*Computer Modelling and New Technologies, 2012, vol. 16, no. 2, 50–59*
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# CREATION OF GRAPHS OF FUNCTIONS WITH USE OF THEOREMS OF ELEMENTARY GEOMETRY

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The solution of tasks with use of compasses and a ruler is one of the most ancient ways of the solution of geometrical tasks. ”Great problems of an antiquity” belong to such tasks. Not all tasks can be solved with use of the set tools. Often at mathematics lessons at school pupils ask to the teacher a question: why this or that theorem is necessary and where it can be used? Not always the teacher finds the convincing answer. However many theorems can be applied not only in a school course of mathematics.

If there is given a function $f(x)$ which is defined graphically then it is possible to draw a graph of a function $\frac{1}{f^2(x)}$ and a graph of a function $\frac{1}{\sqrt{f(x)}}$ in a coordinate system where the length of one unit is given. This can be proven using a theorem typically used in school – altitude-catheter projections theorem.

Methods of using right triangles to draw graphs of the functions $\frac{1}{\sqrt{f(x)}}$, $\frac{1}{f^2(x)}$, $\sqrt{f(x)}$, $f^2(x)$ will be explained.

**Keywords:** rectangular triangle, similarity of figures, average proportional

## 1. Introduction

Among theorems of school geometry one of the main places is occupied by Pythagoras theorem and the conclusions following from it. Signs of similarity of rectangular triangles and possibility of finding of elements of these triangles are such. So, if from top of a right angle in a rectangular triangle on a hypotenuse the perpendicular is lowered, the length of this perpendicular is an average geometrical between hypotenuse pieces into which it is divided. The theorems connected with circles are proved also. So, if from a point out of a circle to a circle are carried out a tangent and a secant, the square of a tangent is equal to work of a secant on her external part.

## 2. Creation of Graphics of Inverse Proportional Relationship and Averages Proportional

Let's address to one of school theorems of the geometry relating to a plane. Let the rectangular triangle $ACB$ from which top $C$ on a hypotenuse the height is carried out is $h$ set (Figure 2.1).

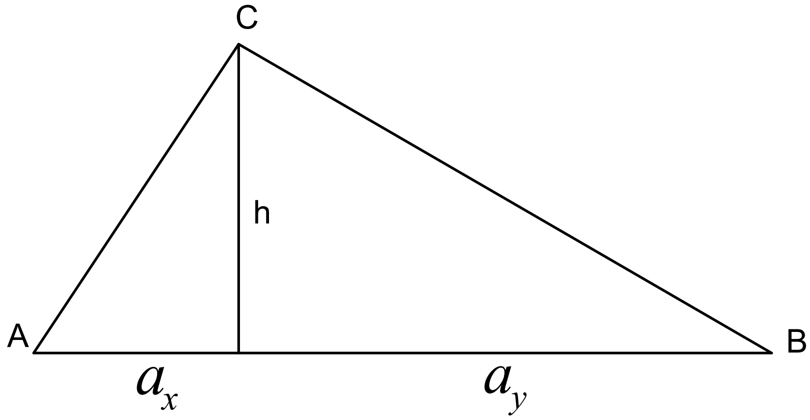


*Figure 2.1*

The hypotenuse is divided into $a_x$, $a_y$ pieces. It is $h^2 = a_x a_y$. Let the height $h$ is 1. Then $a_x a_y = 1 \Rightarrow a_y = \frac{1}{a_x}$. So, if in a rectangular triangle the height is 1, Npieces of a hypotenuse are inversely proportional. Let's apply this theorem to creation of the graphic of function $y = \frac{1}{f(x)}$ if the function $y = f(x)$ graphic is set, a unity and compasses and a ruler are used only (Figure 2.2).

