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How to Find Rare Attractors and Chaos in Nonlinear Dynamical Systems by the Method of Complete Bifurcation Groups

The Collection of Bifurcation Diagrams
in the Pendulum Driven Systems

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