

DYNAMICS OF GRAVITY FEEDER FOR PRISMATIC WORKPIECES

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Abstract: *Feeders are mechanisms of the single- piece feeding for the forced moving of the oriented workpieces. In this work the vertical gravity-impact feeder for moving of the prismatic or plane details is presented. The parts move on inclined guiding plates, free fall and collide with the down plates, the impact phenomena may be used both for decreasing of velocity and for the orientation of the tracking work pieces (lateral reversing). System of equations of plane motion of detail, including stages of sliding on the slope guideway, free flight, impact and motion to the next guideway, are written down. System of equations is solved numerically with help of MathCAD program.*

Key words: forced motion, oblique impact, dry friction, unilateral constraint.

1. INTRODUCTION

Widely used types of technological transport are various kind conveyors, descents, bunkers, feeders. Feeders are a various kind of loading - unloading devices which serves for parts giving in the focused position on the conveyors, or to the processing equipment [1], [2], [3]. In this paper the vertical mechanical feeder with gravitational method of transportation is considered; devices of this kind serve for transportation of light workpieces with small speed of sliding (Fig. 1). The impact phenomenon can be used in feeders both for decreasing of speed of transported details, and for their orientation – turn by the opposite side. The feeder represents a vertical tray with plates attached to walls;

the angle of plate slope to horizon can be regulated (adjusted). The part begins motion on a plate surface when the plate angle of slope to horizon becomes more than angle of a friction. For proper orientation of the parts the guide groove is made in a plate, so part motion on a plate is considered to be plane.

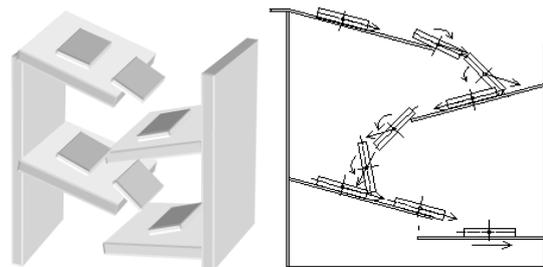


Fig. 1. Model of a gravitational feeder and the scheme of part motion

The part slides on an inclined plate, freely falls on the downstream plate, during the impact with down plate part can change position, turning (upside down) over an underside. The number of plates depends on height to which it is necessary to transfer details, the inclination of plates and distance between them depends on allowable velocity of parts motion and necessity to turn a part. The scheme of movement of flat section of a detail on the guide is shown in Fig.1.

2. ANALYTICAL MODEL

Workpieces, moving on the feeder plate, is considered as perfectly rigid body which motion is interrupted by collisions with surface of fixed perfectly rigid plate. Such problems concern the section of mechanics

studying dynamics of systems with unilateral constraints [4], [5], [6], [7]. The most actual here is modelling and the analysis of the impact phenomenon [8], [5], [19]. In this work impact is modelled as perfectly rigid body impact; the classical theory of impact in a combination with Rauss hypothesis is used for the solution, taking into account a friction at impact in accordance with the Coulomb law. For impact parameters obtaining the general equations of dynamics are drawn up, and the time of impact is divided into two phases –at the end of the first phase normal velocity of contact point of a body is equal to zero, this gives the additional equation for definition of unknown parameters. For the description of the collision process the impact impulse is used. Total impulses of normal reaction in the first and second phases are connected by relation: $S_{IIN} = RS_{IN}$, where R is coefficient of restitution; total impulses of friction forces at sliding are $S_{IF} = -fS_{IN}$, $S_{IIF} = -fS_{IIN}$, where f is friction factor. As it is shown in [8] at plane impact of a body against a fixed obstacle, depending on initial conditions of impact and inertial properties of a body there can be seven qualitatively different cases of impact: 1) full sliding of a body in one direction, generated by pre-impact body motion; 2) the same, generated by pre-impact a body location; 3) non-sliding in a contact point; 4) full sliding at first in one, and then in an opposite direction with a backspacing in the second phase of impact; 5) the same, with a backspacing in the first phase of impact; 6) the sliding termination in the second phase of blow; 7) the same, in the first phase of blow. The second and third cases of impact occur when the contact point has only normal to a surface of impact velocity, that in this case of parts motion is excluded. Impact is considered as perfectly inelastic, i.e. coefficient of restitution $R=0$, so the fourth and sixth cases of impact are also not considered. In this work the equations of dynamics for cases 1, 5, 7 of impact are taken as in [8] and do not cited here.

