USE OF DYNAMICAL STATISTICAL MODELLING FOR INVESTIGATION OF EFFECTIVENESS OF ACTIVITY OF PRODUCTION SYSTEMS IN ECONOMICS

Vitalijs JURENOKS

Riga Technical University, Meža iela 1/7, Riga LV-1048, Latvia
Telephone +37160789855, E-mail vitalijs.jurenoks@rtu.lv

Vladimirs JANSONS

Riga Technical University, Meža iela 1/7, Riga LV-1048, Latvia
Telephone +37160789425, E-mail vladjans@latnet.lv

Konstantins DIDENKO

Riga Technical University, Meža iela 1/7, Riga LV-1048, Latvia
Telephone +37160789447, E-mail konstantins.didenko@rtu.lv

Abstract. *Information technologies provide an opportunity to create, collect, save, process and efficiently use information in the processes under investigation. In contemporary world information technologies are most frequently linked with modelling making use of computer technology and information networks. Modelling is considered to be an indirect investigation method for originals of objects used while researching the substitutes of the objects. The information image of the object (information model) may be used as the object substitute. In economic studies it is important to create a business model.*

Keywords: *Stability of Production systems, statistical modelling, optimization*

Introduction

One of the constituent parts for successful application of production systems in economics (PSE) is optimization of functioning of all production and economic parameters of the given system. The aim of this research work is modelling the behaviour of the criterion of stability of PSE.

When speaking about stability of PSE many authors consider the organisational and economic stability of PSE under changing parameters of external environment. Stable position (functioning) of PSE in the market is understood as the capacity of PSE to maintain or increase the tendency of positive growth, i.e., to maintain (increase) the sales volume of goods or services and the profit margin gained by sales of goods and services rendered under different changes of external and internal environment of PSE functioning.

1. Modelling the behaviour of criterion of stability

Modelling the behaviour of criterion of stability of PSE is based on building a set of alternative strategies, the introduction of which at certain periods of time allow maintaining of PSE functioning in a stable position, i.e., the concept of „criterion of stability” is introduced.[1] Thereby, the main objective of managing stability of PSE is provision of criterion of stability to the given interval of permissible values of the chosen criterion of stability. Taking into account the

[1] Jurenoks, V.; Jansons, V.; Didenko, K. Modelling of Stability of Economic Systems Using Benchmarking and Dynamic Programming. *X International Conference on Computer Modelling and Simulation EUROSIM/UKSim*, Cambridge, United Kingdom, 2008, p. 295-300.

infrastructure of PSE as well as the character of relationship of internal and external factors of the functioning environment of PSE the criterion of integral profit is considered to be the generalized (integral) criterion for stability which may be presented as:

$$P(T_0, T, w) = \int_{T_0}^{T} R(t, w)dt - \int_{T_0}^{T} C(t, w)dt \ \geq \ K, \quad (1)$$

where:

$[T_0, T]$ – the period of modelling time of PSE functioning;

w – the random parameter having a given distribution and affecting the PSE to be investigated;

$P(T_0, T, w)$ – the criterion of integral profit of PSE functioning in the period of modelling time of PSE functioning;

$\int_{T_0}^{T} R(t, w)dt$ - the integral criterion of profit gained from PSE functioning in the period of modelling time of PSE functioning;

$\int_{T_0}^{T} C(t, w)dt$ - the integral criterion of losses linked with the functioning of PSE in the period of modelling time of PSE functioning;

K – the given minimal value of integral criterion of profit due to PSE functioning in the period of modelling time of PSE functioning.

The research work presents the following task of dynamic programming (modelling) for assessing the stability of PSE, that may be shown as:

$$\begin{aligned}
& P(T_0, T) = \int_{T_0}^{T} R(t)dt - \int_{T_0}^{T} C(t)dt \ \geq \ K; \\
& \sum_{t=T_0}^{T} \sum_{i=1}^{n} Q_{i,t} c_{i,j,t} \ \leq \ s_j; \ j = 1,2,\ldots,n; \\
& p^{\min}{}_{i,t} \ \leq p_{i,t} \ \leq \ p^{\max}{}_{i,t}; \\
& Q^{\min}{}_{i,t} \ \leq \ Q_{i,t} \ \leq \ Q^{\max}{}_{i,t}; \\
& C_{i,j,t} \ \leq \ c_{i,j,t}; \\
& i = 1,2,\ldots,n; \ t = T_o,\ldots,T,
\end{aligned} \quad (2)$$

where $p_{i,t}$ – value of the criterion of integral profit of PSE at the moment of modelling time t;

