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PROJECTING ELASTOMERIC SHOCK ABSORBERS WITH MOVING SIDE STOP

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Abstract: *In the design of Vibration isolator it is often necessary to ensure a reduction (or increase) of the value of frequency η_r, that requires the ability to calculate the stiffness characteristics of elastomeric Vibration isolator in the low finite and medium finite deformations. A similar problem arises in the design and calculation of equifrequent rubber-metal compensating devices, which find application in various fields of engineering and construction industries, effectively replacing the hydro- pneumatic - spring compensating devices, working under axial stress-strain. In this case, the stiffness characteristic of "force – settlement" P = P (Δ), even for small finite deformations, will be non-linear (or piecewise linear). In this title proposed a method for determination of rigidity dependence "Force - Settlement" for shock-absorbing elements with absolutely rigid moving (parallel to the vertical axis z) vertical side stops being under pressure, and it is taken into account low compressibility of material of rubber layers. Received solutions can be used to find the dependence „force - settlement" of cylindrical shock absorbers, as well as in projecting such shock absorbers.*

Keywords: rubber, shock-absorber, weak compressibility, side stops, stiffnes.

1. INTRODUCTION

To reduce the harmfulness of vibrations in engineering structures are widely used rubber-metal vibration isolators of different geometry, with unquestionable advantages[2] compared to vibration isolators made from other materials. In the theory of isolation of vibrations attenuation coefficient transfers power from the source of vibration through the vibration isolators on the base is introduced for the frequency $\eta > 2^{0,5}\,\eta_r$, where η_r – mass's resonance frequency of vibration source[1]:

$$\eta_r = \frac{(c/M)^{0.5}}{2\pi} \qquad (1)$$

where: c - stiffness of vibration isolator; M - mass of vibrating mechanism.

The main task in the design of elements of vibration isolator is to provide basic reduction of the resonance frequency below η_r frequency range of vibroactive disturbing forces. At the same time static and dynamic displacements of vibration source do not shall to exceed the permissible values.

For below resonance modes:

$$\eta_r = \frac{(g/\Delta_0)^{0.5}}{2\pi} \qquad (2)$$

where: g - acceleration of gravity;

Δ_0 – static settlement of the elastic elements of vibration isolator, if we assume that the stiffness of vibration isolator does not depends on the frequency and the load value.

From (2) yields, that only for small deformations increase of permissible settlement /Δ_0/ decreases the fundamental frequency η_r (since there is a linear relationship for the rigidity characteristics of the "force - settlement") of elastomeric isolator[1,3]. For large deformations stiffening dependence "force - settlement" of elastomeric vibration isolator has significantly non-linear behavior of the hard type[4] and from (2) yields, that frequency η_r can grow again. For elastomeric vibration isolators minimum attainable frequency η_r, depending on the physical and mechanical properties of elastomeric material, is[2]: 8 Hz - for unreinforced elastomers, 5 Hz - for rubber-metal elements working under compression, 3 Hz - for rubber-metal elements working under a shift loading. In the design of Vibration isolators is often necessary to ensure a reduction (or increase) of the value of frequency η_r, that requires the ability to calculate the stiffness characteristics of elastomeric Vibration isolator in the low finite and medium finite deformations. A similar problem arises in the design and calculation of equifrequent rubber-metal compensating devices, which find application in various fields of engineering and construction industries. In this case, the stiffness characteristic of "force – settlement" P = P (Δ), even for small finite deformations, will be non-linear (or piecewise linear). To solve this problem for the function P = P (Δ), in the general case, it is necessary to design shock absorber that ensures the condition:

$$\frac{dP(\Delta)}{d\Delta}\cdot\frac{1}{P(\Delta)} = f(P(\Delta)) \qquad (3)$$

For the special case f(P(Δ)) = const = C, then from equation (3) we obtain the solution for P(Δ):

$$P(\Delta) = B\cdot\exp(C\cdot\Delta) \qquad (4)$$

The constant of integration B is determined from additional conditions. For example, if shock absorber is multi-layered, an additional condition can be minimization of weight of elastomeric material, providing the required initial settlement Δ_0 of shock absorber. In this case, from (4):

$$B = P(\Delta_0)\cdot\exp(-C\cdot\Delta_0) \qquad (5)$$

and to write down the shock absorber stiffness characteristics P(Δ):

$$P(\Delta) = P(\Delta_0)\cdot\exp[C(\Delta-\Delta_0)] \qquad (6)$$

will provide the desired balance between rigidity and the level of shock absober loading.