3. DYNAMICS EQUATIONS

When body moves on feeder guide plates five stages of motion are differed:

1) Sliding on an inclined plane – translatory motion, 2) planar motion – sliding with rotation in the presence of unilateral constraint, 3) free motion of a body under gravity, 4) collision with an inclined plane, 5) planar motion – sliding with rotation till contact with a plane. Peculiar feature of writing of the equations of dynamics and definition of parameters of movement in is using own coordinates system for each stage of motion, drawing up the formulas of coordinates transform and projection components of velocities transformation from one system into another. Final values of parameters of motion at each stage are initial one in new system of coordinates for a following stage of motion.

1) Translation motion - sliding on an inclined plane (Fig. 2).

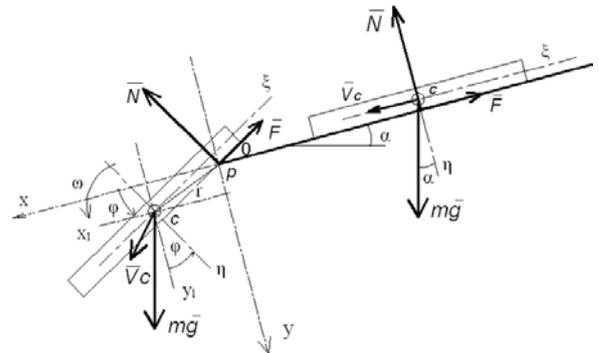


Fig.2. the first and the second stages of motion

Coordinate – an axis x , the equation of motion at the first stage at continuous movement:

$$\ddot{x}_c = g(\sin(\alpha) - f \cos(\alpha)), \quad (1)$$

velocity and time:

$$v = \sqrt{2gL(\sin(\alpha) - f \cos(\alpha))},$$

$$T = \sqrt{\frac{2L}{g(\sin(\alpha) - f \cos(\alpha))}},$$

where α – a plane slope angle to horizon, c – mass centre of a body, f – friction factor (rest and sliding are provided identical).

2) Planar motion - rotation round point P with sliding till separation from surface (fig. 2). Constraint in a point of contact P of a part and a plate is unilateral; reaction force in a contact point is subject to the law of a dry friction, i.e. the Coulomb law:

$$F = -fN \frac{v_p}{|v_p|} \text{ at } v_p \neq 0,$$

$$|F| \leq fN \text{ at } v_p = 0, N \geq 0$$

N - normal force of reaction, F - tangent force of reaction.

Coordinates systems (Fig. 2): xoy - fixed rectangular Cartesian coordinates system, the origin of coordinates - plate edge, x_1cy_1 - passing through the mass centre Cartesian system of the coordinates parallel xoy , $\xi c\eta$ - the rectangular system of coordinates rigidly connected with a body, polar coordinates: r -radius vector, from point P to point c , and φ - rotation angle of a body,

$$r = r(t), \varphi = \varphi(t),$$

$$x = r \cos(\varphi), y = r \sin(\varphi).$$

The equations of motion:

$$\begin{cases} r\ddot{\varphi} + 2\dot{r}\dot{\varphi} = -\frac{N}{m} + g \cos(\alpha + \varphi) \\ \ddot{r} - r\dot{\varphi}^2 = -\frac{F}{m} + g \sin(\alpha + \varphi) \\ J\ddot{\varphi} = Nr \end{cases} \quad (2)$$

Angular velocity and angular acceleration of a body:

$$\dot{\varphi} = \omega, \ddot{\varphi} = \varepsilon,$$

radial and tangential velocities of the mass centre:

$$v_{cr} = \dot{r}, v_{c\varphi} = r\dot{\varphi}.$$

Initial conditions of the second stage:

$$\omega_0 = 0, v_0 = v_r = v, r_0 = 0, \varphi_0 = 0,$$

$$N_0 = mg \cos(\alpha),$$

final parameters of the second stage:

$$\omega_1, \varphi_1, r_1, \dot{r}_1.$$

The end of stage - equality to zero of normal reaction $N_1 \leq 0$, or equality of mass center coordinate to the distance from the mass center to detail edge $r_1 = a/2$.

Initial conditions of the third stage:

$$x_{c0} = r_1 \cos(\varphi_1), y_{c0} = r_1 \sin(\varphi_1),$$

$$\omega_0 = \omega_1,$$

$$\dot{x}_{c0} = \dot{r}_1 \cos(\varphi_1) - r_1 \dot{\varphi}_1 \sin(\varphi_1),$$

$$\dot{y}_{c0} = \dot{r}_1 \sin(\varphi_1) - r_1 \dot{\varphi}_1 \cos(\varphi_1).$$

3) Free movement of a body in the field of gravity (Fig. 3).