$p^{min}{}_{i,t}$ – minimal value of the criterion of integral profit of PSE at the moment of modelling time t (determined by conditions of modelling);

$p^{max}{}_{i,t}$ – maximum value of the criterion of integral profit of PSE at the moment of modelling time t;

$Q_{i,t}$ – volume of sales (services of functioning) of PSE functioning realized at the moment of modelling time t;

$Q^{min}{}_{i,t}$ – minimal volume of sales (services) of PSE functioning realized at the moment of modelling time t (determined by conditions of modelling);

$Q^{max}{}_{i,t}$ – maximum volume of sales (services) of PSE functioning realized at the moment of modelling time t (determined by conditions of modelling on the basis of information characterizing real capacity of the market segment);

$C_{i,j,t}$ – production (services of functioning) price per unit of PSE realized at the moment of modelling time t;

$C_{i,j,t}$ – maximum production (services of functioning) price per unit of PSE realized at the moment of modelling time t (determined by conditions of modelling on the basis of information about the prices of similar production in the market);

n – amount of resources of PSE considered in the process of modeling;

t – modelling time;

T_0 – time for positiveness of the period of stability;

T – time for completing the process of modelling.

2. Modelling process of stability of production system in economics

Fulfillment of the conditions (2) provide appertainment of values of integral profit P (criteria of stability of PSE) to the given boundaries during the period of modelling time t of PES functioning at given constraints (condition 2). The dynamics of the process under investigation takes place during the period of modelling time $[T_0,T]$, see Figure 1.

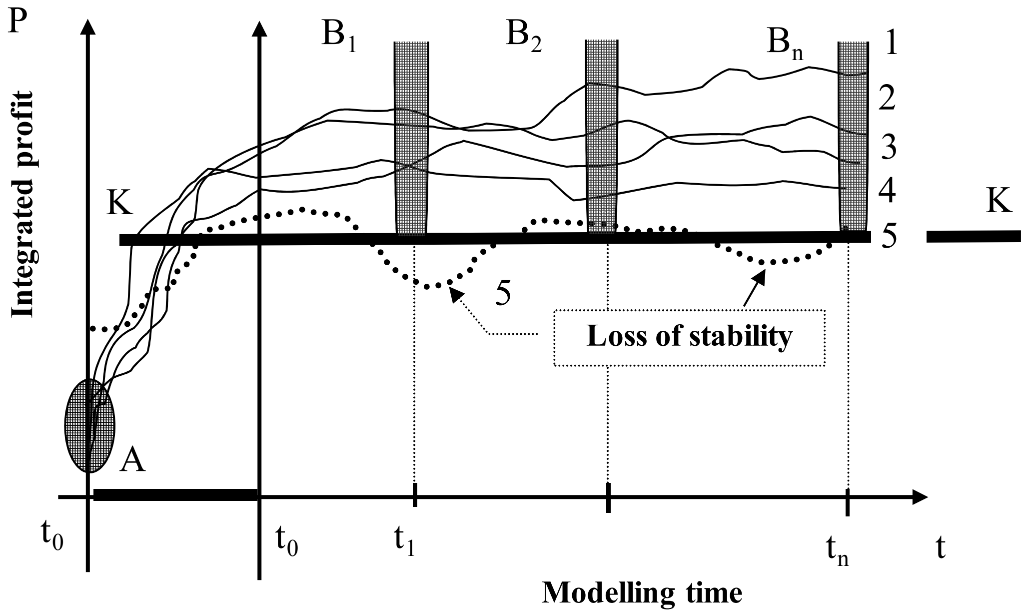

Figure 1. Graphical representation of using algorithm of dynamic programming for ensuring stability of PSE

In Figure 1 trajectory 5 shows violation of the condition of stability (condition 1). As the process of modelling of the stability of PSE is investigated dynamically, the necessity arises to use the methods of dynamic programming. The value of integral profit P (criterion of stability of PSE) is accepted as the target function. During the investigation the dynamic state of PSE is modelled with variable values of parameters of PSE satisfying conditions in system (2).

In Figure 1:

B_n – variety of permissible values of the criterion P after n periods of modelling time of PSE;

A – the interval (set) of values of parameters of PSE (Q, p, c,...) when criterion of stability of PSE falls into interval (set) B_n by optimal trajectories;

B_1, B_2, …, B_n – the interval (set) through which the trajectory of values of criterion P must pass at the moments of time (control points) T_0+t_1, $T_0+t_1+t_2$, …, T. The task of managing the stability of PSE in the process shown in Figure 1 turns into a task of control of trajectory of the values of the criterion P and (if possible) introduction of constructive effects into the process of PSE development at the moments T_0+t_1, $T_0+t_1+t_2$, ...,T using the information about the possible optimal trajectories of the values of criterion P.

