} + \Lambda_2 y^j, \quad b(x) \frac{y^{j+1} - y^{j+1/2}}{\tau/2} = \Lambda_1 y^{j+1/2} + \Lambda_2 y^{j+1}
\]  

(2)

is presented. Its distinctive feature is that it is of the form (2) not only at the internal nodes but at the boundary nodes as well. Therefore while realizing it, the three-point Thomas algorithm is applied not only along internal lines of the mesh as in the traditional ADI scheme, but also at the boundaries of the mesh domain. The scheme has the local approximation error \( O(\tau^2 + |h|^2) \) on a uniform mesh including boundary nodes. It is shown theoretically that scheme (2) is absolutely stable and converges at a rate of \( O(\tau^2 + |h|^{5/2}) \) in the energetic norm \( \| \cdot \|_D \) where \( D = B^{-1} \), \( B = \text{diag}\{b_{11},b_{12}\} \) and \( b_{11},b_{12} \) are mesh values of the function \( b(x) \). At the same time, it is determined on test problems that scheme (2) is of the second order of convergence by the space steps as well.

Finally, we mention that scheme (2) has been applied in [2; 3] when solving ferrohydrostatics problems.

REFERENCES


MODELING, SIMULATION AND ANALYSIS OF NONLINEAR DYNAMICS IN MULTISECTION SEMICONDUCTOR LASERS

M. RADZIUNAS

Weierstrass Institute for Applied Analysis and Stochastics
Mohrenstrasse 39, 10117 Berlin, Germany
E-mail: radziunas@wias-berlin.de

A nonlinear dynamics of narrow-waveguide multisection edge-emitting semiconductor lasers can be described by the Traveling Wave model [1]. It consists of a hyperbolic system of linear one space dimensional first order PDEs nonlinearly coupled with a system of ODEs, and can be formally written as a following system of equations:

\[
\frac{d}{dt}E = H(n)E,
\]

\[
\frac{d}{dt}n = \varepsilon (I - R(n) - g(n)\|E\|^2),
\]

where \(E = E(z, t) \in \mathbb{C}^4, \quad z \in [0, L], \quad n = n(t) \in \mathbb{R}^m\).

For certain laser types we are able to reduce a given system of PDEs to low dimensional Mode Approximation (MA) systems of ODEs [2]. This reduction is based on the projection of the original infinite-dimensional phase space into the subspace spanned only on a few instantaneously changing spectral elements [3]. Depending on the selection of these spectral elements we can achieve a precise approximation of the original model even for large parameter domains [4].

The constructed low dimensional MA systems are accessible to classical numerical continuation and bifurcation analysis tools [5]. The numerical bifurcation analysis of our model equations provides a better understanding of multisection laser dynamics and helps us to develop new laser design concepts for applications in optical communication systems.

REFERENCES

Γ-CONVERGENCE OF PAIRS OF DUAL FUNCTIONALS

U. RAITUMS

Institute of mathematics and Computer Science, University of Latvia

29 Rainis blvd., LV-1459 Riga, Latvia

E-mail: uldis.raitums@lumii.lv

We consider a slightly modified notion of Γ-convergence of convex functionals in uniformly convex separable Banach spaces. More precisely, we follow the approach by V.V. Zhikov [1] and consider a uniformly convex separable Banach space $X$ and its closed subspace $V$, and we say that a sequence $\{F_j\}$ Γ-converges to $\tilde{F}$ if

$$
\tilde{F}(x) = \inf \{ \liminf_{j \to \infty} F_j(x + v_j) \mid \{v_j\} \subset V, v_j \rightharpoonup 0 \text{ weakly as } j \to \infty \}.
$$

(1)

We establish that under standard coercitivity and growth conditions the Γ-convergence of a sequence of convex functionals $\{F_j\}$ to $\tilde{F}$ implies that the corresponding sequence of dual functionals $\{F^*_j\}$ converges in an analogous sense to the dual to $\tilde{F}$ functional $(\tilde{F})^*$.

REFERENCES

LIMIT THEOREMS FOR THE PERIODIC HURWITZ ZETA-FUNCTION

A. Rimkevičienė

Šiaulėnių College
Aušros al. 40, LT-76241, Šiauliai, Lithuania
E-mail: audronerim@gmail.com

Let \( a = \{a_m : m \in \mathbb{N}_0\}, \mathbb{N}_0 = \mathbb{N} \cup \{0\} \), be a sequence of complex numbers with minimal period \( k \in \mathbb{N} \), and \( \alpha, 0 < \alpha \leq 1 \), be a fixed parameter. The periodic Hurwitz zeta-function \( \zeta(s; \alpha; a) \), \( s = \sigma + it \), is defined, for \( \sigma > 1 \), by

\[
\zeta(s; \alpha; a) = \sum_{m=0}^{\infty} \frac{a_m}{(m+\alpha)^s},
\]

and is meromorphically continued to the whole complex plane.

An application of the function \( \zeta(s; \alpha; a) \) for approximation of analytic functions requires probabilistic limit theorems for \( \zeta(s; \alpha; a) \). In the report, we consider the weak convergence, as \( T \to \infty \), of the probability measure

\[
\frac{1}{T} \text{meas}\{t \in [0, T] : \zeta(\sigma + it; \alpha; a) \in A\}, \quad A \in \mathcal{B}(\mathbb{C}),
\]

where \( \mathcal{B}(\mathbb{C}) \) denotes the class of Borel sets of the complex plane \( \mathbb{C} \), and \( \text{meas}\{A\} \) is the Lebesgue measure of a measurable set \( A \subset \mathbb{R} \).

Since the function \( \zeta(s; \alpha; a) \) depends on the parameter \( \alpha \), three cases are considered:

1. The simplest case is of transcendental \( \alpha \). In this case, the set \( \{\log(m+\alpha) : m \in \mathbb{N}_0\} \) is linearly independent over the field of rational numbers \( \mathbb{Q} \).
2. Case of rational \( \alpha \) is reduced to the system \( \{\log p : p \text{ is prime}\} \) which also is linearly independent over \( \mathbb{Q} \).
3. The most complicated case is of algebraic irrational \( \alpha \). In this case, at least 51 percent of the set \( \{\log(m+\alpha) : m \in \mathbb{N}_0\} \) are linearly independent over \( \mathbb{Q} \).

The explicit form of the limit measure is given.

REFERENCES

SPECIAL MULTILAYER PERCEPTRON PARAMETER OPTIMIZATION

L. RINGIENE and G. DZEMYDA

Institute of Informatics and Mathematics
Akademijos 4, LT-08663, Vilnius, Lithuania
E-mail: lauraringiene@gmail.com, dzemyda@ktl.mii.lt

Data coming from the real world are often difficult to understand because of its high dimensionality. For human perception, the multidimensional data must be represented in a low-dimensional space, usually of two or three dimensions. Let the analysed data set is $X_i = (x_{1i}, ..., x_{ni}), i = 1, m$, here $n$ is the initial dimensionality, $m$ is the number of data items. The resulting dimensionality (denote it by $d$) is 2 or 3: the resulting data set is $Y_i = (y_{1i}, ..., y_{di}), i = 1, m$. The goal of the visualization methods is to represent the input data items in a lower-dimensional space so that certain properties (e.g. clusters, outliers) of the structure of the data set were preserved as faithfully as possible. There are many methods for multidimensional data visualization (e.g. principal component analysis, multidimensional scaling, locally linear embedding etc.) [1, 2]. Artificial neural networks find large applications in visualization of multidimensional data [3–6].

In this paper, propose and investigate a special feed forward neural network for multidimensional data visualization. The network consists of Gaussian radial basis functions and multilayer perceptron [7], which is trained using the error back propagation algorithm. The main idea is that the multidimensional data are visualized (presented in two-dimensional or three-dimensional space) using outputs of the last hidden layer of the neural network. The number of neurons in the hidden layer is equal to the desired dimensionality (two or three). The proposed visualization approach includes data clustering, determining the parameters of radial basis functions and forming the data set to train the multilayer perceptron. The neural network parameters that determine visualization results are chosen experimentally.

REFERENCES

SHEET LINER MOTION IN A MAGNETIC COMPRESSOR: ELASTIC, LIQUID AND PLASTIC LINER MODELS COMPARISON

A.S. RODIN1, M.P. GALANIN1 and A.P. LOTOTCKII2

1 Keldysh Institute of Applied Mathematics of RAS
Miuskaya sq., 4, 125047, Moscow, Russia
E-mail: rals@bk.ru
2 SRC RF TRINITI
142190, Troitck, Moscow region, Russia

There are considered magnetic compressor [1], which is one of the stages in the setup “MOL” (Magnetic compression of liners). This setup is intended for research of work at all levels of setup “Baikal” and generation of electrical pulse of mega joule level. The compressor work is based on the magnetic field compression by metallic liner, which is accelerated by electro – dynamical forces to velocity of 1 km/s.

There were constructed various mathematical models of sheet liner motion in the magnetic compressor [2]–[3]. Liquid, elastic, and plastic (case of arbitrary deformations) models of the liner were presented. There were considered two 2D approximations, corresponding to the longitudinal and transverse cross – sections of spatial region. Set of calculations was executed and comparative analysis of liner behavior for different models and different material deformation curves was performed.

Calculations showed that constructed liquid and plastic models reflect the following experimental data [1]: in longitudinal cross – section the central part of liner plate performs plane – parallel motion, in transverse cross – section the width of liner decreases because of edges deformation [2]–[3]. Work was done under partial financial support of RFBR (project No. 09-01-00151).

REFERENCES


ABOUT FULL-MULTIGRID METHOD

C. RODRIGO, F.J. GASPAR, C.W. OOSTERLEE and I. YAVNEH

Department of Applied Mathematics, University of Zaragoza
Pedro Cerbuna 12, 50009, Zaragoza, Spain
E-mail: carmenr@unizar.es

In order to accelerate iterative solution procedures, the use of hierarchies of computational grids of various resolutions is a well-known technique. Multigrid algorithms are among this kind of methods, and they have become one of the most efficient numerical techniques for solving the algebraic linear equation systems arising from the discretization of partial differential equations. By other hand, the obtention of a good initial approximation can benefit the performance of iterative methods. The construction of such an approximation by means of inexpensive computations on coarser grids is known as nested iteration. Typically, the problem is initially discretized and solved on a very coarse grid, and this solution is interpolated to a finer grid, where it serves as an initial approximation. Several iterations of some numerical algorithm are employed, and the result is interpolated to a still finer grid, and so on, until an approximate solution is obtained on the target grid.