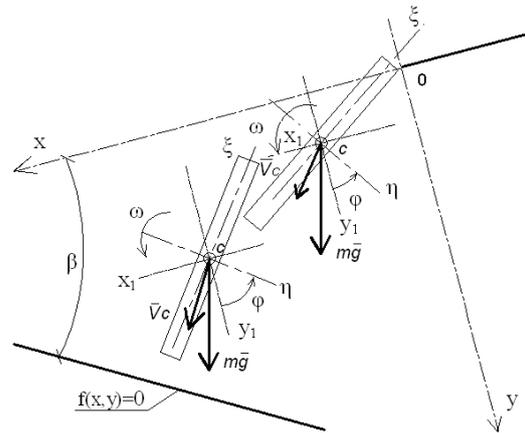


Fig.3. The third stage of motion of body

Cartesian coordinates xoy are used:

$$\begin{cases} \ddot{x}_c = g \sin(\alpha) \\ \ddot{y}_c = g \cos(\alpha) \\ J\ddot{\varphi} = 0 \end{cases} \quad (3)$$

The equation of the lower plate in coordinates xoy : $f(x, y) = 0$, or in the intercept form of the equation of a straight line:

$$\frac{x}{d} + \frac{y}{b} = 1,$$

where d and b - the intercepts, cut by the lower plate on axes x and y .

Coordinates of a point of collision K :

$$x_K = x_c + \frac{a}{2} \cos(\varphi), y_K = y_c + \frac{a}{2} \sin(\varphi).$$

Definition of a point of impact and the impact moment - fulfilling of a condition of unilateral constraint:

$$\frac{x_K}{d} + \frac{y_K}{b} \geq 1.$$

Normal and tangent to a plane velocity of contact point at the beginning of impact:

$$\dot{x}_K = \dot{x}_c - \omega_1 \frac{a}{2} \sin(\varphi),$$

$$\dot{y}_K = \dot{y}_c + \omega_1 \frac{a}{2} \cos(\varphi), \quad \omega_1 = \omega_0,$$

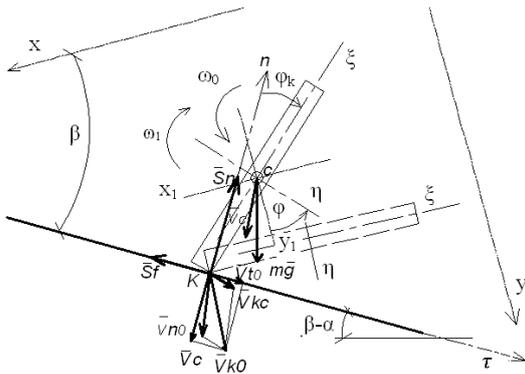
$$v_{Kn} = \dot{x}_K \sin(\beta) + \dot{y}_K \cos(\beta),$$

$$v_{Kt} = -\dot{x}_K \cos(\beta) + \dot{y}_K \sin(\beta),$$

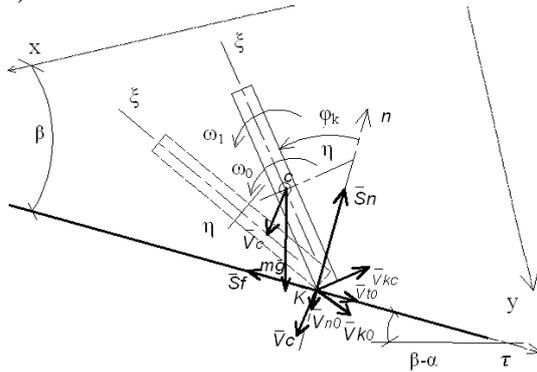
$$v_{Kn} = \dot{x}_c \sin(\beta) + \dot{y}_c \cos(\beta) + \omega_1 \frac{a}{2} \cos(\alpha + \beta)$$

$$v_{Kt} = -\dot{x}_c \cos(\beta) + \dot{y}_c \sin(\beta) + \omega_1 \frac{a}{2} \sin(\alpha + \beta)$$

4) Body impact about the fixed surface of the lower plate (fig. 4), coordinates - a normal and a tangent to a plate, the beginning of coordinates - in a point of collision K: $nK\tau$, coordinates of a body during blow don't change, post-impact velocities are defined from the general equations of dynamics for impact [8].



a) variant I



b) variant II

Fig.4. The fourth and fifth stages of motion

The angle between direction of axis ξ of plates and a normal to a plate:

$$\varphi_K = \beta + \varphi - \frac{\pi}{2}.$$

Angle between a direction of pre-impact velocity and a normal to a plate:

$$\theta = \arctan\left(\frac{v_{Kt}}{v_{Kn}}\right).$$

5) Post-impact motion (Fig. 4) – plane motion with sliding on a plate without separation under condition of perfectly nonelastic impact. Movement of a body after impact is described by the differential equations in a projection to axes τ and n , with the initial conditions received at the previous stage of calculation – angular velocity, mass center velocity and dynamic reaction in the collision point.