If constant $C = \rho^2/g$ (where: ρ - the natural frequency of equipment that is equipped with the projected shock absorber; g - acceleration of gravity), equation (6) describes the stiffness characteristics P (Δ), which provides a description of equifrequent characteristic of compensative shock absorber. It can be several design solutions. In[1, 4] for one of the features (cylindrical shock absorbers with fixed absolutely rigid side stops) the technique of calculation of such shocks absorbers is described.

2. PROBLEM DESCRIPTION

In this paper we propose a design of shock absorber with rigid side boards, which can move parallel to the direction of the applied compressing force. Since due to lateral movement of stops changes free surface of elastomeric layer in a shock absorber, the stiffness of shock absorber changes. In contrast to the shock absorbers with fixed side stops (fixed stops help to implement only the increase of stiffness in the process of loading), the moving side stops make it possible to increase or decrease stiffness (from initial stiffness) in the process of loading. Stiffness of shock absorber can vary from the hard shock (free lateral surface of the elastomeric layer of shock absorber is zero, the precipitation is only due to the weak compressibility of the elastomer) to the stiffness of the shock absorber with no side stops (height of the free elastomeric layer of a shock absorber is the height of the elastomeric layer). Then it can be three cases of moving of side supports.

In the first case, the side stops are moving at the time when the shock absorber is not loaded (fig.1).

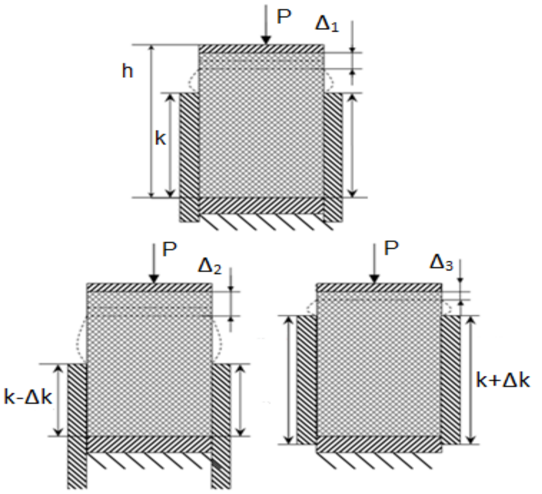

Fig. 1. Shock absorber with moving side stops

In this case we obtain a linear stiffness characteristic “force-settlement” (fig. 2).

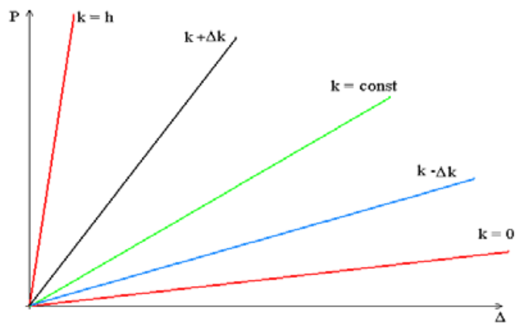

Fig. 2. Dependence “force – settlement” for shock absorber with moving side stops. First case.

In the second case side stops are moving step by step depending on the loading of the shock absorber (fig. 3).

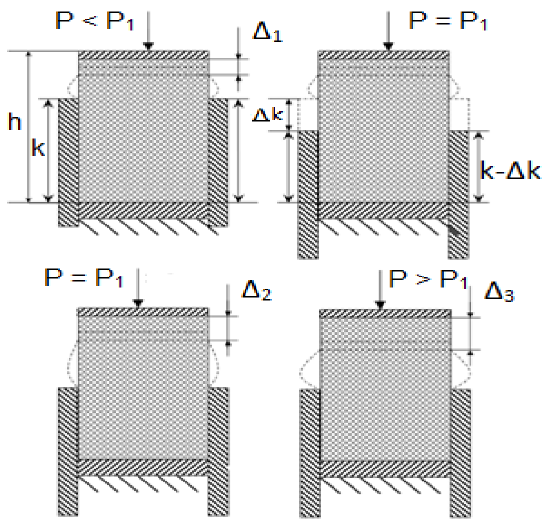

Fig. 3. Shock absorber with moving side stops. Side stops moving “step by step”.