Nested iteration and multigrid computational techniques are combined to yield the so-called full multigrid (FMG) algorithm. In this well-known approach, the iterative solver is a multigrid cycle which employs the very same grid hierarchy to greatly accelerate the convergence of a basic iterative solver (relaxation). FMG is the most efficient approach to multigrid methods, since it is considered to be asymptotically optimal, that is, the number of arithmetic operations required is proportional to the number of grid points, with only a small constant of proportionality.

The goal of the FMG algorithm should be to yield a numerical solution whose error is comparable to the discretization error. Typically, the common lore states that one or two multigrid cycles are sufficient to reach such discretization accuracy. However, the key question then is whether the solution obtained by this algorithm is “sufficiently accurate”, and in practice, it may be quite difficult to assess whether the FMG solution indeed yields discretization-level accuracy. This notion is formalized here by defining a worst-case relative accuracy measure, denoted $E_{\text{FMG}}^\ell$, which compares the total error of the $\ell$-level FMG solution against the inherent discretization error. This measure can be used for tuning algorithmic components so as to obtain discretization-level accuracy. FMG has received relatively little attention in terms of analysis, and in this work, local Fourier analysis (LFA) framework for FMG is also developed for estimating $E_{\text{FMG}}^\ell$. This results in a tool which yields, a-priori, valuable insights into the various components of the FMG algorithm and their effect on the final relative accuracy. Some Fourier analysis results are presented to confirm the theoretical estimates, and numerical experiments illustrate its practical utility.
DEPENDENCE OF HARMONIC MEASURES OF A MULTIPLY CONNECTED DOMAIN ON PERTURBATION OF THE BOUNDARY

S. ROGOSIN
Belarusan State University
Nezavisimosti ave 4, BY-220030, Minsk, Belarus
E-mail: rogosin@gmail.com

Let $D = \bigcap_{k=1}^{n} \text{ext cl } D_k$ be an unbounded multiply connected domain which is an intersection of exteriors to $n$ closed simple Jordan curves $L_k = \partial D_k$. By definition (see [1]) the harmonic measure of the domain $D_k$ (or of the curve $L_k$) is a function $\alpha_k$, harmonic in $D$ and continuous in $\text{cl } D$, satisfying the following boundary condition

$$\alpha_k(t) = \delta_{ks}, \quad t \in \bigcup_{s=1}^{n} L_s. \quad (1)$$

In [2, p. 153] an explicit formula for harmonic measures in the case when $D$ is a multiply connected circular domain $D = \{z \in \mathbb{C} : |z - a_k| > r_k, a_k \in \mathbb{C}, r_k > 0, |a_k - a_j| > r_k + r_j, k = 1, \ldots, n\}$ is found, namely,

$$\alpha_k(z) = \sum_{m=1}^{n} A_m \left[ \text{Re} \left\{ \log \prod_{k \neq m} \frac{w_k(k) - a_m}{z^*_k - a_m} \right\} + \log \prod_{k \neq m} \frac{z^*_k(k_1) - a_m}{w_k(k_1) - a_m} \right.$$

$$\left. \log \prod_{k_2 \neq k_1} \prod_{k_1 \neq k_2} \frac{w(k_1,k_2) - a_m}{z^*_k(k_1,k_2) - a_m} \right\} + \log |z - a_m| \right] + A, \quad (2)$$

where $A_m, A$ are complex constants, uniquely defined by certain system of linear algebraic equations,

$$z^*_k = \frac{r^2_k}{z-a_k} + a_k$$

is a symmetry with respect to the $k$th circle, $z^*_k(k_1,k_2) = \left(z^*_k(k_1)\right)_{(k_2)}$, $w$ is a fixed point in $\text{cl } D$.

In our work we perform a perturbation analysis of conformal measures on local transformation of the boundary of the initial multiply connected circular domain (see [3]).

The work is partially supported by the Belarusian Fund for Fundamental Scientific Research.

REFERENCES

BOUNDARY VALUE PROBLEMS WITH NONLOCAL BOUNDARY CONDITIONS AND GREEN’S FUNCTIONS FOR SUCH PROBLEMS

S. ROMAN and A. ŠTIKONAS

Institute of Informatics and Mathematics, Vilnius University
Akademijos 4, LT-08663, Vilnius, Lithuania
E-mail: sverlan.roman@ktl.mii.lt; ash@ktl.mii.lt

We analyze the second-order linear differential equation with two additional conditions

\[ Lu := -(p(x)u')' + q(x)u = f(x), \]
\[ \langle L_0, u \rangle = 0, \]
\[ \langle L_1, u \rangle = 0, \]

where \( p(x) \geq p_0 > 0, p \in C^1[0, 1], q \in C[0, 1], f \in C(0, 1) \cap L_2(0, 1) \) and \( L_0, L_1 \) are linear functionals.

We find an expression for the solution and construct Green’s function for this problem. Formula

\[ G(x, s) = \frac{\langle L_0 \cdot L_1, D[u_0, u_1, C^1(\cdot, \cdot)][u, w, \cdot] \rangle}{\langle L_0 \cdot L_1, D[u_0, u_1] \rangle}, \]

where \( D[v, w](x, y) := \begin{vmatrix} v(x) & v(y) \\ w(x) & w(y) \end{vmatrix} \)

is valid. It allows to find Green’s function for equation with two additional conditions if we know Green’s function \( C^1 \) for the same equation but with another additional conditions (for example, classical). We apply formulae to Green’s functions for problems with nonlocal boundary conditions. Examples of Green’s functions for problems with nonlocal boundary conditions are proposed.

We can generalized these results for \( n^{th} \)-order boundary problems.

REFERENCES

RECURSIVE BAYESIAN ESTIMATORS FOR SYSTEMS WITH FRACTIONAL OBSERVATION MODELS

M. ROMANOVAS\(^1\), L. KLINGBEIL\(^1\), M. TRÄCHTLER\(^1\) and Y. MANOLI\(^1,2\)

\(^1\)HSG-IMIT – Institute of Micromachining and Information Technology
Wilhelm-Schickard-Straße 10, D-78052, Villingen-Schwenningen, Germany
E-mail: (michailas.romanovas, lasse.klingbeil, martin.traechtler)@hsg-imit.de

\(^2\)Chair of Microelectronics, Department of Microsystems Engineering (IMTEK), University of Freiburg
Georges-Köhler-Allee 101, D-79110, Freiburg, Germany
E-mail: manoli@imtek.de

The work presents Recursive Bayesian Estimators \([1]\) such as Kalman Filters for state-space models with the process observation being a fractional derivative of the variable rather than the variable itself. Such models can serve as reasonable approximations of some systems such as those of complex underlying dynamics due to biomimetic materials \([2]\). As an example of realistic model for the filter performance evaluation we consider the discrete approximation of the neuron dynamics for motion coding in the vestibular system \([3]\):

\[
r(t) = r_0 + \lambda \frac{d^q \omega(t)}{dt^q},
\]

where \(r(t)\) - is the neuron’s firing intensity function, \(r_0\) is the spontaneous firing rate, \(\lambda\) - the sensitivity at unit frequency, \(\omega(t)\) - the angular rate and \(q\) - is the order of fractional differintegral.

We also provide the preliminary results on nonlinear filter modification (Extended Fractional Kalman Filter) and discuss its performance on the advanced nonlinear neuron firing model \([3]\):

\[
r(t) = r_0 + \lambda \tanh \left( \kappa \left( \frac{d^q \omega(t)}{dt^q} - \bar{\omega} \right) \right),
\]

where \(\kappa\) and \(\bar{\omega}\) are used to adjust the saturation level and directional symmetry of the nonlinearity.

The performance of the developed estimators is compared to conventional integral-order estimators. We also discuss the state-space representation for fractional order models, implementation of advanced approximation schemes, results on model observability and stability of the estimator as well as other relevant properties.

REFERENCES


The aim of the work is to develop and investigate fluid and structure codes coupling into one aeroelastic code based on Reduced Order Galerkin Model of the flow and nonlinearity of structural models. This phenomenon has important influence in many aeronautical applications.

Aerodynamic forces from stream of air lead to deformations of the structure and these deformations change aerodynamic forces. More complicated dynamic analysis algorithm, e.g. including hyper-elastic materials, is also supported.

The existing aeroelastic tool, based on high-fidelity flow solver, was successfully tested and the results were compared with wind tunnel results for NACA 0012 profile. The computations were carried out in parallel environment for fluid mesh larger than one million tetra elements.

Structural part of in-house aeroelastic code includes nonlinear material models like Mooney Rivlin and Neo Hooke, but also respects elements like nonlinear springs (geometrical nonlinearities). Structural model was tested for linear and nonlinear case.

Reduced Order Model of the flow for aeroelastic analyses with geometrical nonlinearities, based on Galerkin method, is under development. It requires mode basis, computed in high-fidelity computations (like RANS or Direct Numerical Simulation of Navier-Stokes or Euler equations), to approximate the high-fidelity solution and in the projection of the governing equations.

In the paper the results of the unsteady numerical simulations for flexible delta wing (as an example of applying nonlinear material models) are presented.

Next, the Proper Orthogonal Decomposition of resulting velocity and pressure fields is performed to create mode basis that is required for further Reduced Order Modelling.

REFERENCES


IMPACT OF FACTOR ROTATION METHODS ON SIMULATION COMPOSITE INDICATORS

J. RUKŠĖNAITĖ

Vilnius Gediminas technical university
Sauletekio al. 11, LT-10223 Vilnius, Lithuania
E-mail: jurga.ruksenaite@fm.vgtu.lt

Composite indicators (CI) which compare country performance are useful tool in setting policy priorities and in monitoring performance. The use of CI’s around the world is growing year after year. Composite Indicators is mathematical function-model of indicators and weights from complex different fields. Usually CI’s methodology is used for ranking different countries (worldwide or regional etc) from the best to the worst.

In this work we have defined CI as an additional tool for country’s economy analysis. Further more as showed the empirical results, CI can be used instead of Gross Domestic Product (GDP). GDP is often criticized for the inaccuracy and unreflecting view of economic development and well-being of citizens.