4. NUMERICAL EXAMPLES

The example of the numerical solution of this problem for parts transportation on two guide plates is resulted bellow. Part has following parameters: mass $m=0.3$ kg, length of 0.1 m, the moment of inertia in respect to an axis passing through the mass centre $J_c = 2.5 \cdot 10^{-4}$ kgm². Two variants of part motion are considered – feed-moving from one plate to another without turning of a part and feed-moving with reversing. For this purpose different ways of plates locations are used:

1) in the first variant following parameters are accepted: angles of guide plate inclination $\alpha = 15^\circ$, $\beta = 30^\circ$, the equation of a surface of the lower plate in axes xy , passing through the edge of first the plates:

$$\frac{x}{0.347} + \frac{y}{0.2} = 1.$$

2) in the second variant angles of slope of the upper and lower guide plates are $\alpha = 10^\circ$, $\beta = 35^\circ$, the equation of a surface of the lower plate in axes xy :

$$\frac{x}{0.428} + \frac{y}{0.3} = 1.$$

The sliding friction and statical friction factor are assumed identical and equal $f=0.15$, coefficient of restitution of normal reaction in impact point is accepted zero.

Part movement begins with continuous sliding on the top plate, calculation is executed in program MathCAD, and calculation results are presented in table 1.

As it is shown in table, in the first case angular velocity changes a sign after impact, the velocity of body in contact with surface point, also changes a sign, after impact the body lies down on a plate by the same side; in the second case the velocities don't change the direction, and after impact the body lies down on a plate by reverse side.

Parameter	unit	case 1	case 2
		$\alpha=15^\circ$ $\beta=30^\circ$	$\alpha=10^\circ$ $\beta=35^\circ$
The first stage -slipping			
t	s	0.423	0.887
v_0	m/s	0.473	0.226
The second stage – plane motion till release			
r_0	m	0	0
φ_0	rad	0	0
v_{r0}	m/s	0.473	0.226
$v_{\varphi 0}$	m/s	0	0
ω_0	s^{-1}	0	0
t	s	0.879	1.169
r	m	0.5	0.35
φ	deg	15.18°	45.15°
v_{r1}	m/s	0.746	0.527
$v_{\varphi 1}$	m/s	0.271	0.269
ω_1	s^{-1}	5.490	7.637
The third stage – free plane motion			
x_{c0}	m	0.049	0.032
y_{c0}	m	0.012	0.015
v_{xc0}	m/s	0.653	0.363
v_{yc0}	m/s	0.461	0.467
ω_0	s^{-1}	5.490	7.637
t	s	0.821	1.417
x_{c1}	m	0.111	0.101
y_{c1}	m	0.082	0.178
φ_1	deg	41.02°	87.15°
v_{xc1}	m/s	0.862	0.605
v_{yc1}	m/s	1.239	1.837
The fourth stage – collision with an inclined plane- impact			
φ_K	deg	18.97°	32.14°
τ_{c0}	m	0,016	-0.027
n_{c0}	m	0,047	0.042
v_{cr0}	m/s	-0.127	0.558
v_{cn0}	m/s	-1.504	-1.852
$v_{K\tau 0}$	m/s	0.133	0.881

v_{Kn0}	m/s	-1.593	-1.649
impact type		5	1
v_{ct1}	m/s	0.043	0.408
v_{cn1}	m/s	-0,188	-0.851
ω_1	s^{-1}	-11.722	31.972
$v_{K\tau 1}$	m/s	-0,508	0.408
v_{Kn1}	m/s	0	0
The fifth stage – planar motion before contact with plane			
t	s	0.868	0.031
ω_1	s^{-1}	-18.837	34.50
v_{ct1}	m/s	0.284	0.527
v_{cn1}	m/s	-0.876	-1.072

Table 1. The parameters of the body motion along the feeder guide plates

Plots of dependence of some parameters of motion from time for the first variant are shown in Fig. 5-7