In this case, a piecewise nonlinear stiffness characteristics “force-settlement” (fig.4) “jumps” along the axis of settlement corresponding to the change of height of the side stops in time. At this point force is constant, but by decreasing the free surface of the elastomeric layer settlement increases.

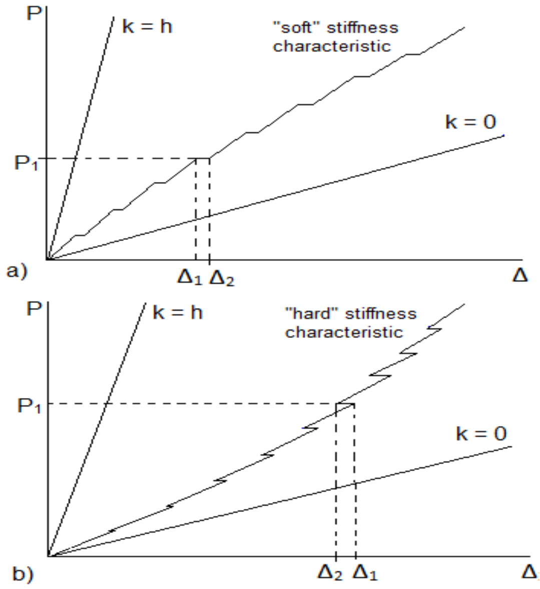

Fig.4. Dependence “force – settlement” for shock absorber with moving side stops. Second case, (a) – “soft” stiffness characteristic, (b) – “hard” stiffness characteristic

In third case, the height of the side stops does not change continuously, depending on the load (fig. 5)

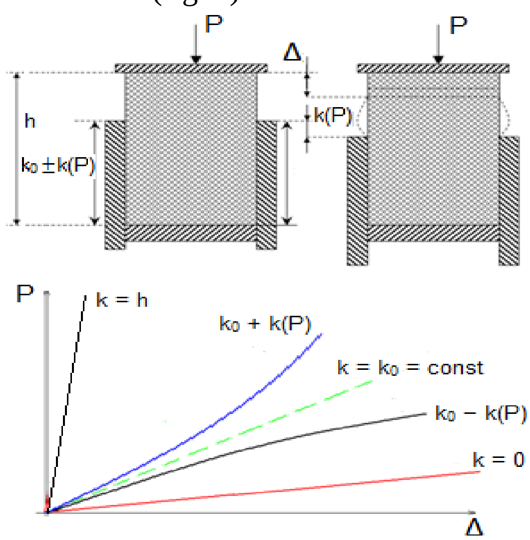

Fig.5 Dependence “force – settlement” for shock absorber with moving side stops. Third case.

3. SOLUTION

To obtain the analytic dependence of P (Δ) "force-settlement" for small finite strains (up to 10% - 15%) shock absorber divided into two parts: Part I ((h - k (P)) ≤ z ≤ h), we have an axisymmetric compression. In Part II (0 ≤ z ≤ (h - k (P)) volume compression. The solution we obtain taking into account the weak compressibility of the elastomer.

For the first part of shock absorber the dependence of the "force - settlement" P(Δ) in the light of weak compressibility of rubber, we find using the principle of minimum total potential energy of deformation $U(u_1,w_1)$ [1,2]:

$$U(u_1, w_1) = J(u_1, w_1) - P\Delta_1 \qquad (7)$$

where:

$$J = 2\pi G \int_{-\frac{h}{2}}^{\frac{h}{2}} \int_0^b \left[\left(\frac{\partial u}{\partial r}\right)^2 + \left(\frac{u}{r}\right)^2 + \left(\frac{\partial w}{\partial r}\right)^2 + \frac{1}{2}\left(\left(\frac{\partial u}{\partial z}\right) + \left(\frac{\partial w}{\partial r}\right)\right)^2 + \frac{3\mu}{1+\mu} s \left(\left(\frac{\partial u}{\partial r}\right) + \frac{u}{r} + \left(\frac{\partial w}{\partial z}\right)\right) - \frac{9\mu\,(1-2\mu)}{4\,(1+\mu)^2} s^2 \right] r\,dr\,dz \qquad (8)$$

Where: G - is modulus of rigidity, μ - is Poisson's ratio.