The choice of single indicators weights is one of the steps in compiling CI. Different methods produce different view of CI. The mathematical problem is setting weights that reflect both theoretical framework and statistical data properties.

The aim of the work is to analyse impact on the compilation of CI applying different weights for the indicators. Factor analysis with orthogonal factor rotation methods has been used. In social science often researches allow some correlation among factors, therefore oblique rotation has been involved in analysis too. Additional restriction - variables are short time series (n < 50).

Lithuania’s social-economic indicators which strongly correlate to GDP have been chosen for the simulation of CI. After applying factor rotation methods, particular technique for estimation weights of individual indicators has been used. The results have proved the hypotheses that selection of factor rotation method has the crucial impact on the CI compilation.

REFERENCES

We continue to develop a theory of $L$-fuzzy valued integral $\int_{E} f d\mu$ of a function $f : X \rightarrow \mathbb{R}$ over an $L$-fuzzy set $E \in L^X$ with respect to an $L$-fuzzy valued $T$-measure $\mu$. We assume that $L$ is a completely distributive lattice, operations for $L$-fuzzy sets and $L$-fuzzy real numbers are based on a t-norm $T$, $f$ is a measurable function with respect to a finite measure $\nu$ defined on a $\sigma$-algebra of crisp sets, $\mu$ is the t-norm based extension of $\nu$, an $L$-fuzzy set $E$ is measurable with respect to $\mu$.

The t-norm based construction of fuzzy valued $T$-measure was considered in [1] and [2]. The concept of fuzzy valued integral first defined at the previous conference MMA 2009 was developed in our talk at the conference FSTA - "Fuzzy Sets: Theory and Applications" in 2010 in Slovakia.

At the current stage our main focus area refers to the basic properties and possible applications of $L$-fuzzy valued integrals.

REFERENCES


THE AERODYNAMICS ASPECTS OF MORPHING CAR 
REAR SPOILER - PRELIMINARY STUDY

M. RYCHLIK and M. NIEZGDKA

Institute of Combustion Engines and Transport, Division of Methods of Machine Design, Poznan University of Technology
Piotrowo 3, 60-965 Pozna, Poland
E-mail: rychlik.michal@poczta.fm

The newest investigations from area of aircraft aerodynamics are oriented on increasing flight parameters (such as: level of maneuvers, range, comfort and others) with simultaneous reduction of fuel consumption. This goal is achieved by control of aerodynamics properties and changing shape of wings or all structure of plane [1]. The concept of such control is named as ”morphing”. Morphing is based on small actuation mechanisms (or special materials) [2, 3] using the many different techniques for reshaping of geometry.

Morph can be defined as ”to transform or be transformed completely in appearance or character” [4]. Mostly morphing is realized for shape of wings. The inspiration for that concept was observation of birds flight. Dependent from air conditions birds can adapt many parameters of their geometry to minimize energy of flight.

That morphing technique can be used not only in aircraft industry. This high technology can be very important in automotive manufacturing - especially in high speed racing cars. These days’ generally car spoilers have inflexible structures without any mechanization. That rigid setting gives optimal parameters of flow only in very tight range of speed. Because speed of car is changing very dynamically, such solution is not efficient.

Force generated by rear spoiler is very important not only for decreasing fuel consumption but primarily for increasing safety and control of the car in full range of speed.

This article presents aerodynamics analysis of morphing car rear spoiler. In the first step review of airfoil profiles for car application (as a rear spoiler) was analyzed. Authors investigate the airfoil profile which generates maximum lifting force with the lowest drag force. The idea of authors is developed the morphing car spoiler with constant pressure force in widely range of speed. In this paper the preliminary study of numerical fluid flow analysis of morphing car spoiler are present and discussed.

REFERENCES

THE FINITE-DIFFERENCE SCHEME FOR
TWO-DIMENSIONAL PARABOLIC EQUATION WITH
NONLOCAL INTEGRAL CONDITIONS

S. SAJAVIČIUS

Vilnius University
Naugarduko 24, LT-03225, Vilnius, Lithuania
E-mail: svajunas.sajavicius@mif.vu.lt

We construct and analyse the weighted finite-difference scheme for two-dimensional parabolic equation
\[
\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + f(x, y, t), \quad 0 < x < 1, \quad 0 < y < 1, \quad 0 < t \leq T,
\]
subject to nonlocal integral conditions
\[
\begin{align*}
&u(0, y, t) = \gamma_1 \int_0^1 \alpha(x) u(x, y, t) \, dx + \mu_1(y, t), \\
&u(1, y, t) = \gamma_2 \int_0^1 \beta(x) u(x, y, t) \, dx + \mu_2(y, t),
\end{align*}
\]
boundary conditions
\[
\begin{align*}
&u(x, 0, t) = \mu_3(x, t), \quad u(x, 1, t) = \mu_4(x, t), \quad 0 < x < 1, \quad 0 < t \leq T,
\end{align*}
\]
and initial condition
\[
u(x, y, 0) = \varphi(x, y), \quad 0 \leq x \leq 1, \quad 0 \leq y \leq 1,
\]
where \( f(x, y, t), \mu_1(y, t), \mu_2(y, t), \mu_3(x, t), \mu_4(x, t), \alpha(x), \beta(x), \varphi(x, y) \) are given functions, \( \gamma_1, \gamma_2 \) are given parameters, and \( u(x, y, t) \) is an unknown function.

The main attention is paid to the stability of scheme. We use the well-known stability analysis technique which was used in order to analyse the finite-difference schemes for differential problems with other types of nonlocal conditions (see, e.g., [1]–[3]). We demonstrate the efficiency of the considered finite-difference scheme by solving several test problems with different types of weight functions \( \alpha(x) \) and \( \beta(x) \) and calculating the maximum norm of computational errors. All numerical experiments were performed using the technologies of grid computing.

REFERENCES


ON SOME PROPERTIES OF THE SPECTRA

N. SERGEJEVA

Daugavpils University, DMF
Parades iela 1, LV-5400, Daugavpils, Latvia
E-mail: natalijasergejeva@inbox.lv

Asymmetric oscillator of the type

\[-x'' = \mu x^+ - \lambda x^-, \quad x^\pm = \max\{\pm x, 0\}, \quad (1)\]

is considered under the boundary conditions of the integral type.

The spectrum of the problem is studied.

Some interesting properties of the spectrum are discussed.

REFERENCES

ROBUST FINITE DIFFERENCE SCHEMES FOR SINGULARLY PERTURBED PARTIAL DIFFERENTIAL EQUATIONS

G.I. SHISHKIN and L.P. SHISHKINA

Institute of Mathematics and Mechanics, Russian Academy of Sciences
S. Kovalevskaya Str., 16, GSP-384, Ekaterinburg, Russia
E-mail: shishkin@imm.uran.ru

In singularly perturbed boundary value problems, for small values of the perturbation parameter $\varepsilon$, boundary and interior layers appear whose width can be arbitrarily small. Derivatives of solutions for such problems in a neighbourhood of the layers grow without bound as $\varepsilon \to 0$. Standard numerical methods developed for regular problems turn out to be nonapplicable for singularly perturbed problems; errors of the discrete solutions in the maximum norm can increase unboundedly as $\varepsilon \to 0$. The standard numerical methods — under a condition their formal convergence — become ill conditioned by virtue of loss of solution stability to disturbances during computations.

Robust numerical methods are special methods for solving of singularly perturbed problems, errors of solutions for which and also their conditioning are independent of values of the perturbation parameter $\varepsilon$. Accuracy of their solutions is specified only by the number of nodes in the meshes used.

In the present talk, special grid approximations of boundary value problems are considered for singularly perturbed elliptic and parabolic equations. Robust finite difference schemes are constructed which inherit the monotonicity property of differential problems [1] that allows us to avoid appearance of nonphysical effects; convergence is considered in the maximum norm [1] which is — opposite to $L_1$, $L_2$ and energy norms — adequate for problems with boundary and interior layers. Objects for our study are problems with boundary layers and other singularities generated by nonsmooth data, unboundedness of domain and also problems for systems of equations and problems with a perturbation vector-parameter [2].

Modern trends in the development of robust difference methods for singularly perturbed problems are discussed such as methods of high-order accuracy, conditioning of special finite difference schemes etc.

REFERENCES


1This research was supported by the Russian Foundation for Basic Research under grant 10-01-00726.
ROBUST FINITE DIFFERENCE SCHEME
OF THE SOLUTION DECOMPOSITION METHOD
FOR A SINGULARLY PERTURBED PARABOLIC
CONVECTION-DIFFUSION EQUATION

L.P. SHISHKINA and G.I. SHISHKIN

Institute of Mathematics and Mechanics, Russian Academy of Sciences
S. Kovalevskaya Str., 16, GSP-384, Ekaterinburg, Russia
E-mail: Lida@convex.ru, shishkin@imm.uran.ru

In the constructing robust numerical methods for singularly perturbed problems an approach
based on the condensing mesh method (on the Bachvalov/Shishkin meshes) is used rather often;
the stepsize in such meshes changes sharply in a neighbourhood of the boundary layer that leads
to difficulties when using economic numerical methods [1; 2]. In the fitted operator method (in the
Il’in scheme), it is possible to apply uniform meshes; however, coefficients in the discrete equations
depend on the singular component of the problem solution that significantly constricts the region
for applicability of this method [3]. Here, a new method is proposed to construct difference schemes
which is different from distinguished above.

An initial-boundary value problem is considered for a one-dimensional singularly perturbed para-
bolic convection-diffusion equation; parabolic boundary layer appears in a neighbourhood of the
output part to the boundary for small values of the perturbation parameter \( \varepsilon \) which takes arbitrary
values in \((0, 1]\). For the differential problem, a new approach is developed to construct \( \varepsilon \)-uniformly
convergent difference schemes, namely, the method based on decomposition of the discrete solution
relating to the method of asymptotic constructs. In this method of the decomposition of the discrete
solution, its regular and singular components are solutions of discrete subproblems on uniform
meshes. The constructed difference scheme converges \( \varepsilon \)-uniformly in the maximum norm at the rate
\( \mathcal{O}(N^{-1} \ln N + N_0^{-1}) \), where \( N + 1 \) and \( N_0 + 1 \) are the number of nodes in the spatial and time
meshes, respectively; for fixed values of the parameter, the scheme converges with the first order of
accuracy.