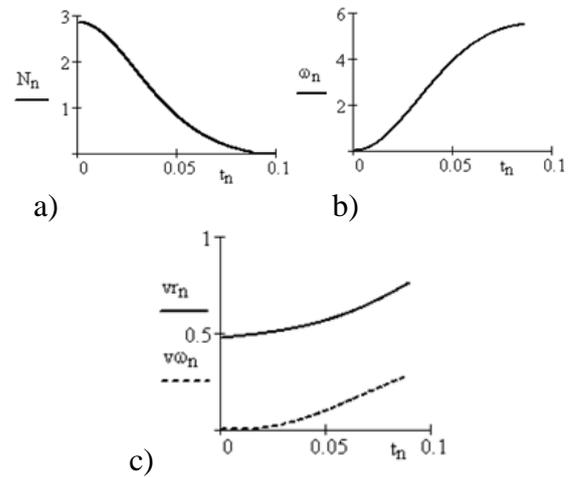


Fig.5. Motion in the second stage. Plots of dependence on time: a) $N=N(t)$, b) $\omega=\omega(t)$ c) $v_{cr}=v_{cr}(t)$ and $v_{c\omega}=v_{c\omega}(t)$

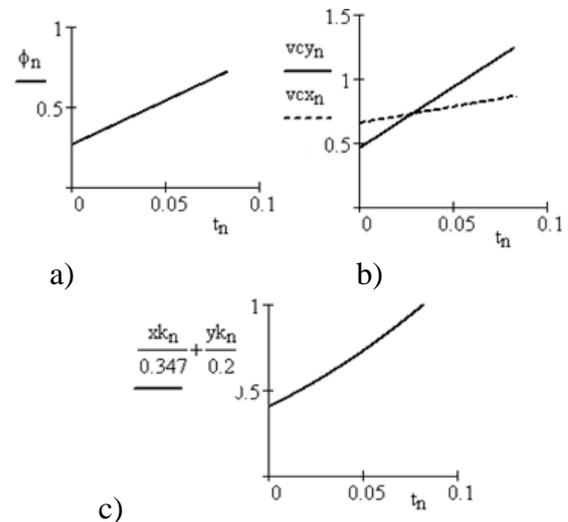


Fig. 6. Motion in the third stage. Plots of dependence on time: a) $\varphi = \varphi(t)$, b) $v_{xc} = v_{xc}(t)$, $v_{yc} = v_{yc}(t)$, c) constraint function $x_k/0.347 + y_k/0.2 = 1$

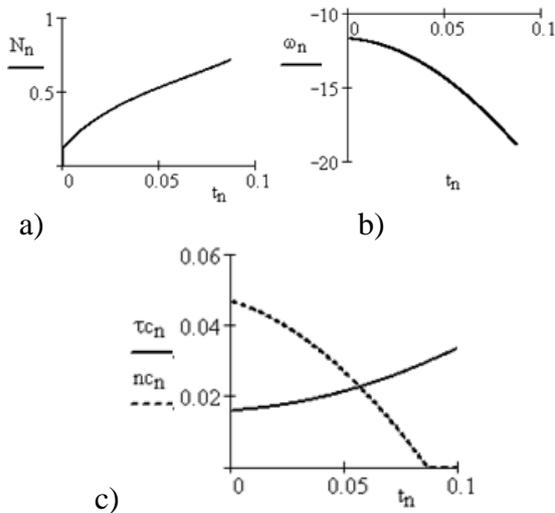


Fig. 7. Motion in the fifth stage. Plots of dependence on time: a) $N = N(t)$, b) $\omega = \omega(t)$ c) $\tau_c = \tau_c(t)$, $n_c = n_c(t)$

5. CONCLUSION

In this paper the the variant of gravity-operated feeder is presented and dynamics of workpieces motion in this device is investigated. The dynamic equations are written down for each stage of motion with using eigensystem of coordinates for every type of motion and development of formulas for coordinates conversion and velocity projections transform from one coordinate system into another. The conditions of turning of part as a result of its collision with rigid guide plate are found: in order to a body turned over it is necessary, that angular speed must not change a direction after impact and, in addition, to be sufficient for body rotation in necessary direction. At inclination of plate from left to right downward, positive initial angular velocity and positive initial angle between surface normal and part φ_k - greater, than inclination angle of plate to horizon - angular velocity after impact remains positive and perfectly inelastic impact will be of the 1-st type (with complete sliding in one direction to the right downward); a negative initial angle

between normal to the surface and a part results in reversing of rotation, here perfectly inelastic impact occurs of the 5-th type (with the complete sliding and changing of sliding direction). The changing of this angle is possible by the increase of angular velocity of free motion or by increasing of time of free motion. There is a parameters domain of initial angle ($0 \leq \varphi_k \leq \beta - \alpha$), in which the impact may be 7-th type with stopping of sliding and angular velocity reversing. The numeral examples of workpiece motion are given; equations of motion are solved with help of MATHCAD program.

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