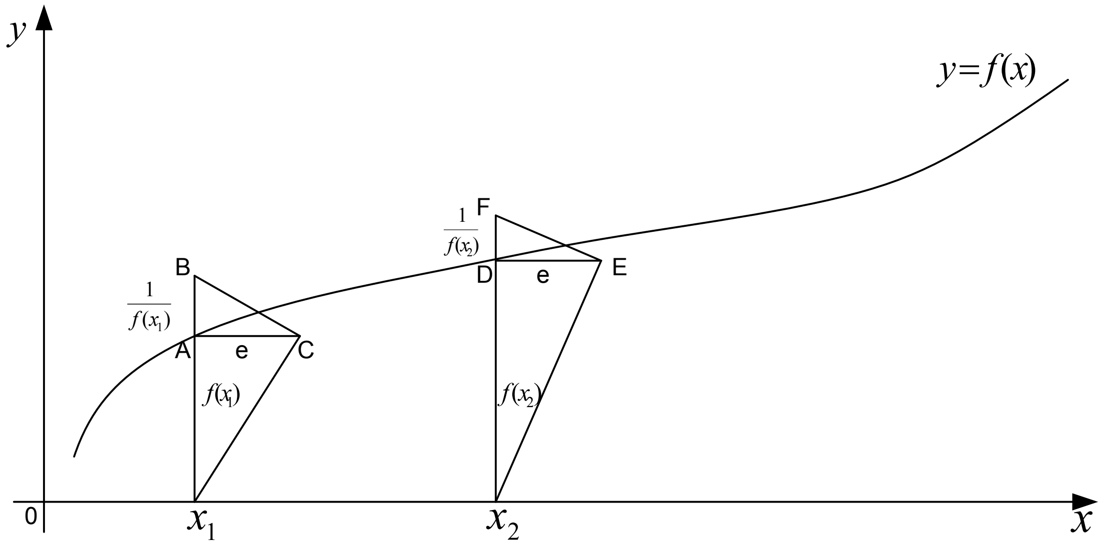


*Figure 2.2*

In each point of the set graphic of function $y = f(x)$ In each point of the set graphics of function $y = f(x)$ design rectangular triangles $x_1CB, x_2EF,...$, in which heights $AC = 1, DE = 1,...$ Then $AB, DE,...$ pieces are automatically equal to values of required function. Them postpone from points $x_1, x_2,...$, and then connect.

Let's consider one more school theorem (Figure 2.3).

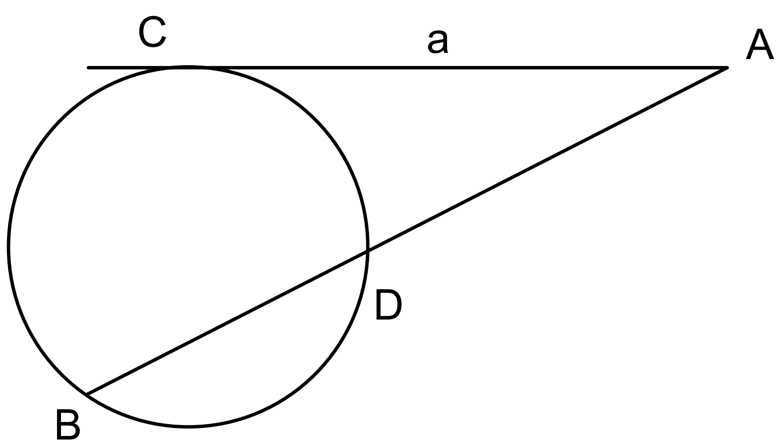


*Figure 2.3*

Let from A point out of a circle to this circle are carried out a tangent of $AC$ and a secant $AB$. Then the square of a tangent of $AC$ is equal to work of a secant of $AB$ on its external part of $AD$. If it will appear that an external part of $AD = 1$, then $AC^2 = AB$ and $AC = \sqrt{AB}$. We use this theorem for creation of the graphics of function $y = \sqrt{f(x)}$ if the function $y = f(x)$ graphic is known is unity piece compasses and a ruler is set (Figure 2.4).

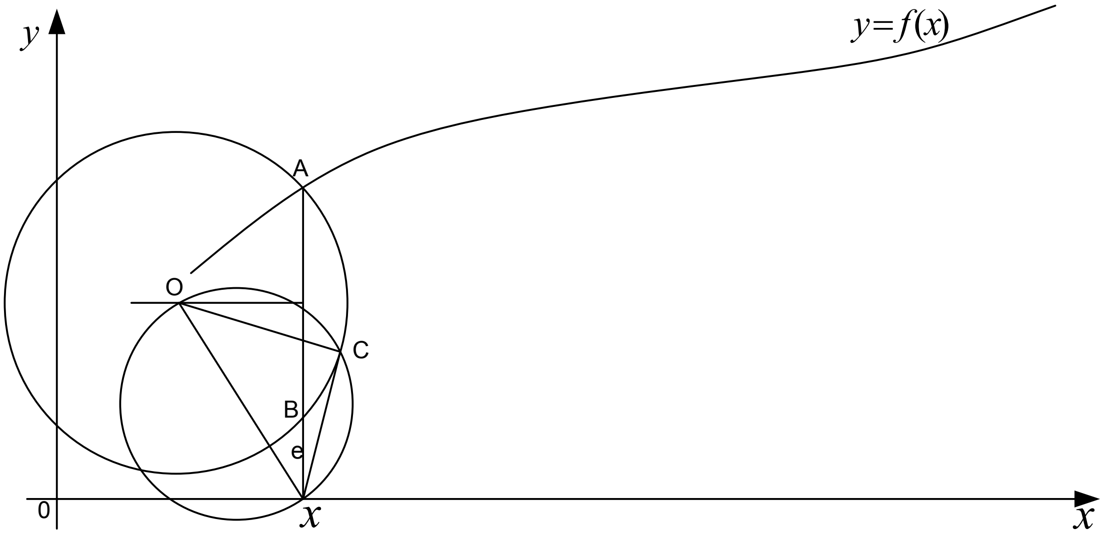


*Figure 2.4*

In a point $x$: $Ax = f(x)$. Let's find $\sqrt{f(x)}$ in this point. From a $x$ point we postpone $xB = 1$. $AB$ we halve. Let's out a middle perpendicular. From any point of this perpendicular O through points A and B we carry out a circle. Points A and $x$ it is connected. On a piece of $Ox$ we build a circle. The piece of $Ox$ is diameter, therefore $OCx$ triangle – rectangular. And therefore the piece $xC$ is a tangent to the first circle. The length of this piece and also is equal $\sqrt{f(x)}$ in a $x$ point. Such operation is made in all points of function $y = f(x)$.