Unlike from known \( \varepsilon \)-uniformly convergent difference schemes of the fitted operator (in the Il’in
scheme) and the method of meshes condensing in the boundary layer (on the Bachvalov/Shishkin
meshes), here discrete subproblems are solved on uniform meshes, moreover, the coefficients in the
discrete equations do not depend on the singular component of the problem solution.

REFERENCES


\(^1\)This research was supported by the Russian Foundation for Basic Research under grant 10-01-00726.
DIRECT METHOD FOR STATIC ANALYSIS OF FLEXIBLE CURVED CANTILEVER UNDER FOLLOWER FORCE

B. SHVARTSMAN

Tallinn University of Technology, Faculty of Economics and Business Administration
Akadeemia tee 3 - 476, 12618, Tallinn, Estonia
E-mail: boris.shvartsman@tseba.ttu.ee

Nowadays there are many research studies related to large deflection problem of a cantilever beam subjected to a follower forces. The current study deals with flexible curved uniform cantilever beams under a tip-concentrated follower force. The angle of inclination of the concentrated force with respect to the deformed axis of the beam remains unchanged during deformation. The mathematical formulation of such a problem yields the second order nonlinear two-point boundary value problem. Usually, the similar nonlinear boundary value problem is solved using the shooting method [1]. According to the shooting method, the nonlinear boundary value problem is converted to a set of initial-value problems and the unknown initial value is then determined iteratively. The convergence of this iterative procedure depends upon how close the initial guess values are to the solution sought for.

At the present paper the considered boundary value problem is reduced to an initial value problem for pendulum equation by change of variables. As a result, method needs only one time of integration of the initial value problem. Thus, in contrast to the shooting method [1] the problem considered is solved successfully without iterations. Since the solution of initial value problem is unique, deformed shape of the curved cantilever is unique for any initial curvature and angle of inclination of the follower force. Therefore, it is shown that there exist no critical loads in the Euler sense (divergence). These conclusions generalize the same results for straight cantilever beams under concentrated follower forces [2; 3]. Some equilibrium configurations of the curved cantilever beam under a tip-concentrated follower force are presented. The initial value problem was integrated numerically by the fourth order Numerov’s method. The results were compared with solutions obtained by shooting method [1].

The method proposed is comparatively simple. Besides, the direct method needs less computational time than the time required by the shooting method.

REFERENCES

APPROXIMATION OF ANALYTIC FUNCTIONS BY DISCRETE SHIFTS OF THE PERIODIC ZETA-FUNCTION

D. ŠIAUČIŪNAS

Šiauliai University
P. Višinskio 19, LT-77156, Šiauliai, Lithuania
E-mail: siauciunas@fm.su.lt

Let \( A = \{a_m : m \in \mathbb{N}\} \) be a periodic sequence of complex numbers with the least period \( k \in \mathbb{N} \). The periodic zeta-function \( \zeta(s; A) \), \( s = \sigma + it \), is defined, for \( \sigma > 1 \), by

\[
\zeta(s; A) = \sum_{m=1}^{\infty} \frac{a_m}{m^s},
\]

and is analytically continued to the whole complex plane, except, maybe, for a simple pole at \( s = 1 \) with residue \( a = k^{-1} \sum_{m=1}^{k} a_m \). If \( a = 0 \), then \( \zeta(s; A) \) is an entire function.

Approximation of analytic functions by shifts of zeta-functions comes back to S. M. Voronin, who proved the universality of the Riemann zeta-function. J. Steuding began to study the universality of the function \( \zeta(s; A) \). The first result on approximation of analytic functions by discrete shifts \( \zeta(\sigma + imh; A) \) with a fixed number \( h > 0 \) such that \( \exp\{\frac{2\pi m}{h}\} \) is rational for all \( m \in \mathbb{Z} \setminus \{0\} \) has been obtained in [1]. In this report, we present the following result. We recall that the sequence \( A \) is multiplicative if \( a_1 = 1 \) and \( a_{mn} = a_m a_n \) for all \( (m, n) = 1 \).

THEOREM 1. Suppose that the sequence \( \mathfrak{A} \) is multiplicative such that, for all primes \( p \),

\[
\sum_{\alpha=1}^{\infty} \frac{|a_{p^\alpha}|}{p^{\alpha \sigma}} < 1,
\]

and \( h > 0 \) is any fixed number. Let \( K \) be a compact subset of the strip \( \{s \in \mathbb{C} : \frac{1}{2} < \sigma < 1\} \) with connected complement, and let \( f(s) \) be a continuous non-vanishing on \( K \) function which is analytic in the interior of \( K \). Then, for every \( \varepsilon > 0 \),

\[
\liminf_{N \to \infty} \frac{1}{N+1} \# \left\{ 0 \leq m \leq N : \sup_{s \in K} |\zeta(\sigma + imh; \mathfrak{A}) - f(s)| < \varepsilon \right\} > 0.
\]

REFERENCES

MODELLING AN AMPEROMETRIC BIOSENSOR USED FOR SYNERGISTIC SUBSTRATES DETERMINATION

D. ŠIMELEVICİUS and R. BARONAS

1Department of Software Engineering, Vilnius University
Naugarduko 24, LT-03225 Vilnius, Lithuania
2Institute of Mathematics and Informatics
Akademijos 4, LT-08663 Vilnius, Lithuania
E-mail: dainius.simelevicius@mif.vu.lt

An amperometric biosensor is a sensing device used for measuring the concentration of some specific chemical or biochemical substance in a solution [1]. Biosensors use specific biochemical reactions catalyzed by enzymes immobilized on electrodes. Once a reaction product reaches an electrode it oxidizes or reduces producing the faradaic current which is measured.

A special case of biosensors employing the synergistic reactions of substrates was modelled numerically. The mathematical model is based on non-stationary diffusion equations containing non-linear terms related to biochemical kinetics of the reactions taking place during the biosensor operation [2],

\[
\frac{\partial u_i}{\partial t} = D_i \frac{\partial^2 u_i}{\partial x^2} + \sum_{j,l=1}^{n} \alpha_{ijl} k_{jl} u_j u_l, \quad i = 1, 2, ..., n, \tag{1}
\]

where \( x \) stands for the space, \( t \) is the time, \( u_i(x, t) \) is the concentration of substance \( U_i \), \( D_i \) is the diffusion coefficient of the substance \( U_i \), \( k_{jl} \) is the reaction rate coefficient of the reaction between reactants \( U_j \) and \( U_l \), \( \alpha_{ijl} \) is integer coefficient which defines stoichiometrics of reaction between reactants \( U_j \) and \( U_l \), whereas sign of \( \alpha_{ijl} \) coefficient indicates the reaction direction with regard to \( U_i \).

The governing equations (1) together with appropriate initial, boundary and matching conditions form a boundary-value problem, which was numerically solved by applying the finite difference technique [3]. The mathematical model and the numerical solution were validated using experimental data [2]. The model was used to investigate the synergistic effect and other peculiarities of the biosensor action.

REFERENCES

INVESTIGATION DISCRETE STURM–LIOUVILLE PROBLEMS WITH NONLOCAL BOUNDARY CONDITIONS

A. SKUČAITĖ¹, K. SKUČAITĖ-BINGELĖ² and A. ŠTIKONAS³

¹² Vytautas Magnus University, ¹³ Institute of Informatics and Mathematics, Vilnius University
Akademijos 4, LT-08663, Vilnius, Lithuania
E-mail: Agne.Skucaite@fc.vdu.lt, Kristina.Skucaite-Bingele@fc.vdu.lt, ash@ktl.mii.lt

We investigate Sturm–Liouville problem with one classical boundary condition

\[-u'' = \lambda u, \quad t \in (0, 1), \quad u(0) = 0, \quad (1)\]

and another NBC:

\[u(1) = \gamma u(\xi) \text{ or } u(1) = \gamma u'(\xi) \text{ or } u(1) = \gamma \int_0^\xi u(t) \, dt \quad (2)\]

with parameters \(\gamma \in \mathbb{R}\) and \(\xi \in (0, 1)\).

On the interval \([0, 1]\), we introduce uniform grid \(\omega^h = \{x_j = jh, j = 0, 1, ..., N; Nh = 1\}\). Also, we make the assumption, that \(\xi\) coincide with any grid-point, i.e. \(\xi = mh = m/N\), here \(0 < m < N\).

We approximate differential problems (1)–(2) by the following discrete problem:

\[\frac{y_{j-1} - 2y_j + y_{j+1}}{h^2} + \lambda y_j = 0, \quad j = 1, N - 1, \quad y_0 = 0, \quad (3)\]

and one of these NBC:

\[y_n = \gamma y_m; \quad y_n = \gamma \frac{y_{m+1} - y_{m-1}}{2h}; \quad y_n = \gamma \left( \sum_{i=1}^{m-1} y_i + \frac{y_m}{2} \right); \quad y_n = \gamma \left( \sum_{i=m+1}^{N-1} y_i + \frac{y_{m+1} + y_N}{2} \right). \quad (4)\]

We analyze such discrete Sturm–Liouville problems (3)–(4) and investigate how the spectrum of these problems in the complex plane depends on the NBC parameters \(\gamma\) and \(\xi\).

REFERENCES

INVESTIGATION OF COMPLEX EIGENVALUES FOR STATIONARY PROBLEMS WITH NONLOCAL BOUNDARY CONDITION

K. SKUČAITĖ–BINGELĖ¹, A. SKUČAITĖ² and A. ŠTIKONAS³

¹,² Vytautas Magnus University, ²,³ Institute of Informatics and Mathematics, Vilnius University
Akademijos 4, LT-08663, Vilnius, Lithuania
E-mail: Kristina.Skucaite-Bingele@fc.vdu.lt, Agne.Skucaite@fc.vdu.lt, ash@ktl.mii.lt

Let us investigate the Sturm–Liouville problem

\[-u'' = \lambda u, \quad t \in (0,1),\]

with one classical (the first or the second type) boundary condition: \(u(0) = 0; u'(0) = 0\), and another two-point boundary condition \((0 \leq \xi \leq 1)\):

\[u'(1) = \gamma u(\xi); \quad u'(1) = \gamma u'(\xi); \quad u(1) = \gamma u'(\xi); \quad u(1) = \gamma u(\xi)\]

(1)

or integral nonlocal boundary condition:

\[u(1) = \gamma \int_0^1 u(t) \, dt, \quad 0 \leq \xi < 1; \quad u(1) = \gamma \int_0^\xi u(t) \, dt, \quad 0 < \xi \leq 1\]

(2)

with parameter \(\gamma \in \mathbb{R}\).