Taking into account the nature of interaction of internal and external factors of environment of PSE functioning, the necessity arises to include the incidental parameter w with a given distribution into the model. The condition (1) now looks like as follows:

$$P(T_0,T,w)=\int_{T_0}^{T} R(t,w)dt-\int_{T_0}^{T} C(t,w)dt \ \geq \ K \ , \qquad (3)$$

where:

w – the incidental parameter (vector of incidental parameters) with the given distribution affecting the behaviour of PSE to be investigated;

$P(T_0,T,w)$ – the criterion of integral profit of PSE functioning during the period of modelling time t;

$\int_{T_0}^{T} R(t,w)dt$ – the integral criterion of profit due to PSE functioning in the period of modelling time t;

$\int_{T_0}^{T} C(t,w)dt$ – the integral criterion of losses linked with PSE functioning in the period of modelling time t. In this case we consider the stochastic modelling of stability of PSE during the modelling time t of PSE functioning. The task for stochastic modelling of PSE functioning is presented as follows:

$$\begin{aligned}
&P(T_0,T,w)=\int_{T_0}^{T} R(t,w)dt-\int_{T_0}^{T} C(t,w)dt \ \geq \ K;\\
&\sum_{t=T_0}^{T}\sum_{i=1}^{n} Q_{i,t}c_{i,j,t} \ \leq \ s_j; \ j=1,2,...,n;\\
&p^{\min}{}_{i,t} \ \leq p_{i,t} \ \leq \ p^{\max}{}_{i,t};\\
&Q^{\min}{}_{i,t} \ \leq \ Q_{i,t} \ \leq \ Q^{\max}{}_{i,t};\\
&C_{i,j,t} \ \leq \ c_{i,j,t};\\
&i=1,2,...,n; \ t=T_o,...,T.
\end{aligned} \qquad (4)$$

3. Graphical representation of the results of statistical modelling of stability of the production system in economics

The process of statistical modelling may be realized with the help of modelling programmes. Modelling of PSE stability was performed by changing the parameters characterizing the basic features of the financial reserve of PSE, as well as changing the actual parameters of payments among participants of PSE. For modelling purposes the authors have used MS Excel software[2]. The table illustrating the changes of the parameters to be modelled is shown in Figure 2.

[2] Jurenoks, V.; Jansons, V.; Didenko, K. Modelling of financial stability in logistics in conditions of uncertainty. *21-st European Conference on Modelling and Simulation*, Prague, Czech Republic, 2007, p. 30-36.

	Income		Payment
Rezerve	**10000**		
delay_min =	**7**	**delay_min =**	**3**
w1 =	**25**	**w2 =**	**20**
delay_max =	**32**	**delay_max =**	**23**

max_time_negat =	**58**

Figure 2. Graphical representation of changing parameters to be modelled of PSE stability by using the Monte Carlo methods

Using the results of statistical modelling of the PSE state makes it possible to determine the probability characteristics of its stability at the given moments of time, including the identification of risks linked with PSE functioning in the given periods of time. Figure 1 shows the results of statistical modelling of PSE transfer to a stable state.

Table 1. First time positiveness depending on w1 and w2

w	$f_1(w)$	w	$f_2(w)$
1	58	1	58
2	5	2	85
3	5	3	67
4	0	4	85
5	3	5	84
.....	.....	.....	.....
97	85	97	2
98	85	98	6
99	83	99	7
100	85	100	3

Graphical illustration of the results of statistical modelling of PSE stability by using Monte Carlo methods and changing parameters w1 and w2, is presented in Figures 3 and 4.