## 3. Creation of Graphics of Various Functions with Use of Theorems of Elementary Geometry

Theorem 1: the altitude-catheter projections theorem – if an altitude is dropped from the vertex with the right angle to the hypotenuse then the product of the catheter projections on the hypotenuse is equal to the length of the altitude squared.

If there is a right triangle where $a,b$ – catheter, $c$ – hypotenuse, $a_c, b_c$ – catheter projections, $h$ – altitude, then:

$a_c \cdot b_c = h^2$. (***Theorem 1***)

If one of the catheter projections is equal to one unit then the other catheters projection is equal to the altitude squared:

$$a_c = h^2 \text{ or } b_c = h^2. \quad (1)$$

$$\sqrt{a_c} = h \text{ or } \sqrt{b_c} = h. \quad (2)$$

If the altitude is equal to one unit then product of catheter projections is equal to one unit or each catheters projection is equal to 1 divided by the other catheters projection:

$$a_c \cdot b_c = 1 \text{ or } a_c = \frac{1}{b_c} \text{ or } b_c = \frac{1}{a_c}. \qquad (3)$$

Let's say that there is given a graph of a function $f(x)$.
If a point of graph is chosen and perpendiculars to axis dropped, then perpendicular dropped to *x*-axis represents value of function $f(x)$.

The right triangle can be drawn using this perpendicular as the altitude or the catheters projection. Using perpendicular as the altitude $h$ and one of the catheter projections $a_c$ equal to one unit, the other catheters $b_c$ would be equal to the altitude squared. As the altitude in this case represents value of function then the catheters projection $b_c$ represents the value of function squared $f^2(x)$ (Figures 3.1 and 3.2).