We analyze such Sturm–Liouville problems and investigate how the spectrum in the complex plane of these problems depends on the nonlocal boundary conditions parameters \(\gamma\) and \(\xi\). These problems with nonlocal boundary condition are not self-adjoint so, the spectrum has points in complex part \([1; 2]\). In the case of two-point boundary conditions complex eigenvalues exist for any value of \(\xi\) (for large \(\gamma\)). We investigate the characteristic function \(\gamma = \gamma(\sqrt{\lambda}, \xi)\), [3] and bifurcation points in respect of the parameter \(\xi\). In the first case of integral nonlocal boundary conditions only real eigenvalues exist. The second integral boundary condition we described two types of bifurcation. First type: when two different complex curves join each other in critical point. Second type: loop curve disappears when zero and pole meet with critical point, i.e., we have constant eigenvalues. All results of simulation are presented as graphs of complex-real and real characteristic functions.

REFERENCES


INVESTIGATION OF THIN CONDUCTING SHEETS INTERACTING WITH $TE_{10}$ WAVE

G. ŠLEKAS$^1$, Č. KANCLERIS$^2$ and R. ČIEGIS$^3$

$^1,^2$ Semiconductor Physics Institute
Goštauto 11, LT-01108, Vilnius, Lithuania
$^3$ Vilnius Gediminas Technical University
Saulėtekio al. 11, LT-10223 Vilnius, Lithuania
E-mail: gediminas.slekas@pfi.lt

Rapid progress of high-frequency electronics stimulates a wide use of structures with 2D electron gas. In fact, this is a very thin high conductivity semiconductor layers in which electrons are moving without collisions with impurities. Microwave detectors are one of examples where such layers are used. In [1] a thin asymmetrically shaped semiconductor structure for the measurement of microwave pulse power has been proposed. The asymmetrical shape leads to the non-uniform distribution of electric field in it and consequently to the non-uniform electron heating in the structure when it interacts with microwaves. As a result, a DC voltage appears on the ends of the structure enabling microwave power measurement. Since electron heating effect is put on a basis of the proposed device performance, it can be used in a wide frequency range, where a traditional semiconductor diode based on microwave current rectification is of a little use.

Although the detector is in general a multilayered structure grown on a semi-insulating substrate, the thin layer conducting current through the structure is of a great importance for the performance of the device. Therefore, we have investigated the interaction of the electromagnetic wave with the conductive layer and calculated the electric field elucidating conditions leading to the non-uniform electric field distribution in it. Our findings will be important to determine the characteristics of the proposed sensor or engineer the sensor with desirable features.

For the calculation of the electromagnetic field components in the waveguide segment with modeled structure, we have used the finite-difference time-domain (FDTD) method, originally proposed by Yee [2]. At the edges of the waveguide non-reflecting Mur’s boundary conditions were applied. Since the thickness of the active layer is much less than the wavelength of the electromagnetic wave, direct account of it in the FDTD calculation procedure needs a very fine grid. Due to complicated geometrical shape of the mesa, the fine grid in the transverse direction is also desirable. To account the thin material layer in the FDTD grid, we have used the sub-cell method proposed by Maloney and Smith [2]. Even by using this approach, each electromagnetic field component should be calculated at more than $10^9$ points. The parallel version of the algorithm was developed, using ParSol data parallelization tool.

REFERENCES


97
RECENT DEVELOPMENT OF DIFFERENT BRANCHES OF THEORETICAL AND APPLIED MATHEMATICS IN THE CONTEXT OF FUZZY SETS

A.ŠOSTAKS

Department of Mathematics, University of Latvia
Zelīnu iela 8, LV - 1002 Riga, Latvia
E-mail: sostaks@lanet.lv

After the inception of the concept of a fuzzy set in 1965 by L. Zadeh [1] and its extension in 1967 to the concept of a lattice-valued, or an L-fuzzy set by J. A. Goguen [2] the attention of many mathematicians working both in theoretical and applied branches of mathematics was directed to the development of the counterparts of these branches in the context of (L-)fuzzy sets. On the other hand, the researchers working in other fields of science, found the concept of a fuzzy set as an important alternative to the probabilistic methods of research. Methods of research based on fuzzy sets are effective in particular in cases when one has to encounter problems caused by vagueness or uncertainty, whose nature is not of the probabilistic origin. In this talk we shall give a very brief survey of theoretical aspects of fuzzy sets and their role in different branches of mathematics and its applications. In particular, we plan to touch the following items:

1. A short introduction into the prehistory and the history of fuzzy sets.
2. The concept of a fuzzy set. Fuzzy numbers, fuzzy quantities and operations with them.
3. Fuzzy sets as the basis for fuzzy logics.
4. Topological and algebraic structures in the context of fuzzy sets.
5. Fuzzy measures and integrals.
8. Applications of fuzzy sets in other fields of science and in industry.
9. Research work done in the field of "Fuzzy Mathematics" at the University of Latvia.

REFERENCES

A SIMPLE FORTH-ORDER SPECTRAL-DIFFERENCE SCHEME FOR NUMERICAL SIMULATIONS OF NONLINEAR DYNAMICS OF OPTICAL PULSED BEAMS

A. STANKEVICH and V. VOLKOV

Belarus State University
Nezavisimosty 4, Minsk, Belarus
E-mail: v.volkov@tut.by

We have considered the problem of optical pulsed beam propagation in nonlinear media. The 3D + 1 nonlinear Schrodinger type equation governing the problem in its general form is (see, for example, [1])

\[ i \frac{\partial E}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial E}{\partial r} + \frac{1}{r^2} \frac{\partial^2 E}{\partial \varphi^2} + \frac{\partial^2 E}{\partial t^2} + F(E)E = 0, \]  

(1)

The use of the cylindrical coordinates provides essential advantages at simulations of optical beams which are close to axially symmetrical. However, the most efficient split-step Fourier method is not applied directly to numerical treatment of such problem.

For numerical analysis of the considered problem we propose a modified split-step scheme based on combination of the Fourier transformation of the linear part of Eq. (1) with respect to \( t \) and \( \varphi \) coordinates, and forth-order finite-difference approximation of the resulting equations in \( r \) direction. The main problem during constructing the high-order finite difference schemes in cylindrical coordinates is the result of the singularity of the differential operator at \( r \to 0 \). In order to avoid this problem we offer the transformation of coordinates \( r^2 = \rho \). By dint of this transformation the singularity is eliminated from the radial part of the differential operator. Moreover, this transformation maps the uniform grid along \( r \) direction to a quasi-uniform grid along \( \rho \). The quasi-uniform grid is refined near the cylinder axis where requirement to the accuracy is usually stronger.

The results of the numerical experiments confirm forth-order accuracy of the proposed scheme for wide range of the angular modes of the solution. This demonstrates the advantage of proposed method upon the similar one, using the standard second-order finite difference scheme. In particular, in order to get a resonable accuracy of the approximate solution of the linear part of the problem the forth-order scheme admits a few times larger discretization step along \( r \) direction in comparison with the second-order scheme.

REFERENCES


GENETIC ALGORITHM-BASED CALIBRATION OF REDUCED ORDER GALERKIN MODELS

W. STANKIEWICZ, R. ROSZAK and M. MORZYŃSKI

Institute of Combustion Engines and Transport, Poznan University of Technology
ul. Piotrowo 3, 60-965 Poznań, Poland
E-mail: witold.stankiewicz@put.poznan.pl

Numerical analyses of fluid flow, based on the solution of Navier-Stokes, Euler or even Reynolds-Averaged Navier-Stokes (RANS) equations are very time-consuming - especially when complex geometries (like full aeroplane configuration) are analysed. These high-fidelity models require computational grids in order of thousands (2D) or millions (3D) of degrees of freedom to cover all important vertex scales.

In the cases when an approximate flow solution is satisfactory (e.g. aeroelastic analyses [1]), or solution time is crucial (e.g. real-time, feedback flow control [2]), low-fidelity, reduced order models (ROMs) might be used.

One of the most widely used methods of flow model reduction is projection of (approximated) governing equation onto space spanned by empirical, physical or mathematical modes, called Galerkin Projection [2; 3]. Usually the mode basis is truncated, that results in system of a few ordinary differential equations (Galerkin System). The lack of smaller eddies in mode basis caused by the truncation influences the quality loss and affects dynamical behaviour of the system.

To improve the Reduced Order Model, minimise the difference between ROM and high-fidelity solutions and balance the energy flow, calibration of the Galerkin System’s terms [4; 5] might be applied.

In this paper reduced order model based on the modes from Proper Orthogonal Decomposition (POD) of the fluid flow is presented. The neglect of small-scale modes is balanced by the calibration of the existing terms of Galerkin System. This calibration is based on genetic algorithm. In each iteration (generation), a number of values of Galerkin System’s terms (chromosomes) are tested to find the minimum of differences between the mode amplitudes computed by Galerkin Model and projected for high-fidelity model. Next, the best solutions are selected. Finally, roulette selection, crossover and mutation procedures are used to create new generation of chromosomes.

REFERENCES

FOURTH-ORDER ADI METHOD FOR POISSON EQUATION WITH WEIGHTED INTEGRAL CONDITIONS

O. ŠTIKONIENĖ and M. SAPAGOVAS

Institute of Mathematics and Informatics
Akademijos 4, LT-08663, Vilnius, Lithuania
E-mail: olgast@ktl.mii.lt, m.sapagovas@ktl.mii.lt

We consider two dimensional Poisson equation in a rectangle domain with a weighted nonlocal boundary condition in one coordinate direction:

\[
\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x, y), \quad 0 < x < 1, \quad 0 < y < 1,
\]

\[
u(0, y) = \gamma_1 \int_0^1 \alpha(x)u(x, y)dx + \mu_1(y),
\]

\[
u(1, y) = \gamma_2 \int_0^1 \beta(x)u(x, y)dx + \mu_2(y),
\]

\[
u(x, 0) = \mu_3(x), \quad \nu(x, 1) = \mu_4(x),
\]

where \(\gamma_1, \gamma_2\) are given parameters and \(\alpha(x), \beta(x)\) are given weight functions.