Choosing the displacement functions u_1 (r, z), w_1 (r, z), respectively, along the axes r and z:

$$u_1 = A_1 r \left(1 - \frac{z}{h}\right)\left(1 - \frac{z}{h^*}\right),\; h^* = h - k(P)$$

$$w_1 = -\frac{3\Delta_1}{2h^*\left(1 + 2\frac{h}{h^*}\right)}\left(z - \frac{z^2(h + h^*)}{hh^*}\right) + \frac{z^3}{3hh^*} - \frac{4h^{*2}}{3h} \qquad (9)$$

that satisfy the geometric boundary conditions:

$$u_1(r,h) = 0, \quad u_1(r,\ h - k(P)) = 0$$
$$w_1(r,h) = -\Delta_1, \qquad (10)$$
$$w_1(r,\ h - k(P)) = 0$$

From the principle of minimizing the functional $U(u_1,w_1)$ the dependence of the "force - settlement" $P(\Delta_1)$:

$$\Delta_1 = \frac{P\cdot(h - k(P))}{\pi b^2 G}\left[1.8 + \frac{1{,}2 + 1{,}5\alpha^2}{\left(1 + 3\frac{1-2\mu}{2\mu}\alpha^2\right)}\right]^{-1}, \quad \alpha = b/\left(h - k(P)\right) \qquad (11)$$

For the second part of the shock absorber, assuming that the mobility of the side constraint is absolutely rigid, the dependence of the force - of sediment P (Δ_{II}), without taking into account the frictional forces on the contact surface elastomer – metal, defined as in the case volumetric compression [1]:

$$\Delta_{II} = \frac{3Ph}{2\pi b^2 G}\cdot\frac{(1-2\mu)}{(1+\mu)} \qquad (12)$$

For all the shock absorber dependence "force – settlement" P (Δ), with (9) and (10) takes the form:

$$\Delta = \Delta_1 + \Delta_{II} \qquad (13)$$

When using the dependences (9) - (13), note that they are obtained for small finite deformations, when:

$$0 \le \Delta_1/(h - k(P)) \le 0{,}1 \div 0{,}15 \qquad (14)$$

The thickness of a free (without side boards) elastomeric layer h_0 (P = 0) in the process of loading can vary:

$$0 \le h_0(P=0) = h - k(P=0) \le h \qquad (15)$$

Based on the requirements of exploitation the shock absorber can vary the thickness of the elastomeric layer is free depending on the size of loading, or can be given any requirement to change the stiffness characteristics of the shock absorber. For the two algorithms condition of smallness of final deformation must be satisfied (11). The most simple to constructing dependence of "force - settlement", If you specify the intervals for changing the load P and the thickness of the free layer of elastomeric shock absorber. In this case, the algorithm for constructing the function P (Δ) will be the following. Let the interval values of the compressive of force P is split into N (not necessarily equal) intervals and set requirements for the change in the thickness of the free elastomeric layer, that is a function k (P) for each interval of the force P. For i - th interval:

$$P_{i-1} \le P_i \le P_{i-1} + P_i^* \qquad (16)$$

Where: P_i*- step of force P (if these steps are the same, P_i* = P / N)

The thickness of the free (without side boards) elastomeric layer to the i-th interval of loading takes values:

$$h_{i-1}(P = P_{i-1}) \le h(P = P_i) = h(P = P_{i-1}) - k(P = P_i) \le h \qquad (17)$$

The total settlement $\Delta_{\Sigma i}$ shock absorber on the i - th step of loading is given by:

$$\Delta_{\Sigma i} = \Delta_{\Sigma(i-1)} + \left(\Delta_I\left(h_0 - k_i(P_i), P_i^*\right) + \Delta_{II}(k_i(P_i), P_i^*)\right) \qquad (18)$$

where the values of Δ_I ((h_0 - k_i (P_i), P_i *) and Δ_{II} (k_i (P_i), P_i*) calculated by formulas (11) and (12) and substituting the values of the required thickness of the free elastomeric layer is at each stage of loading. From (8) - (14) that if k (P = P_i) is negative, the thickness of the elastomeric layer is free to grow and shock absorber stiffness will decrease, that is dependence of the force - settlement be precipitate a "soft" type. For positive values of k (P = P_i) elastomeric layer thickness of free will decrease and damping increases, that is dependence of the force - settlement be precipitate a "hard" type.