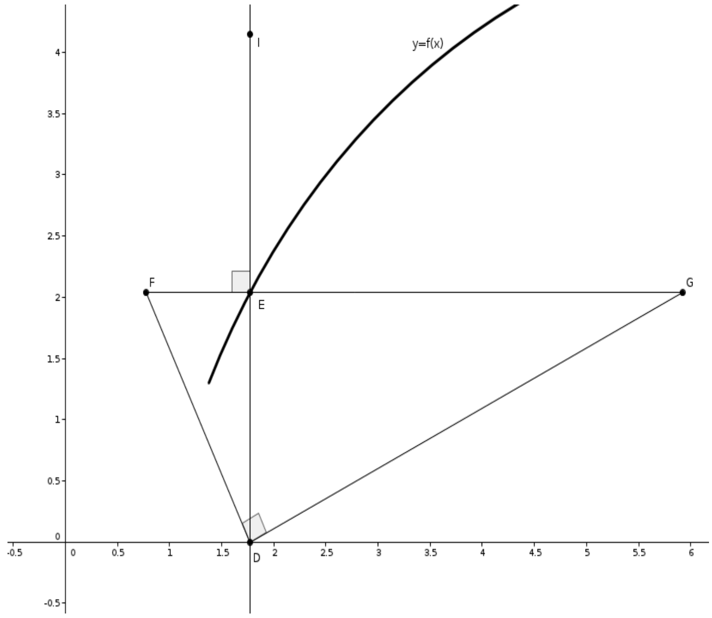


*Figure 3.1*

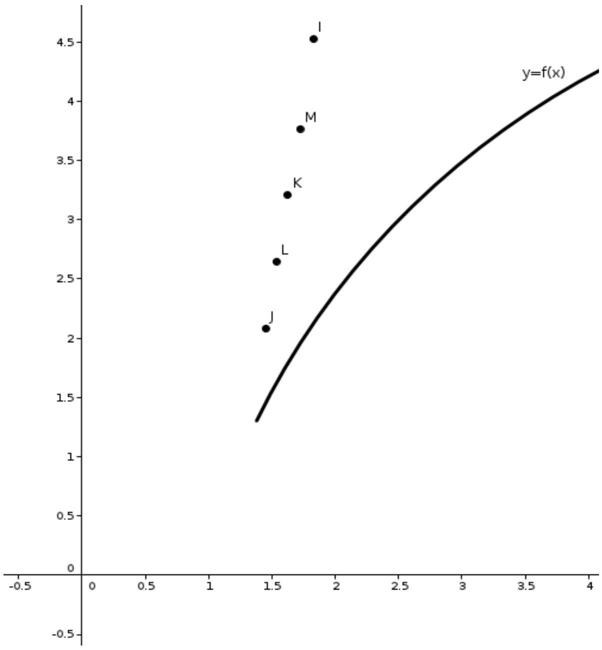


*Figure 3.2*

As we see in the *Graph 1*, there is the right triangle *DFG* and the altitude *DE* represents the value of the function – $f(x)$. Also, the segment *EF* is equal to one unit. Using *Theorem 1* and *Conclusion* (*1*) we get that the segment *EG* is equal to $f^2(x)$. The segment *EG* can be used to get point *I* so that *DI* = *EG*.

Drawing more right triangles at the other points of the function $f(x)$ we can get other points of the function $f^2(x)$ (Figure 3.2).

The right triangle can also be drawn using the perpendicular to the *x*-axis as a catheters projection. If the perpendicular is the catheters projection $a_c$ and the altitude $h$ is equal to one unit, the other catheters projection $b_c$ is equal to $\frac{1}{a_c}$. As the catheters projection $a_c$ represents the value of function $b_c$ represents the value of 1 divided by value of function $\frac{1}{f(x)}$ (Figures 3.3. and 3.4).

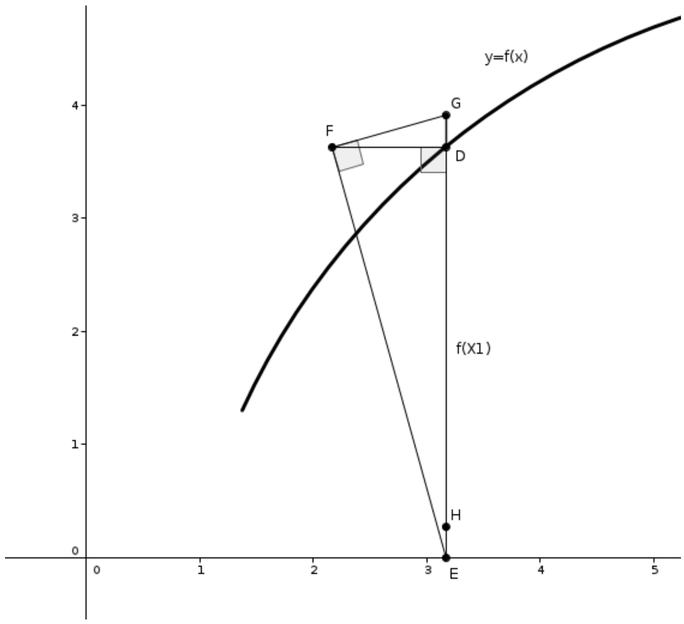


*Figure 3.3*

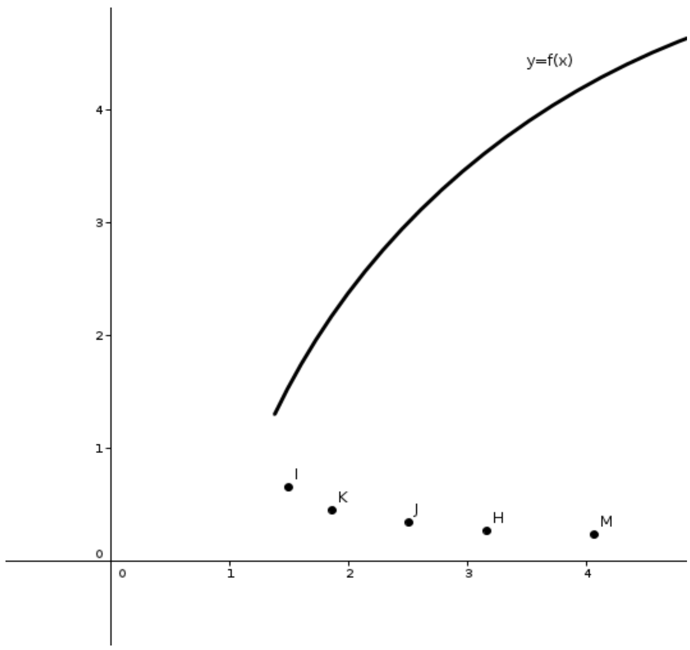


*Figure 3.4*

As we see on Figure 3.3. , there is right triangle $EFG$ and the altitude $DE$ represents the value of the function $-f(x)$. Also, the segment $DF$ is equal to one unit. Using *Theorem 1* and *Conclusion* (*3*) we get that the segment $GD$ is equal to $\frac{1}{f(x)}$. The segment $GD$ can be used to get point $H$ so that $EH = GD$.

Drawing more right triangles at the other points of the function $f(x)$ we can get other points of the function $\frac{1}{f(x)}$ (Figure 3.4).

