Proceedings of the 12th International Conference "Reliability and Statistics in Transportation and Communication" (RelStat'12), 17–20 October 2012, Riga, Latvia, p. 78-84. ISBN 978-9984-818-49-8
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INSPECTION PROGRAM FOR THE CASE OF TWO RANDOM PARAMETERS OF FATIGUE CRACK

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Development of inspection program for limitation of fatigue failure probability (FFP) of fatigue-prone aircraft (AC) and fatigue failure rate (FFR) of airline (AL) is a problem of high priority. This problem is aggravated by stringent limitation of number of full-scale fatigue tests (only one or two airframes are tested). In previous authors publications [1-2] minimax solution of the problem was offered in assumption that in exponential approximation of fatigue crack size growth function, $a(t)=a_0\exp(Qt)$, only one parameter of fatigue crack trajectory (PFCT) is unknown. Here we consider the case of two unknown parameters.

Keywords: Minimax, inspection program, Markov chains, reliability

1. Introduction

Limitation of fatigue failure probability (FFP) of fatigue-prone aircraft (AC) and fatigue failure rate (FFR) of airline (AL) is a problem of high priority. A lot of papers and books are devoted to this problem. In [1] the Markov Chains (MC) and Semi-Markov process with rewards (SMPW) theories are offered for its solution using exponential approximation of fatigue crack size growth function, $a(t)=\alpha\exp(Qt)$, where α, Q are parameters of fatigue crack trajectory (PFCT). The value α is called equivalent initial flow size (EIFS). (Note, it is not real initial flow size; it is only a parameter of exponential approximation of fatigue crack trajectory!) The value Q defines the speed of fatigue crack size growth in logarithm scale: $\log(a(t))=\log\alpha+Qt$. PFCT are random variables. It is supposed that the cumulative distribution function (cdf) of the vector (α , Q) is known but some parameter of this cdf , θ, is not known. Estimation of θ and the choice of inspection program under condition of limitation FFP up to specified life (AC retirement live), t_{SL}, or limitation of FFR of AL can be made using minimax processing of results of observation of some random fatigue cracks during AC type full-scale fatigue approval test. Specific feature of approval test is a decision to redesign the new AC type if some reliability requirements are not met. In [1] it has been assumed that α is some constant. In this paper this assumption is eliminated.

2. Minimax Choice of Inspection Program

Despite of all the simplicity, the equation $a(t)=\alpha\exp(Qt)$ gives us rather comprehensible result in the interval (t_d,t_c), where t_d is a time when the crack becomes detectable ($a(t_d)=a_d$) and t_c is a time when the crack reaches its critical size ($a(t_c)=a_c$) and fatigue failure takes place. Corresponding random variables are defined by equations: $T_d=(\log a_d-\log\alpha)/Q=C_d/Q$, $T_c=(\log a_c-\log\alpha)/Q=C_c/Q$.

Let us denote $X = \log Q$ and $Y = \log C_c$, where $C_c = \log a_c - \log \alpha$. From the analysis of the fatigue test data it can be assumed, that the $\log T_c = \log C_c - \log Q$ is distributed normally. It comes from the additive property of the normal distribution that it can take place if either both $\log C_c$ and $\log Q$ are normally distributed or if one of these components is normally distributed while another one is constant. Contrary to [1] in this paper we consider the first case: vector $(X, Y) = (\log(Q), \log(C_c))$ has two dimensional normal distribution with vector-parameter $\theta = (\mu_X, \mu_Y, \sigma_X, \sigma_Y, r)$. It is worth to note, that for the case when a_c and a_d are constants then cdf of C_d is completely defined by the distribution of C_c because $C_d = C_c - \delta$, where $\delta = \log(a_c / a_d)$.

For known θ there are two decisions d_0 and d_1: the aircraft is good enough and the operation of this aircraft type can be allowed (d_0) and redesign of aircraft should be made (d_1). In the case of first decision the vector $\vec{t} = (t_1, ..., t_n)$, where t_i is the time moment of *i*-th inspection, should be defined also.

If θ is known the different rules can be offered for the choice of structure of vector $\vec{t}$: 1) every interval between inspections is equal to $t_{SL} / (n+1)$, 2) probability of failure in every interval is equal to $P(T_C < t_{SL}) / (n+1)$ … In this paper we suppose that (just as in both mentioned examples) the vector $\vec{t}$ is defined by the fixed t_{SL} and choice of n.

For substantiation of the choice of inspection number we should know FFP of AC and FFR and gain (GL) of AL as functions of n. For this purpose the process of operation of AC can be considered as absorbing MC with $(n+4)$ states. The states $E_1, E_2, ..., E_{n+1}$ correspond to AC operation in time intervals $[t_0, t_1), [t_1, t_2), ..., [t_n, t_{SL})$, States E_{n+2}, E_{n+3}, and E_{n+4} are absorbing states: AC is discarded from service when the SL is reached or fatigue failure (FF), or fatigue crack detection (CD) takes place.