We investigate the generalization of the alternating direction method for a system of difference equations that approximates Poisson equation with integral boundary conditions with the fourth-order accuracy. The integral in the nonlocal boundary conditions is approximated by the Simpson formula with error \(O(h^4)\).

The convergence conditions for fourth-order ADI method are established. Results of numerical investigation of the sufficient conditions of stability for this method are presented.
MATHEMATICAL MODELLING OF A SPARK-GAP SWITCH

S. SYTOVA and V. TIKHOMIROV

Research Institute for Nuclear Problems, Belarusian State University
Bobruiskaya 11, 220030 Minsk, Belarus
E-mail: sytova@inp.minsk.by, vvtikh@inp.minsk.by

A sharpening spark-gap is a critical component of different electrotechnical devices including Marx generators and flux compression generators (FCGs) designed for a variety of applications [1]. Both design and experimental investigation of different devices should be preceded by the theoretical study and computer simulation. The main function of a spark-gap is to sharpen the electrical impulse that is applied to the resistive load. There exist a lot of different physical models of a spark-gap [2].

We propose the universal model describing spark-gap breakdown in an arbitrary electrical circuit. It is the generalization of similar equation [2] which is applicable to simplest electrical circuit only. Our model has the following form:

\[ U\ddot{U} + \frac{1}{C} \left( \dot{U}I - \dot{I}U \right) - \dot{U}^2 + \xi(U)U^2 \left( \frac{1}{C}I - \dot{U} \right) = 0, \quad U(0) = U_0, \quad \dot{U}(0) = U_0^* \tau, \]

where \( U \) is a spark-gap voltage, \( I \) is a current in the circuit, \( C \) is a spark-gap interelectrode capacitance, \( U_0 \) is its breakdown voltage, \( \tau \) is a characteristic time of electrical discharge initiation. \( \dot{U} \) and \( \ddot{U} \) stand for the first and second derivatives of \( U \). Coefficient \( \xi(U) \) has the form:

\[ \xi(U) = a(bU/U_0 - c)^2 \]

with some constant \( a, b, c \) determined by gas pressure inside the spark-gap, discharge gap and other parameters.

Proposed numerical method for solving introduced equation allowed to obtain a good agreement between our numerical results and [2]-[4] in the particular case of simple electrotechnical scheme consisting of a capacity, spark-gap and resistive load. Comparison between different physical models of spark-gap is adduced. Our computer program [5] allows to simulate with sufficient accuracy different devices containing spark-gap.

REFERENCES

The work is aimed at development of a new approach for simulation of two-phase flows in porous media using explicit difference schemes. Logical simplicity of explicit schemes allows them to be implemented efficiently on modern high-performance computer systems which combine shared and distributed memory, multicore CPUs and different accelerators.

The new model is based on kinetic approach and considers liquids as slightly compressible. The kinetically-consistent finite difference schemes and the related QGD system [1] were developed for simulation of gas dynamics flows and were applied successfully to solve a wide range of industrial and ecological problems. In [2; 3] this approach was generalized to the case of single-phase flows in porous media. The main idea is: there is no sense to consider scales less than the minimal reference length \( l \). In porous media \( l \) equals approximately to a hundred rock grains. At that the continuity equation includes an additional diffusion term with the small parameter \( l \). It provides the stability of the scheme with central-difference approximation of the convective term.

Further development of this investigation is construction of the model for the two-phase flow case taking into account the capillary and gravity forces. The system of equations includes the continuity equations, the equations for Darcy velocities and also the pressure - density state relations for both phases. In contrast to the widely used IMPES method there is no necessity to solve the elliptic pressure equation. The regularizing diffusion terms in phase continuity equations differ from that in the single-phase equation and depend on the corresponding phase saturation. Furthermore small parameters may be different for the phases.

To provide the high solution accuracy at the sufficient scheme stability the continuity equation is transformed from the parabolic to hyperbolic type. Thus it includes the second order time derivation and can be approximated by the three-level scheme similar to the absolutely stable Duffort-Frankel scheme. The scheme obtained is conditionally stable with the relation \( \Delta t \leq \Delta h^{3/2} \) in contrast to the classical restriction \( \Delta t \leq \Delta h^2 \).

Adequacy of the models developed has been validated by a number of test problems. Computations were performed using the kinetic approach and traditional methods. Good agreement of results has been observed while the kinetic approach allowed to increase the time step and to reduce significantly computational costs.

REFERENCES


COMPARING THE PERFORMANCE OF MULTICORE AND MULTIPROCESSOR COMPUTERS FOR 3D PROBLEMS IN NONLINEAR OPTICS AND GAS DYNAMICS

V.A. TROFIMOV, O.V. MATUSEVICH and I.A. SHIROKOV

Lomonosov Moscow State University, Faculty of Computational Mathematics and Cybernetics

Leninskye Gory, 119992, Moscow, Russia

E-mail: vatro@cs.msu.ru

As it is well-known, to describe the modern problems of physics one needs the computer simulation of 3D problems in such diverse areas as: nonlinear optics, hydrodynamics, field theory, plasma physics, etc. However, most of such problems require large amount of computations and substantial computer resources. Therefore, it is significant to implement 3D algorithms efficiently taking in consideration the processor microarchitecture and platform features. Besides, as multi-core processors are widespread it is critical to use parallel computing to improve applications performance.

In this paper we consider two examples of 3D problems: second harmonic generation problem described by the set of nonlinear Schrodinger equations and laser plume expansion problem described by of quasi gas dynamics equations. Both of these problems require a large amount of computation. Our study examines several microprocessor designs among which are Intel Core2 Duo, Xeon and Itanium2 processors and IBM processors (Power4 and PowerPC 450) used in high performance computing systems of our faculty like IBM eServer pSeries 690 (Regatta) and IBM Blue Gene/P.

First, in our research we examine the optimization technique across a single processor by the example of SHG problem. We show the advantage of using performance libraries which can significantly reduce the execution time of the program. Next, we suggest how to reorganize computations to take full advantage of memory hierarchies. Total speedup of application due to suggested optimizations for SHG problem solution executing in sequential mode can run up to 8 times on Intel architectures and up to 5.5 times on IBM architecture. In our paper we also use SHG application as a benchmark to compare different computer architectures used for high performance computing. We show that one Itanium processor can execute computations as fast as 3 processors of Power4 processors of IBM Regatta.

Finally, we discuss the problems that arise for parallel computing on systems with shared memory. For this purpose we parallelized our SHG problem using OpenMP technology. We had to conclude that not all parallelization results in speed-up. To find out the reason of performance degradation we use a sample code that performs arithmetic operations on elements of two arrays in a loop. We gradually increased the number of operations in the loop and observed the growth of efficiency of parallelization. This experiment allowed us to assume that shared memory bandwidth may limit the efficiency of parallel computations.

For complete analysis we also show the use of another technology for parallel computing based on message passing between parallel processes (MPI) by example of another 3D problem - expansion of laser plume. We have achieved the linear growth of performance for parallel execution of the application.

This work is partly supported by the Russian Foundation for Basic Research (grant number 08-01-00107-a).
COMPARATIVE ANALYSIS OF FINITE DIFFERENCE DECOMPOSITION SCHEMES FOR A SINGULARLY PERTURBED REACTION–DIFFUSION EQUATION

I.V. TSELISHCHEVA and G.I. SHISHKIN

Institute of Mathematics and Mechanics, Ural Branch of the Russian Academy of Sciences
S. Kovalevskaya 16, 620219, GSP-384, Ekaterinburg, Russia
E-mail: tsi@imm.uran.ru, shishkin@imm.uran.ru

We consider grid approximations of a domain decomposition method applied to the Dirichlet problem for a singularly perturbed reaction–diffusion equation. The equation involves the perturbation parameter $\varepsilon^2$ multiplying the highest order derivatives, $\varepsilon$ takes arbitrary values in the half-open interval $(0,1]$. With an example of an ordinary differential reaction–diffusion equation, we construct and analyze finite difference schemes based on an overlapping domain decomposition method and using piecewise uniform meshes. A comparative analysis of the efficiency of the difference decomposition schemes for sequential and parallel computations is made. We give conditions that ensure the $\varepsilon$-uniform convergence for solutions of the decomposition schemes with increasing the number of iterations. It is shown (in contrast to the time–dependent convection–diffusion equation examined in [1]) that the increase in the number of solvers in parallel schemes on piecewise uniform meshes leads to the acceleration in solving the parallel method in comparison with the sequential method, without loss in the accuracy of the solution of the decomposed scheme. Lower and upper bounds for the error of the decomposition method and for the number of iterations are obtained. Conditions are found under which the parallel scheme solves the problem faster than the sequential one; moreover, the error in the solution of the parallel scheme does not exceed the error in the case of the sequential scheme. Namely, the time required for the solution reduces practically $P$ times, where $P$ is the number of parallel processors; and computational costs are close. We discuss an approach how these results can be generalized to the case of the singularly perturbed elliptic reaction–diffusion equation on a rectangle considered in [2], which calls for further study.

REFERENCES


This research was supported by the Russian Foundation for Basic Research under grant No. 10–01–00726.
ON MANY–VALUED BORNOLOGICAL STRUCTURES

I. ULJANE and A. ŠOSTAKS

Department of Mathematics, University of Latvia

Zellu iela 8, LV–1002 Riga, Latvia

E-mail: uljane@mail.com, sostaks@lanet.lv

In [1] S.T. Hu studied the problem of the possibility to define the concept of boundedness in a topological space. To do this he introduced a system of axioms which later gave rise to the concept of a bornology and a bornological space. In a certain sense a bornological space can be viewed as a counterpart of a topological space if one is mainly interested in the property of boundedness of mappings and not in their property of continuity. At the first stage of research bornological structures were mainly considered on Banach or, more general, on linear topological spaces, but later the research was extended to topological spaces without any linear structure, see e.g. [2].

In paper [6] the concept of a bornology on the $L^X$-exponent of a set $X$ was introduced and some basic facts about the category of $L$-bornological spaces and their bounded mappings were proved. The aim of this talk is to introduce the alternative approach to the study of bornological structures in the context of $L$-sets: Namely we define a many-valued, or, more precisely, an $L$-valued bornology on the ordinary exponent $2^X$ of a set $X$. While the approach in [6] can be considered as a bornological counterpart of Chang-Goguen fuzzy topologies [3], [4], our approach here is a bornological counterpart of Mingsheng Ying’s fuzzifying topologies [5].