If the perpendicular is the catheters projection $a_c$ and the other catheters projection $b_c$ is equal to one unit, the altitude $h$ is equal to the square root of value of catheters projection $a_c$ which represents the value of the function, it means, the altitude is equal to $\sqrt{f(x)}$.

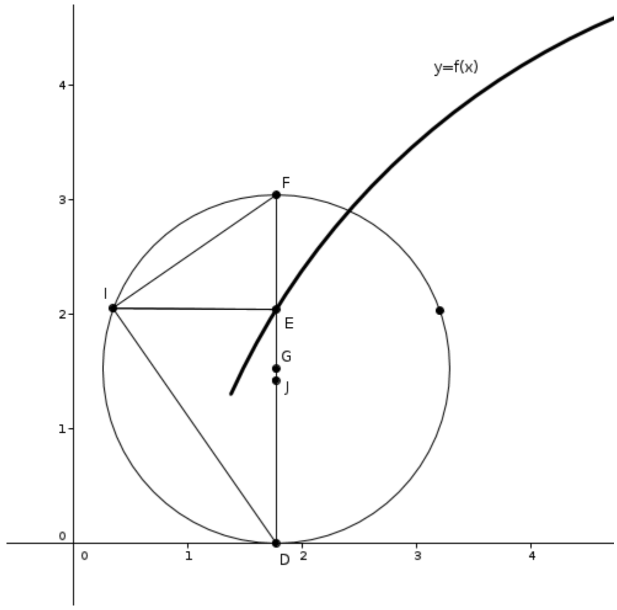


*Figure 3.5*

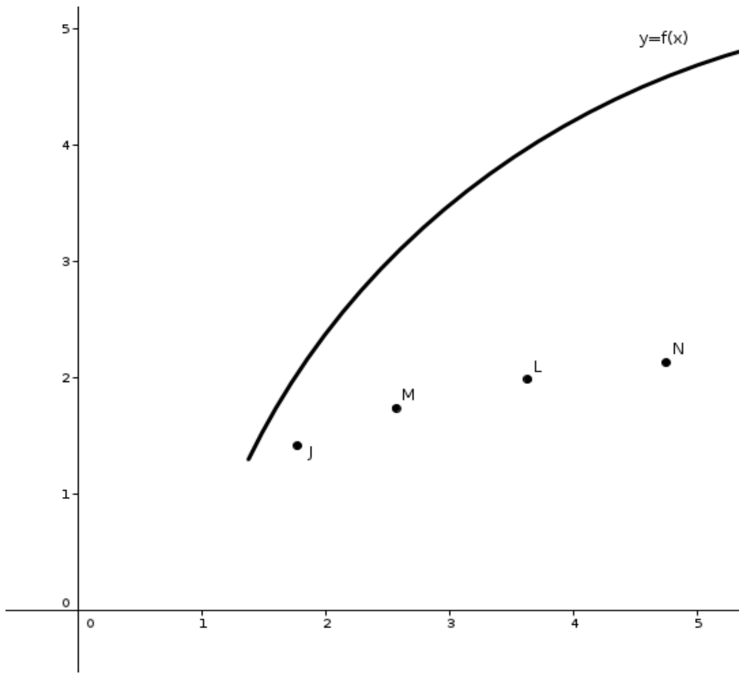


*Figure 3.6*

As we see in the *Graph 5*, there is the right triangle *DIF* and the catheters projection *DE* represents the value of the function – $f(x)$. Also, the segment *EF* is equal to one unit. Using *Theorem 1* and *Conclusion* (*2*) we get that the segment *EI* is equal to $\sqrt{f(x)}$. The segment *EI* can be used to get point *J* so that *EI* = *DJ*.

Drawing more right triangles at the other points of the function $f(x)$ we can get other points of the function $\sqrt{f(x)}$ (Figure 3.6).

This shows that it is possible to get a segment of line equal to value of function in power of $2$ or $-1$ or $\frac{1}{2}$ by drawing a right triangle.

If the segment of line equal to the value if function in power of $\frac{1}{2}$ is got, then this segment of line can be used to draw another right triangle and get another segment of line which value is equal to the value of previous segment of line in power of $-1$, in other words, equal to the value of function in power of $-\frac{1}{2}$.

