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Transport and Telecommunication Institute, Lomonosov 1, LV–1019, Riga, Latvia

NETWORKS OF NONLINEAR PROJECTORS

A. Kovantsov

Riga Technical University, Department of Engineering Mathematics
Kalku Str.1, Riga LV-1658, Latvia
Phone: 67089528. E-mail: akovancova@baltinet.lv

Differentiation of system of the differential equations by a task becomes much easier, if system to simplify, i.e. to lead it to such look when it breaks up to blocks, in each of which – the independent system of unknown functions. In this case use special matrixes – projectors. Such projectors generate a network of surfaces in Euclidean space: surfaces of projections and projecting networks. For each set system of the equations there are networks, and not the unique. Properties of such networks are considered in work.

Keywords: systems of the differential equations, a projecting network of surfaces, Jordan's cell

1. Introduction

Both in the theory, and in many practical questions, the all–important role is played by problems of the solution of systems of the ordinary differential equations. One of such problems consists in splitting of this system on blocks, i.e. in such subsystems, each of which contains smaller number of the functions entering at the same time and under a sign of a derivative, and in the right parts. Abundantly clearly that integration of such subsystems is a task simpler, than integration of initial system. Splitting of this system on blocks appeared equivalent to reduction of a square matrix to a normal Jordan form. To Jordan's each cell there corresponds a certain block of the split system. In certain cases such splitting can be carried out by means of special degenerative matrixes – projectors.

2. Geometry of Splitting of Systems

Set of the degenerative matrixes P_α, satisfying to conditions

$$P^2{}_\alpha = P_\alpha, \quad P_\alpha P_\beta = 0, (\alpha \neq \beta), \quad \sum_{i=1}^{\alpha} P_i = E \quad (E - \text{unity matrix})$$

is called as system of linear (nonlinear) projectors.

However, thus there is open a question of the geometrical processes occurring at such splitting.

The independent system of the differential equations is considered

$$\frac{dy^i}{dt} = f^i(y^j),\ i, j = 1,...,n\ . \tag{1}$$

Variables y^i are considered as rectangular coordinates of a point in n – measured Euclidean space.

Possibility to break system (1) on blocks, each of which contains smaller number of the functions entering at the same time and under a sign of derivatives in the left parts, and in the right parts of each block, conducts to simplification of initial system, and thereby – to simplification of its integration. Let

$$y^i = F^i(z^j) \quad \rightarrow \quad z^j = \overline{F^j}(y^i) \tag{2}$$

- the no degenerative transformation bringing system (1) to the split look

$$\begin{cases} \frac{dz^{i_1}}{dt} = f_1^{i_1}(z^{j_1}), & i_1, j_1 = 1,2,3,..,s_1 \\ \frac{dz^{i_2}}{dt} = f_2^{i_2}(z^{j_2}), & i_2, j_2 = s_1 + 1,...,s_1 + s_2, \\ ------------------------- \\ \frac{dz^{i_p}}{dt} = f_p^{i_p}(z^{j_p}), i_p, j_p = s \ + s_2 + ... + 1,...,s_1 + ... + s_p \end{cases}, \tag{3}$$

- where $s_1 + ... + s_p = n.$ Each group of the equations of the system (3), containing independent system of variables call the block. No degenerative transformation (2), reformative system (1) to a look (3), generates p degenerative transformations, each of which transfers all space to some surface of dimension s_α $(\alpha = 1,...,p)$ and system (1) – to the corresponding block of system (3). Really, having substituted values from (2) in (1), we will come to system of the equations $\frac{\partial F^i}{\partial z^j}\frac{dz^j}{dt} = f^i(F^j(z^k)).$

Designating through $\frac{\partial \overline{F^j}}{\partial y^i}$ matrix elements, to a return matrix $\left(\frac{\partial F^i}{\partial z^j}\right)$ (owing to a transformation (2) no degenerative, the last also no degenerative), we have

$$\frac{dz^j}{dt} = \frac{\partial \overline{F^j}}{\partial y^i} f^i . \tag{4}$$

On a condition the last system looks like (3), i.e. equalities take place

$$\begin{aligned} &\frac{\partial \overline{F^1}}{\partial y^i} f^i = f_1^{i_1}(\overline{F^{j_1}}) \\ &----------- \\ &\frac{\partial \overline{F^{i_p}}}{\partial y^i} f^i = f_p^{i_p}(\overline{F^{j_p}}). \end{aligned} \tag{5}$$

Let's break nondegenerate transformation (2) into p of blocks

$$\begin{aligned} &z^{j_1} = \overline{F^{j_1}}(y^i), \\ &-------, \\ &z^{j_p} = \overline{F^{j_p}}(y^i). \end{aligned} \tag{6}$$

With each line of transformation (6) we will connect degenerative transformation which we will determine by equalities

$$\begin{aligned} &z^{j_1} = \overline{F_1^{j_1}}(y^i), && z^{j_p} = \overline{F^{j_p}}(y^i), \\ &z^{j_2} = \varphi_1^{j_2}(z^{j_1}), && z^{j_1} = \varphi^{j_1}(z^{j_p}), \\ &------------------, \\ &z^{j_p} = \varphi_1^{j_p}(z^{j_1}), && z^{j_{p-1}} = \varphi_p^{j_{p-1}}(z^{j_p}), \end{aligned} \tag{7}$$

where $\varphi_1{}^{j_2},...,\varphi_1{}^{j_p},...,\varphi_p{}^{j_1},...,\varphi_p{}^{j_{h-1}}$ – some functions of the corresponding number of variables. Each transformation defined by the corresponding column of system (7), we will designate the transformation, associated to the corresponding block of system (3). The associated transformations (7) we will designate p_α $(\alpha = 1,...,p)$. Each of transformations P_α transfers all space E_n to the corresponding surface which dimension is equal respectively $s_1,...,s_p$. Let's designate these surfaces symbols σ_α. The equations of surfaces σ_α have respectively a look

$$\begin{aligned} &z^{j_2} = \varphi_1{}^{j_2}(z^{j_1}), \qquad z^{j_1} = \varphi_p{}^{j_1}(z^{j_2}), \\ &-------------------, \\ &z^{j_p} = \varphi_1{}^{j_p}(z^{j_1}), \qquad z^{j_{p-1}} = \varphi_p{}^{j_{p-1}}(z^{j_p}). \end{aligned} \tag{8}$$