4. EXPERIMENT

4.1 Description

As an example, consider a step change in stiffness of the shock absorber. Parameters of the shock absorber: b = 18 mm, h = 40 mm, μ = 0.493, G = $5.2 \cdot 10^5$ N/m^2.

A cylindrical rubber shock absorber loaded with an axial compressive force and the height of the side stop has changed “step by step”, step size is 6.5 mm. The experiment was performed on the test machine Zwick/Roell Z150.

4.2 Experimental Results

The results obtained in the experiment are shown in Table 1.

Force, N	k, mm	Stiffness, N/mm
0 - 20	0	39.24
20 - 40	6.5	46.46
40 - 60	13	56.65
60 - 80	19.5	70.22
80 - 100	26	101.76
100 - 120	32.5	152.72
120 - 140	37	332.40

Table 1. The dependence of the stiffness of rubber shock absorber on the height of the side stops. The experimental results.

4.3 Analytical Solution

The analytical solution consider using formula (13). The calculation was made using "MathCad" software. We have two analytical solutions. In first solution (solution “a”) rubber is considered as weakly compressible material. Results are shown in Table 2.

Force, N	k, mm	Stiffness, N/mm
0 - 20	0	39.41
20 - 40	6.5	46.67
40 - 60	13	57.23
60 - 80	19.5	71.19
80 - 100	26	103.46
100 - 120	32.5	154.8
120 - 140	37	338.05

Table 2. The dependence of the stiffness of rubber shock absorber on the height of the side stops. Analytical solution “a”.

In second analytical solution (solution “b”), rubber is considered as non - compressible material and in formula (13) we use Poisson’s ratio μ = 0.5. Results are shown in Table 3.

Force, N	k, mm	Stiffness, N/mm

0 - 20	0	39.43
20 - 40	6.5	47.09
40 - 60	13	58.43
60 - 80	19.5	76.96
80 - 100	26	112.69
100 - 120	32.5	210.36
120 - 140	37	529.91

Table 3. The dependence of the stiffness of rubber shock absorber on the height of the side stops. Analytical solution “b”.

4.4 Comparsion of Results

Force, N	Error ε, % (solution “a”)	Error ε, % (solution “b”)
0 - 20	0.43	0.48
20 - 40	0.45	1.36
40 - 60	1.02	3.14
60 - 80	1.38	9.60
80 -100	1.67	10.74
100 - 120	1.36	37.74
120 - 140	1.70	59.42

Table 4. Comparison of both obtained results - experimental and analytical.

Then we compare the results obtained analytically with the results obtained during the experiment and calculate the error. Results are shown in Table 4.

5. CONCLUSION

In this title proposed a method for determination of rigidity dependence "Force - Settlement" for shock-absorbing elements with absolutely rigid moving (parallel to the vertical axis z) vertical side stops being under pressure. It's also taken into account low compressibility of material of rubber layers. Received solutions can be used to find the dependence „force - settlement” for cylindrical shock absorbers, as well as in projecting such shock absorbers. You can design a shock absorber with a given non-linear („hard” or „soft”) stiffness characteristics. As an example, considered the technique of constructing a rubber shock absorber with variable stiffness (the first example of a “soft” stiffness, the second with “hard” stiffness). The height of the side stops depends on the applied force, and changes “step by step”. As can be seen from a comparison of the results (Table 4.) neglecting the weak compressibility of rubber leads to a large error when the thickness of rubber decreases.

7. ACKNOWLEDMENTS

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8. REFERENCES

[1] Lavendel E.E. *Calculation of rubber products*. Машиностроение, Moscow, 1976.
[2] Lyapunov V.T., Lavendel E.E., Schlyapochnokov S.A. *Rubber vibration isolators.* L., Судостроение, 1988.
[3] Lavendel E.E., Dymnikov S.I., Pavlovskis A.A., Sniegs M.I. *Applied methods of calculating the product of highly elastic materials*. Zinatne, Riga, 1980.
[4] Gonca V., Švabs J. *Projecting Elastomeric Shock Absorbers with Variable Stiffness* In Journal of Vibroengineering. - Vol.12, Issue 3. (2010) pp 347-354.

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