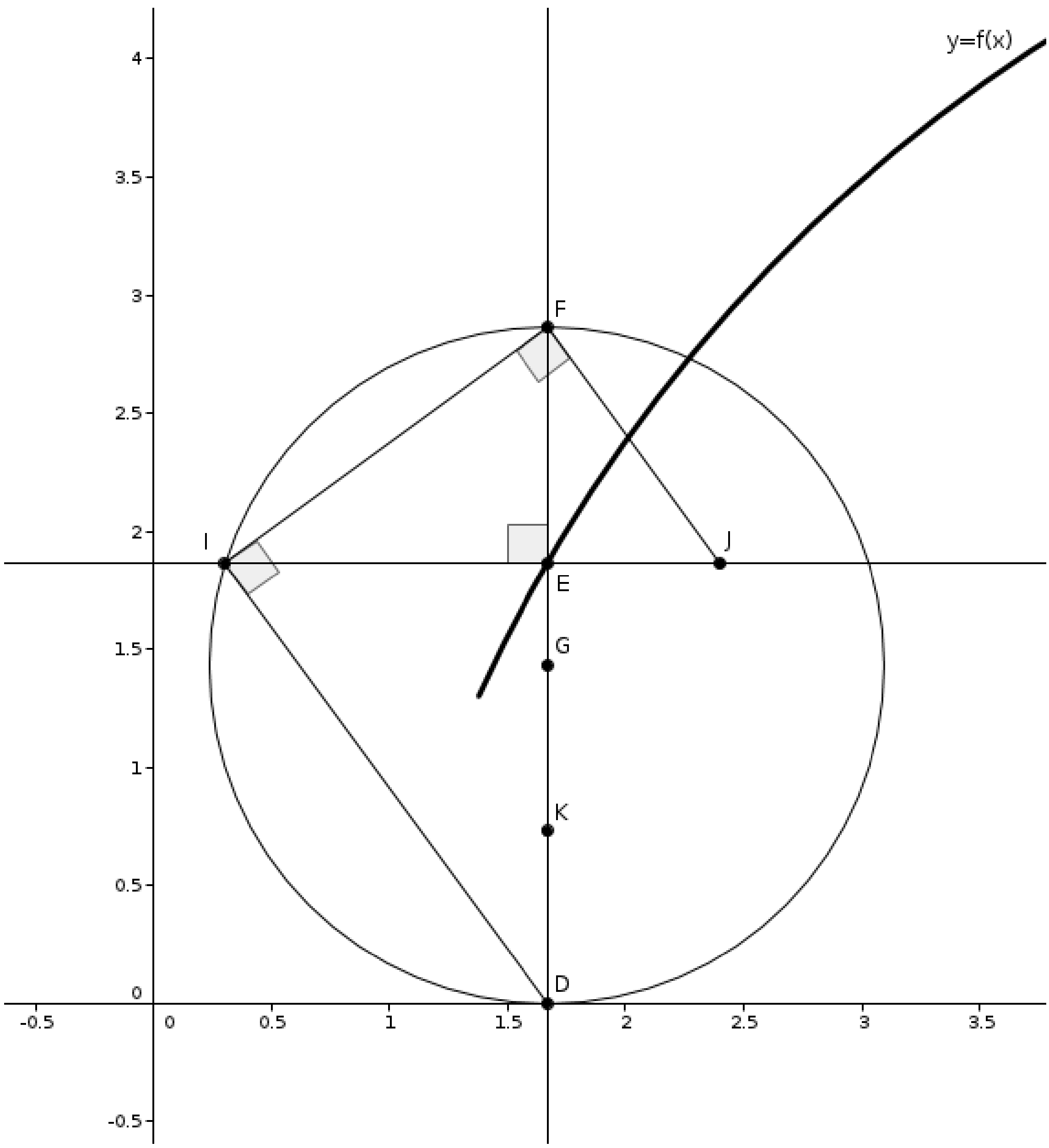


*Figure 3.7*

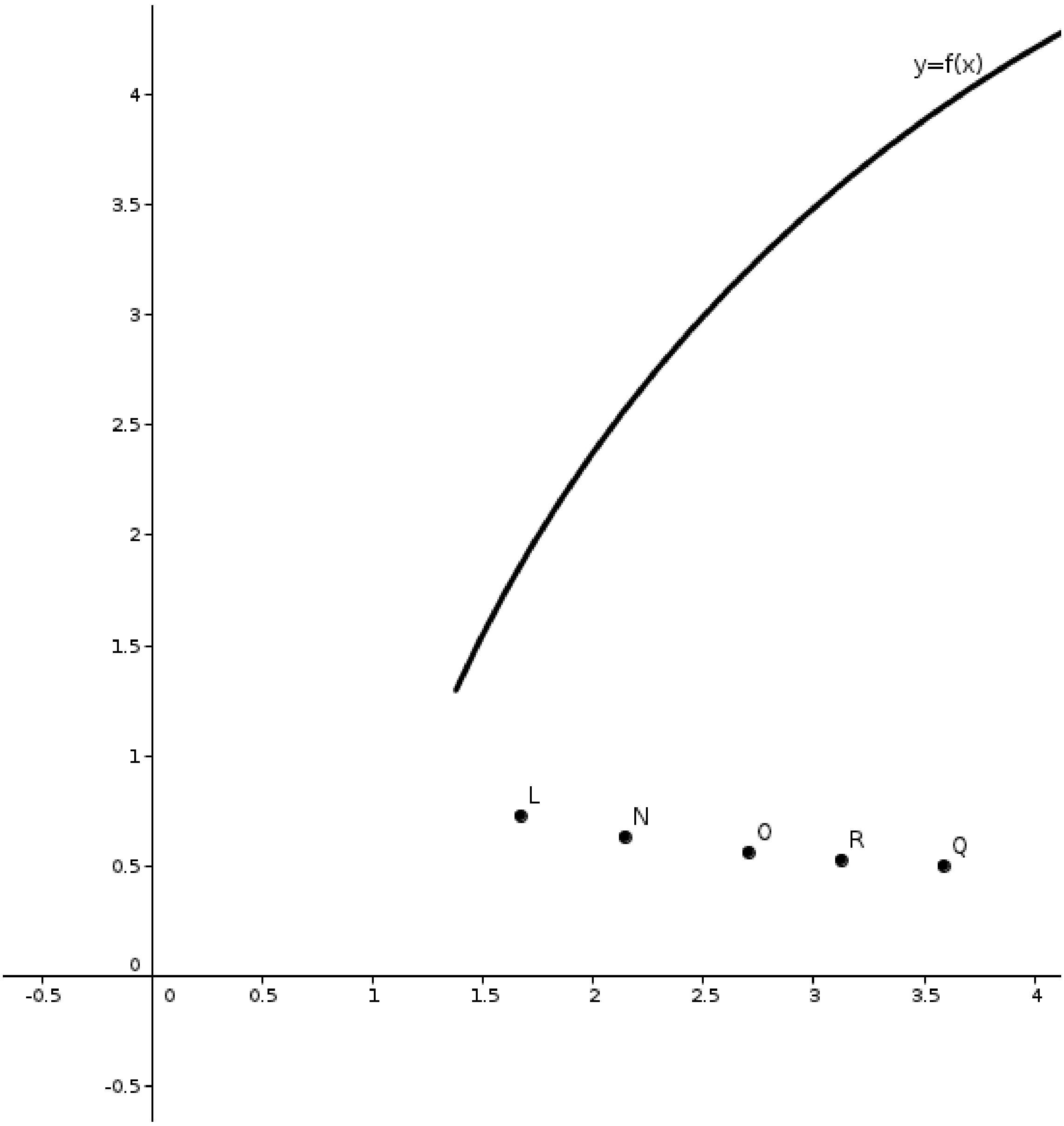


*Figure 3.8*

As we see on Figure 3.7, there is the right triangle *DIF* and the catheters projection *DE* represents the value of the function – $f(x)$. Also, the segment *EF* is equal to one unit. As proved, *EI* is equal to $\sqrt{f(x)}$. There is also the right triangle *IJF* and the catheters projection *EI* represents the value $\sqrt{f(x)}$. Using *Theorem 1* and *Conclusion* (*3*) we get that the segment *EJ* is equal to $\frac{1}{\sqrt{f(x)}}$. The segment *EJ* can be used to get point *K* so that *DK* = *EJ*.

Repeating this at the other points of the function $f(x)$ we can get other points of the function $\frac{1}{\sqrt{f(x)}}$ (Figure 3.8).

If the segment of line equal to the value of function in power of 2 is got, then this segment of line can be used to draw another right triangle and get another segment of line which value is equal to the value of previous segment of line in power of –1, in other words, equal to the value of function in power of –2.