**Definition** Let $L$ be a $cl$-monoid, in particular a complete infinitely distributive lattice with lower and upper bounds 0 and 1 respectively. An $L$-valued bornology on a set $X$ is a mapping $B : 2^X \to L$, such that (1) $B(\{x\}) = 1 \ \forall x \in X$; (2) $B(U_1 \cup U_2) \geq B(U_1) \cap B(U_2)$; (3) $B(\bigvee_{i \in I} U_i) \geq \bigwedge_{i \in I} B(U_i) \forall U_i \in 2^X$. Given two $L$-valued bornological spaces $(X, B_X, (Y, B_Y)$, a mapping $f : X \to Y$ is called bounded if $B_X(U) \leq B_Y(f(U)) \forall U \in 2^X$.

We are studying properties of the category $L$–BORN of $L$-valued bornological spaces and their bounded mappings. In particular we show that the family $B(X)$ of all $L$-valued bornologies on a set $X$ ordered in the natural way is a complete infinitely distributive lattice; prove that the category $L$–BORN is topological over the category SET; construct initial and final structures for a family of mappings in this category.

Finally a construction of fuzzification of the category $L$–BORN will be briefly mentioned.

REFERENCES

A HIERARCHICAL APPROACH TO THE GENERATION OF PARETO POINTS FOR COMPLEX SYSTEMS

O. VAARMANN and A. LEIBAK

Institute of Mathematics at Tallinn University of Technology
Ehitajate tee 5, 19086, Tallinn, Estonia
E-mail: vaarmann@staff.ttu.ee, alar@staff.ttu.ee

Most real-life problems in economics, business and engineering are complex and large scale attempting to simultaneously optimize multiple objectives. Multi-objective optimization deals with finding and evaluating a number of trade-off optimal solutions. Numerous different algorithms have been developed to address computationally complex optimization problems. A standard way for handling such problems is to convert them into a problem with a single objective function. Frequently, multi-objective optimization problems are solved by the scalarization or weighting method which is based on the idea that each objective will be associated with a weighting factor and thereafter the weighted sum of objectives will be optimized. This, of course, requires that someone choose the weights properly reflect the relative importance of the individual objective functions. The aim is then to find the Pareto optimal point for the numerical values of target functions. When such weights are difficult or impossible to obtain then a preferred approach is to provide the decision maker with a set of trade-off solutions, from which particular solutions can be selected on the basis of other external factors. Incorporating decision makers preference into the solution process has played a major role in Multiple Decision Making (MCDA). The generation of Pareto optimal points for complex systems can be easier if the problem can be decomposed and solved as a set of smaller coordinated subproblems which may be treated independently. The idea of hierarchical approach based on decomposition-coordination schemes is to reduce the overall complex problem through the introduction of a set of coordination parameters into a simpler harmonized subproblems which can be distributed over a large number of processors or computers, e.g designating the processors as master and slaves. In this talk we present some results concerning the study of decomposition-coordination approach based on the generation of non-feasible points. In this case only the values reached at the end of the procedure are assured to be feasible. For finding the proper values of coordination parameters some iterative methods are developed. Efficiency, convergence properties and computational aspects of the methods under consideration are analyzed and polyalgorithmic strategy of their implementation is explored [1]. What is an appropriate or satisfactory solution for MCDA? is discussed as well [2].

REFERENCES

COMPUTER MODELLING AND DIAGNOSING
METHODS OF AIRCRAFT AND AVIATION ENGINES

V. VALAVIČIUS

Vilnius Gediminas Technical University
Saulėtekio 11, LT-2040, Vilnius, Lithuania
E-mail: vinval@vgtu.lt

Methods of computer modelling and diagnosing of passenger aircraft, functional systems and aviation engines were carried out by using laboratory research and present scientific sources of aviation enterprises.

Have been studied:

• to make calculations and modelling of diagnostic characteristics of aviation engine by using information gained automatically during the flight;
• to carry out calculation and modelling of aviation engines compressors; to prepare modeling of aerodynamic characteristics of aircraft;
• to perform the modelling and calculation of operational characteristics of passenger aircraft.

The method of diagnosis of linear diagnostic matrixes was prepared. The method was established to be applied when using compressor characteristics which are obtained after the trial and adjustment of engines. The faults appear in the air tract of the engine compressor because the primary characteristics which are taken into consideration when making calculations for unfairly engines vary.

Precise parameters of an engine are necessary for the method classifying damages. This method gives good results only testing engines in the stand while during the operation great errors are obtained.

By using attraction and hourly consumption of fuel the method determining economical operation of the engine was prepared. It allows establishing operational regime of every particular engine.

The new diagnosing principle in the research is based upon the parallel use of parameter diagnosis and non-destructive testing information, i.e. optic, eddy current, ultrasonic and capillary.

Time reduction and increase of accuracy of computer calculations were gained in the research when compared with the existing methods.

REFERENCES

METHODS OF OPEN PIT PRODUCTION OPTIMIZATION BASED ON ITS SECTORAL MODELS

A.M. VALUEV

Moscow State Mining University

MSMU, Leninsky prospect 6, Moscow 119991 Russia

E-mail: amvaluev@online.ru

Optimization of open pit production requires the use of models representing its geometrical shape. The stepped shape of an open pit board may be represented with the set of horizontal curves serving as lower and higher edges of bench slopes which maximum curvature is restricted to satisfy transportation process requirement. These set of these curves may be represented in the form of ODE with control variable $k_l$ subject to simple restriction

$$
\frac{dx_l}{ds} = \cos \varphi_l, \quad \frac{dy_l}{ds} = \sin \varphi_l, \quad \frac{d\varphi_l}{ds} = k_l, \quad |k_l| \leq \frac{R_{\min}}{s}, \quad (1)
$$

that is similar to traditional dynamic systems. However, they must satisfy very specific restrictions set for all $l = 1, \ldots, L$ expressing conditions of sufficient width of bench grounds and slopes stability. In the most simple case these conditions may be expressed as

$$
\rho(x_l(s), y_l(s); x_l(s'), y_l(s')) \geq g \text{ where } s, s' \in [0, S_l], \text{ and } |s - s'| \geq \pi R_{\min}, \quad (2)
$$

$$
\rho(x_l(s), y_l(s); x_{l+1}(s'), y_{l+1}(s')) \geq d_l \text{ where } s \in [0, S_l] \text{ and } s' \in [0, S_{l+1}], l < L. \quad (3)
$$

Therefore principally the problem of production optimization must include the set of expressing in some way the above (1)–(3) relationships. There are three ways to formulate and solve the problem in question. Accuracy of discrete representation is sufficient for the problem of determining the final limits of the open pit. On the contrary, direct finite-difference approximation of (1)–(3) representation must be the most flexible and exact but leads to great amount of calculations dealing with the set of restriction (2)–(3) as well as with the deposit model.

Due to idea first put forward by I.B. Tabakman in the USSR that recently became more popular [1], the mining area is separated into a set of sectors and the polyline approximations of curves satisfying (1) is linked to this sectors set. Original model by I.B. Tabakman yields a very limited accuracy. The author proposed to use more flexible form sector-wise representation with different set of sectors for different bench that enabled the presentation of much more vast set of variants. In the simplest case the set of restrictions is reduced to

$$
x_{ij} \leq x_{i-1l} + d_{ij} - D_{i-1} \text{ for all } i > 1, j, l \in I_l(i, j) \text{ and } |x_{ij} - x_{ij-1} + l_{ij}| \leq D_0 \text{ for all } i, j.
$$

With the use of this representation and feasible directions algorithm implemented using sparse matrices technique valuable practical results for some East Siberian coal quarries were obtained.

REFERENCES

MAXIMUM PRINCIPLE AND THE FOURTH ORDER BOUNDARY VALUE PROBLEM

I. YERMACHENKO
Daugavpils University
Parādes 1, LV-5400, Daugavpils, Latvia
E-mail: inara.jermacenko@du.lv

We investigate the equations of the form

\[ x^{(4)}(t) + \lambda x''(t) = f(t, x(t)), \quad t \in I := [a, b] \]  \tag{1}

together with the boundary conditions

\[ x(a) = x(b) = x'(a) = x'(b) = 0. \]  \tag{2}

Based on the maximum principles for the fourth order linear operators discussed in the papers [1], [2] and [3], we state a maximum principle for a function satisfying the differential inequalities.

**Theorem 1.** Let \( u \in C^4[a, b] \) satisfies the differential inequalities

\[
\begin{align*}
  u^{(4)}(t) + ku''(t) &\geq 0, \quad t \in (a, b) \quad \tag{3} \\
u(a) &\geq 0, \quad u(b) \geq 0, \quad \tag{4} \\
u'(a) &\geq 0, \quad u'(b) \leq 0. \quad \tag{5}
\end{align*}
\]

Then \( u \) cannot assume a non-positive minimum value at an interior point of \((a, b)\) unless \( u \) is identically zero.

Although this maximum principle seems interesting in itself, our main purpose is to apply it to prove the existence and approximation of solutions to the problem (1), (2), in presence of properly ordered lower and upper functions.

**REFERENCES**


DEPTH FIRST SEARCH IN PARALLEL OPTIMIZATION ALGORITHMS

J. ŽILINSKAS

Institute of Mathematics and Informatics
Akademijos 4, LT-08663 Vilnius, Lithuania
E-mail: julius.zilinskas@mii.lt

Branch-and-bound and partitioning algorithms for global and combinatorial optimization are considered in this talk. An iteration of such algorithms processes a node in the search tree representing a not yet explored sub-space of the solution space. The iteration has three main components: selection of the node to process, its evaluation and branching of the search tree. Sub-spaces which cannot contain a global minimum are discarded from further search pruning the branches of the search tree. The rules of initial covering and branching depend on type of partitions used.

There are three main strategies of selection:

- Best first – select the node with the best value of a criterion.
- Depth first – select the node which is most far from the root node.
- Breadth first – select the node which is most near to the root node.