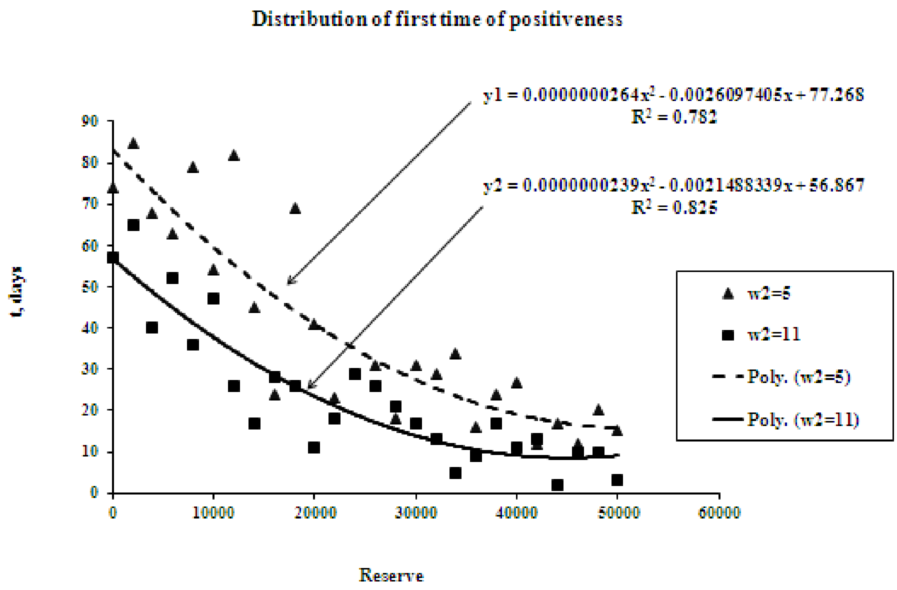

Figure 3. Graphical representation of the results of statistical modelling of PSE stability by using Monte Carlo methods (Distribution of first time of positiveness)

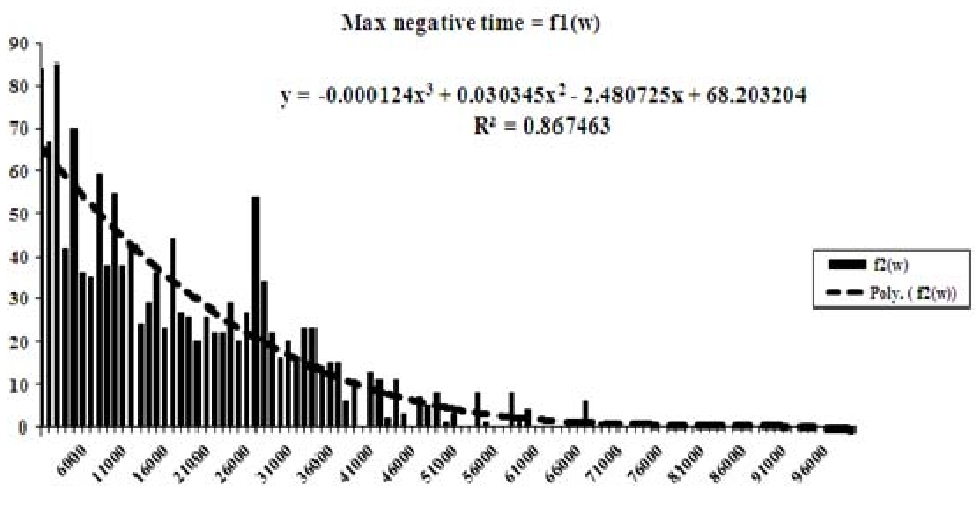

Figure 4. Graphical representation of the results of statistical modelling of PSE stability by using Monte Carlo methods (polynomial approximation of PSE stability)

Figure 4 shows that cubic polinom describes the changes in PSE stability well enough when changing the basic parameters of the system to be modelled. In the case of the well-known distribution of incidental parameter w it may be possible to use the parametric methods of modelling of incidental values. In real life only rarely it is possible to gain a sufficiently good description of behaviour of incidental parameters. In this case nonparametric methods of modelling may be used by applying histograms, methods of local non-parametric regression etc. The authors have described in detail the non-parametric methods of modelling in other research papers (see references).

Conclusions

The application of modelling is connected with the fact that frequently it is not possible to provide a definite description of the behaviour of the economic system being investigated. When investigating the dynamic behaviour of the economic system, i.e. by making definite changes of parameters of the system under investigation, we frequently observe the existence of incidental factors affecting the character of the behaviour of the system. In addition, it should not be forgotten that the very character of the research also brings its incidental elements into the research process. By using statistical dynamic programming as well as the Monte Carlo method for modelling it has become possible:

1) to set alternative strategies of PSE performance;

2) to model PSE stability using nonparametric techniques;

3) to model the "risk zones" in which the stability of PSE has been distorted;

4) to identify the amount of the financial reserves required for PSE stability in the "risk zones";

5) to manage functioning of PSE using the integrated criterion of PSE stability.

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