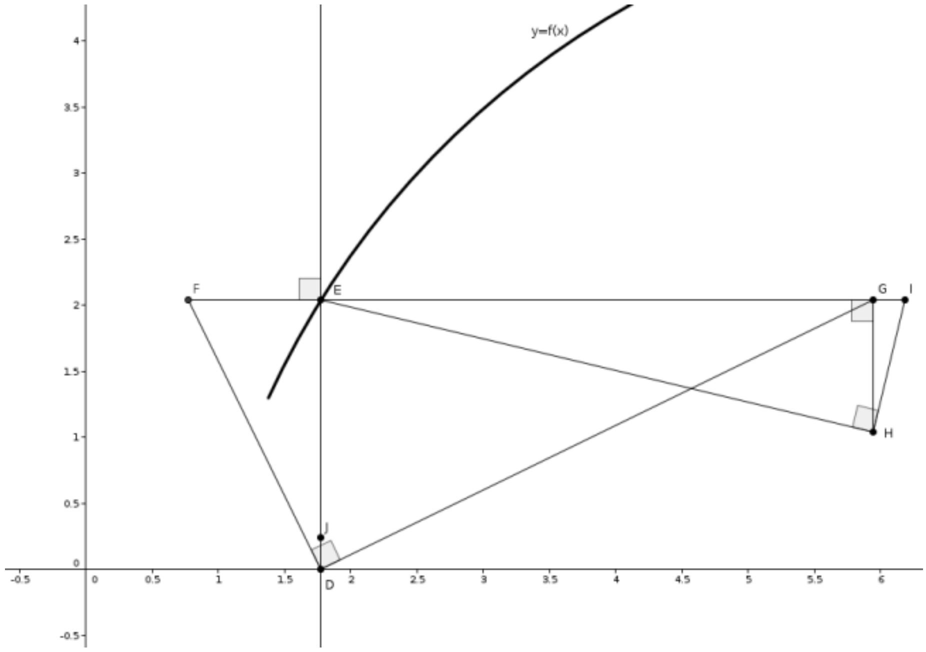


*Figure 3.9*

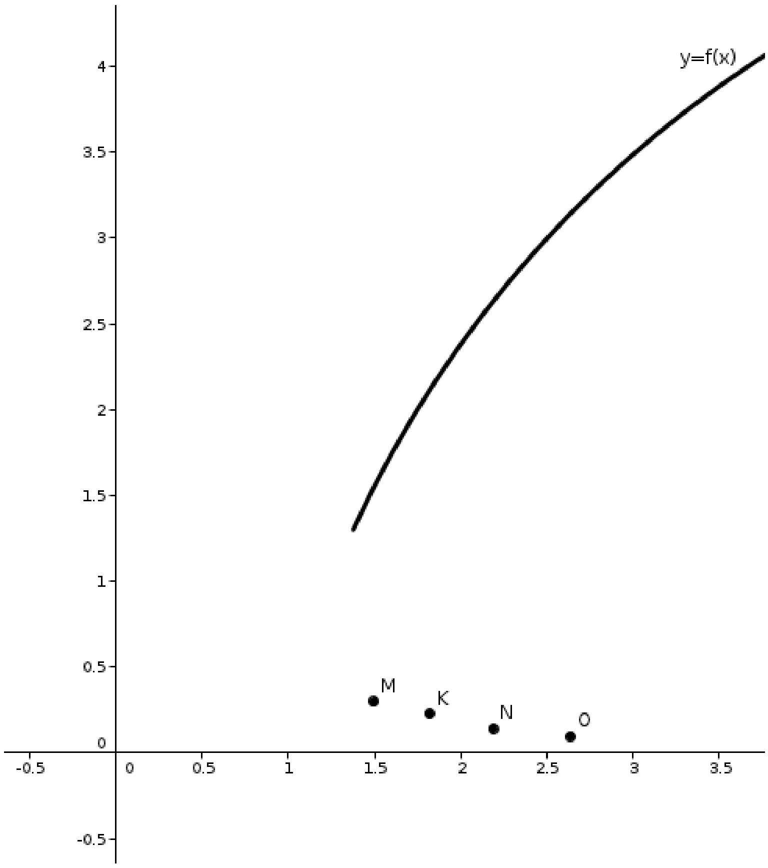


*Figure 3.10*

As we see on Figure 3.9, there is the right triangle *DFG* and the altitude *DE* represents the value of the function $-f(x)$. Also, the segment *EF* is equal to one unit. As proved, *EG* is equal to $f^2(x)$. There is also the right triangle *EHI* and the catheters projection *EG* represents the value $f^2(x)$. Using *Theorem 1* and *Conclusion* (*3*) we get that the segment *GI* is equal to $\frac{1}{f^2(x)}$. The segment *GI* can be used to get point *J* so that *DJ* = *GI*.

Repeating this at the other points of the function $f(x)$ we can get other points of the function $\frac{1}{f^2(x)}$ (Figure 3.10).

## 4. Conclusions

As shown previously, it is possible to draw function $f(x)$ in the power $-\frac{1}{2}$, –1, –2, $\frac{1}{2}$ or 2 using only a ruler and a compass if the function $f(x)$ is given graphically. Theoretically, it is possible draw functions in powers of:

$$-\frac{1}{2}, -\frac{1}{4}, -\frac{1}{8}, -\frac{1}{16} \dots - \frac{1}{2^n}\ , \ n = 1,2,3 \dots$$

$$-2, -4, -8, -16 \dots - 2^n\ , \quad n = 1,2,3 \dots$$

$$2, 4, 8, 16 \dots\ 2^n\ , \quad n = 1,2,3 \dots$$

It can be done using the idea that by drawing one right triangle it is possible to power the function in the power $-1$, $\frac{1}{2}$ or 2. For example function $f^{-\frac{1}{8}}(x)$ can be drawn in 4 steps:

1. Draw the right triangle to get function $f^{\frac{1}{2}}(x)$.
2. Use the function $f^{\frac{1}{2}}(x)$ and draw the right triangle to get function $f^{\frac{1}{4}}(x)$.
3. Use the function $f^{\frac{1}{4}}(x)$ and draw the right triangle to get function $f^{\frac{1}{8}}(x)$.
4. Use the function $f^{\frac{1}{8}}(x)$ and draw the right triangle to get function $f^{-\frac{1}{8}}(x)$.

## References

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*Received on the 23rd of March 2012*