The advantage of the depth first strategy is that the search tree may be constructed sequentially to avoid storing of unbranched nodes. This is very important when the search tree is big. The breadth first search requires storing of unbranched nodes in a first-in-first-out structure. The best first search requires storing of unbranched nodes in a priority structure, which not only requires memory resources, but insertion and removal of nodes takes at least logarithmic time to the number of nodes in the queue in the worst case.

Experience on parallelization of depth first search algorithms is presented. Parallelization of several optimization algorithms are discussed:

- Lipschitz optimization with simplicial partitions;
- branch-and-bound based on interval arithmetic;
- two-level optimization algorithm for multidimensional scaling with city block distances based on branch-and-bound and quadratic programming;
- combinatorial branch-and-bound for topology optimization of truss structures;
- copositivity checking and copositive programming by simplicial partitioning.

Results of experimental investigation are discussed.
GLOBAL OPTIMIZATION BASED ON A SIMPLICIAL STATISTICAL MODEL OF MULTIMODAL FUNCTIONS

A. ŽILINSKAS and J. ŽILINSKAS

Institute of Mathematics and Informatics
Akademijos 4, LT-08663 Vilnius, Lithuania
E-mail: antanasz@ktl.mii.lt, julius.zilinskas@mii.lt

A well-recognized one-dimensional global optimization method [1] is generalized to the multidimensional case. The generalization is based on a multidimensional statistical model of multimodal functions constructed by generalizing computationally favorable properties of a popular one-dimensional model — the Wiener process. A simplicial partition of a feasible region is essential for the construction of the model. The basic idea of the proposed method is to search where improvements of the objective function are most probable; a probability of improvement is evaluated with respect to the statistical model. Some results of computational experiments are presented.

The main difficulty in the development of statistical-models-based multidimensional algorithms is due to time-consuming computations needed to update the model characteristics including, the information acquired during the search. Similar difficulties are also inherent in algorithms that process the current search information in the frame of other models of objective functions: radial basis functions, response surfaces, and kriging. Therefore such algorithms are mainly oriented to “expensive” objective functions.

A known disadvantage of global optimization algorithms aimed at some optimality with respect to a model of objective functions is time-consuming auxiliary computations needed for planning of the current iteration. The time needed for optimal planning of the current iteration depends on the complexity of computing of the optimality criterion for this iteration. For the standard models of statistical and radial basis functions these computations include inversion of a matrix of the order equal to the iteration number (which is equal to the number of objective function values computed during the previous search iterations). The most severe computational difficulty, caused by the inversion of large matrices, can be avoided using simplified multidimensional statistical models [2], however auxiliary computations remain intensive so far. The reduction of auxiliary computations is an urgent and challenging problem of statistical-models-based global optimization.

Although the general theory of one-dimensional and multidimensional statistical models is identical, a sub-class of one-dimensional models, namely the Markovian stochastic functions, has computationally favorable properties that enable the development of efficient one-dimensional global optimization algorithms; an exceptionally favorable member of this sub-class is the Wiener process [2]. In the present talk, a multidimensional statistical model is constructed generalizing favorable properties of the one-dimensional case; such a generalization is aided by a simplicial partition of a feasible region.

REFERENCES

## INDEX OF AUTHORS

<table>
<thead>
<tr>
<th>Author</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annunziato, M.</td>
<td>1</td>
</tr>
<tr>
<td>Asmuss, S.</td>
<td>2</td>
</tr>
<tr>
<td>Avenhaus, R.</td>
<td>53</td>
</tr>
<tr>
<td>Balkys, G.</td>
<td>3</td>
</tr>
<tr>
<td>Baronas, R.</td>
<td>94</td>
</tr>
<tr>
<td>Bartoszewski, Z.</td>
<td>4</td>
</tr>
<tr>
<td>Belovas, I.</td>
<td>5</td>
</tr>
<tr>
<td>Bikulciene, L.</td>
<td>6</td>
</tr>
<tr>
<td>Birgelis, K.</td>
<td>7</td>
</tr>
<tr>
<td>Bobinska, T.</td>
<td>8</td>
</tr>
<tr>
<td>Bosiakov, S.</td>
<td>9</td>
</tr>
<tr>
<td>Bras, M.</td>
<td>10</td>
</tr>
<tr>
<td>Breidaks, J.</td>
<td>2</td>
</tr>
<tr>
<td>Budkinsa, N.</td>
<td>2</td>
</tr>
<tr>
<td>Buike, M.</td>
<td>8</td>
</tr>
<tr>
<td>Buikis, A.</td>
<td>8, 42, 61</td>
</tr>
<tr>
<td>Bula, I.</td>
<td>11</td>
</tr>
<tr>
<td>Cepitis, J.</td>
<td>12</td>
</tr>
<tr>
<td>Chetverushkin, B.</td>
<td>13, 103</td>
</tr>
<tr>
<td>Churbanova, N.</td>
<td>13, 103</td>
</tr>
<tr>
<td>Čiegis, R.</td>
<td>14, 36, 57, 97</td>
</tr>
<tr>
<td>Čiupaila, R.</td>
<td>38</td>
</tr>
<tr>
<td>Danilenko, S.</td>
<td>15</td>
</tr>
<tr>
<td>Dementev, A.</td>
<td>16</td>
</tr>
<tr>
<td>Deveikis, A.</td>
<td>17</td>
</tr>
<tr>
<td>Dubatovskaya, M.</td>
<td>18</td>
</tr>
<tr>
<td>Dubickas, A.</td>
<td>19</td>
</tr>
<tr>
<td>Dzemyda, G.</td>
<td>3, 20, 78</td>
</tr>
<tr>
<td>Eglite, I.</td>
<td>, 48</td>
</tr>
<tr>
<td>Elkins, A.</td>
<td>, 21</td>
</tr>
<tr>
<td>Filatovas, E.</td>
<td>22</td>
</tr>
<tr>
<td>Funaro, D.</td>
<td>23</td>
</tr>
<tr>
<td>Furmanov, I.</td>
<td>13</td>
</tr>
<tr>
<td>Galanin, M.P.</td>
<td>79</td>
</tr>
<tr>
<td>Garbuza, T.</td>
<td>24</td>
</tr>
<tr>
<td>Garunktis, R.</td>
<td>25</td>
</tr>
<tr>
<td>Gaspar, F.J.</td>
<td>80</td>
</tr>
<tr>
<td>Grigorenko, O.</td>
<td>26</td>
</tr>
<tr>
<td>Gritsans, A.</td>
<td>27</td>
</tr>
<tr>
<td>Gudynas, G.</td>
<td>28</td>
</tr>
<tr>
<td>Ideon, E.</td>
<td>29</td>
</tr>
<tr>
<td>Igumenov, V.</td>
<td>30</td>
</tr>
<tr>
<td>Iliev, O.</td>
<td>55</td>
</tr>
<tr>
<td>Iltina, M.</td>
<td>31</td>
</tr>
<tr>
<td>Iltins, I.</td>
<td>31</td>
</tr>
<tr>
<td>Ivanauskas, F.</td>
<td>65</td>
</tr>
<tr>
<td>Ivanovienė, I.</td>
<td>32</td>
</tr>
<tr>
<td>Jachimavičienė, J.</td>
<td>33</td>
</tr>
<tr>
<td>Jackiewicz, Z.</td>
<td>34</td>
</tr>
<tr>
<td>Jackiewicz, Z.</td>
<td>4</td>
</tr>
<tr>
<td>Jakubelienė, K.</td>
<td>35</td>
</tr>
<tr>
<td>Janavičius, A.J.</td>
<td>40</td>
</tr>
<tr>
<td>Jankevičiūtė, G.</td>
<td>36</td>
</tr>
<tr>
<td>Janno, J.</td>
<td>37</td>
</tr>
<tr>
<td>Jasinevičius, R.</td>
<td>54</td>
</tr>
<tr>
<td>Jesevičiūtė, Ž.</td>
<td>38</td>
</tr>
<tr>
<td>Jodra, P.</td>
<td>39</td>
</tr>
<tr>
<td>Jucikaitė, K.</td>
<td>62</td>
</tr>
<tr>
<td>Jurgaitis, D.</td>
<td>40</td>
</tr>
<tr>
<td>Kačinskaitė, R.</td>
<td>41</td>
</tr>
<tr>
<td>Kalis, H.</td>
<td>42</td>
</tr>
<tr>
<td>Kamenchenko, S.</td>
<td>43</td>
</tr>
<tr>
<td>Kancleris, Ž.</td>
<td>97</td>
</tr>
<tr>
<td>Kandratsiuk, A.</td>
<td>44</td>
</tr>
<tr>
<td>Kareiva, A.</td>
<td>65</td>
</tr>
<tr>
<td>Kirjackis, J.</td>
<td>45</td>
</tr>
<tr>
<td>Klimova, E.</td>
<td>46</td>
</tr>
<tr>
<td>Klingbeil, L.</td>
<td>83</td>
</tr>
<tr>
<td>Koliskina, V.</td>
<td>47</td>
</tr>
<tr>
<td>Kolyshkin, A.</td>
<td>48</td>
</tr>
<tr>
<td>Konopelko, O.A.</td>
<td>50</td>
</tr>
<tr>
<td>Koroleva, A.</td>
<td>49</td>
</tr>
<tr>
<td>Korzynuk, V.I.</td>
<td>50, 51</td>
</tr>
<tr>
<td>Kozlovskaya, I.</td>
<td>51</td>
</tr>
<tr>
<td>Kriauzienė, R.</td>
<td>52</td>
</tr>
<tr>
<td>Krieger, Th.</td>
<td>53</td>
</tr>
<tr>
<td>Krylovas, A.</td>
<td>52</td>
</tr>
<tr>
<td>Krušinskiene, R.</td>
<td>54</td>
</tr>
<tr>
<td>Kurasova, O.</td>
<td>22</td>
</tr>
<tr>
<td>Lakdawala, Z.</td>
<td>55</td>
</tr>
<tr>
<td>Lätt, K.</td>
<td>56</td>
</tr>
<tr>
<td>Laukaitytė, I.</td>
<td>57</td>
</tr>
<tr>
<td>Laukaitytė, L.</td>
<td>66</td>
</tr>
<tr>
<td>Laurinčikas, A.</td>
<td>58</td>
</tr>
<tr>
<td>Lebedinska, J.</td>
<td>59</td>
</tr>
<tr>
<td>Leetma, E.</td>
<td>60</td>
</tr>
</tbody>
</table>