Surfaces σ_α we will call surfaces of projections (at $s_\alpha = 1$ we have the line of projections, at $s_\alpha = n-1$ – a hyper surface of projections. Generally we speak $- s_\alpha$ – surfaces of projections). Let's assume that all surfaces pass through the beginning of coordinates. Let $M(y^i)$ – any point of space E_n. Let's take any of surfaces of projections, for example, σ_1. Let's include σ_α surfaces in a network of the Σ_α surfaces having the same dimensions, as σ_α surfaces. Let's call such network a projecting network. Let's designate it a symbol S. Through a point $M(y^i)$ of space pass p surfaces Σ_α (through the beginning of coordinates pass the σ_α surfaces which are also belonging to a network). Let's take the $\Sigma_2,...,\Sigma_p$ surfaces passing through a point M. They lie on some $H_{n-s_{12}}$ surface. Surface H_{n-s_1} we will call a projecting surface. It crosses a σ_1 surface in some point N_1. In a N_1 point all points of E_n space belonging to the projecting H_{n-s_1} surface are projected. A point N_1 we will call a projection H_{n-s_1} of a H_{n-s_1} surface to a surface σ_1 of projections. Projections $N_2,...,N_p$ on other surfaces of projections can be similarly defined.

How the projecting network is connected with the set system of the differential equations (1) split by system (3) and system of σ_α surfaces?

Theorem. At the set system of the equations (1) and beforehand set its split look (3) (functions f^i and $f_\alpha{}^{i_\alpha}$ – are any) exists (and thus not unique) the projecting network S containing σ_α surfaces as that it's forming, which pass through the beginning of coordinates.

Really, let the transformation bringing system (1) to the split look (3) is presented by equalities

$$y^i = y^i(z^j) \tag{9}$$

or, the equalities resolved z^j relatively

$$z^j = z^j(y^i). \tag{10}$$

In that case the first line of system (5) takes a form

$$\frac{\partial z^{i_1}}{\partial y^i}\frac{dy^i}{dt} = f_1^{i_1}(z^{j^1})$$

or

$$\frac{\partial z^{i_1}}{\partial y^i} f^i(y^j) = f_1^{i_1}(z^{j_1}). \tag{11}$$

It is system of the equations in partial derivatives of the first order concerning s_1 functions z^{i_1}. Let's carry out through surfaces $\sigma_2,...,\sigma_p$ any hyper surface H_1. Let its equation $y^n = H_1(y^1,...,y^{n-1})$. We will assume that $f^n(0) \neq 0$. The equations (11) we will write down in a look

$$\frac{\partial z^{i_1}}{\partial y^n} = \frac{f_1^{i_1}}{f^n} - \frac{f^1}{f^n}\frac{\partial z^{i_1}}{\partial y^1} - ... - \frac{f^{n-1}}{f^n}\frac{\partial z^{i_1}}{\partial y^{n-1}}. \quad (12)$$

It is Cauchy type system. Let's integrate it so that equalities took place

$$z^{i_1} = z^{i_1}(y^1,...,y^{n-1},H_1(y^1,...,y^{n-1})) = y^{i_1}.$$

According to the theorem Cauchy–Kovalevsky such solution $z^{i_1} = z^{i_1}(y^j)$ is unique.

As the hyper surface H_1 contains $\sigma_2,...,\sigma_p$ surfaces that

$$\varphi_2{}^n(y^{i_2}) = H_1(y^{i_2},\varphi_2{}^{i_1}(y^{i_2}),\varphi_2{}^{i_3}(y^{i_2}),...,\varphi_2{}^{i'_p}(y^{i_2})), \quad i'_p = s_1 + ... + s_{p-1,} + 1,..n-1.$$

In particular, equalities (12) will be executed and at the following private values

$$z^{i_1} = z^{i_1}(y^{j_1},\varphi_1{}^{j_2}(y^{j_1}),...,\varphi_1{}^{j_{p-1}}(y^{j_1}),\varphi_1{}^{j_p}(y^{j_1}) = z^{i_1}(y^{j_1},\varphi 1^{s_1+1}(y^{j_1}),\varphi_1{}^n(y^{j_1})) = y^{i_1}.$$

Solutions of the equations entering into other lines of system (5) can be similarly found. Thus it is necessary to take other hyper surfaces $H_2,...,H_p$.

Collecting all found solutions, we will receive transformation (10) (or that the same, (9)), bringing system (1) to the split look (3).

The special place occupies a case $n = 2$. In this case we have splitting on two equations. σ_1, σ_2 are curves on the planes passing through the beginning of coordinates. Hyper surfaces H_1, H_2 coincide according to curves σ_2 and σ_1. The system's (5) solution – is unique.

References

1. Kovantsov, A. (2000). Integration of systems of the nonlinear differential equations. *Computer Modeling and New Technologies,* 4(1)*,* 50–52.

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