Let in the transition probability matrix, P_{AC} for corresponding process of AC operation the probability of crack detection during the inspection number *i* be denoted as v_i; probability of failure in service time interval $t \in (t_{i-1}, t_i]$, as q_i and probability of successful transition to the next state as $u_i = 1 - v_i - q_i$. In our model we also assume that an aircraft is discarded from service at t_{SL} even if there are no cracks discovered by inspection at the time moment t_{SL}. This inspection at the end of (n+1)-th interval (in state E_{n+1}) does not change the reliability but it is made in order to know the state of aircraft (whether there is a fatigue crack or there is no fatigue crack). It can be shown that

$$u_i = P(T_d > t_i \mid T_d > t_{i-1}) = P(Q < C_d / t_i) / P(Q < C_d / t_{i-1}) = a_i / a_{i-1},$$

$$q_i = P(t_{i-1} < T_d < T_c < t_i \mid Td > t_{i-1}) =$$

$$= \begin{cases} 0, & if \quad t_{i-1} C_c / C_d > t_i, \\ b_i / a_{i-1}, & if \quad t_{i-1} C_c / C_d \le t_i, \end{cases} \qquad i = 1, ..., n+1,$$

where

$$a_i = P(Q < C_d / t_i) = \int_{\ln \delta}^{+\infty} \left(g_{ai}(y) \right) d\Phi\left(\frac{y - \mu_Y}{\sigma_Y} \right),$$

$$g_{ai} = P(Q < C_d / t_i) = \Phi\left(\frac{\left(\log\left(e^y - \delta \right) - \log t_i \right) - \mu_{X/y}}{\sigma_{X/y}} \right),$$

$$b_i = P(C_c / t_i < Q < C_d / t_{i-1}),$$

$$= P\left(\log C_c - \log t_i \le \log Q < \log(C_c - \delta) - \log t_{i-1}\right),$$

$$= \int_{\ln\delta}^{+\infty} \left(g_{bi}(y)\right) d\Phi\left(\frac{y-\mu_Y}{\sigma_Y}\right),$$

$$g_{bi}(y) = \max\begin{pmatrix} 0, \Phi\left(\dfrac{\left(\log\left(e^y - \delta\right) - \log t_{i-1}\right) - \mu_{X/y}}{\sigma_{X/y}}\right) \\ -\Phi\left(\dfrac{\left(y - \log t_i\right) - \mu_{X/y}}{\sigma_{X/y}}\right) \end{pmatrix},$$

$$\mu_{X/y} = \mu_X + r\frac{\sigma_X}{\sigma_Y}\left(y - \mu_Y\right),$$

$$\sigma_{X/y} = \sigma_X\sqrt{1-r^2}.$$

These probabilities can be calculated using Monte Carlo method also. The following equations can be used for modelling r.v. $Y = \log C_c \sim N(\mu_Y, \sigma_Y^2)$ and $X = \log Q \sim N(\mu_X, \sigma_X^2)$ with some coefficient of correlation r:

$$Y = \eta_1\sigma_Y + \mu_Y,\ X = \eta_1\sigma_X r + \eta_2\sigma_X\sqrt{1-r^2} + \mu_X,$$

where r.v. η_1 and η_2 have standard normal distribution.

Recall that in matrix, P_{AC} there are three units in three last lines in matrix diagonal because states E_{n+2}, E_{n+3}, and E_{n+4} are absorbing states: AC is discarded from service when the SL is reached or fatigue failure (FF), or fatigue crack detection (CD) take place.

In corresponding matrix for operation process of AL the states E_{n+2}, E_{n+3} and E_{n+4} are not absorbing but correspond to return of MC to state E_1 (AL operation returns to first interval). The other lines of P_{AC} and P_{AL} are the same.

For SMPW version of problem using P_{AL} we can get the airline gain $g(n) = \sum_{i=1}^{n+4} \pi_i g_i(n)$, where $\pi = (\pi_1, ..., \pi_{n+4})$ is the vector of stationary probabilities, which is defined by the equation system $\pi P = \pi, \quad \sum_{i=1}^{n+4} \pi_i = 1$; AL operation rewards are defined in following way

$$g_i(n) = \begin{cases} a_i \cdot u_i + b_i \cdot q_i + c_i \cdot v_i, & i = 1, ... n+1, \\ d_i, & i = n+2, ..., n+4, \end{cases}$$

where a_i is the reward related to successful transition from one operation interval to the following one and the cost of one inspection; b_i, c_i and d_i are related with transition to states E_{n+3} (FF), E_{n+4} (CD) and E_1. Let us note that if $a = b = c = 1$, $d = 0$ and time transition to state E_1 are equal to zero, then $\pi_{tj} = \pi_j g_j(n) / g(n)$ defines the part of time which SMP spends in state E_j, $j = 1,...,n+1$; $L_j = g(n)/\pi_j$ defines the mean return time for state E_j. Specifically, L_{n+3} is the mean time between FF; so $\lambda_F = 1/L_{n+3}$ is the FFR. It worth to mention also, that the same value can be calculated and in another way. This value is equal to the ratio of aircraft failure probability, p_F to the mean life of new aircraft, $L_1 = g(n)/\pi_1$ (the mean time of renewal of AC (renewal operation of AL in the first interval)). There are two very similar versions of reliability requirement: requirement_A corresponds to limitation of FFR of AL, requirement _B corresponds to limitation of FFP of AC. Solution of one of the problem version gives unambiguous solution of the other one. First, we consider requirement_A. If θ is known we calculate the gain as function of n, $g(n,\theta)$, and choose the number n_g corresponding to the maximum of gain: $n_g(\theta) = \arg\max_n g(n,\theta)$. Then we calculate FFR as function of n, $\lambda_F(n,\theta)$, and choose n_λ in such a way that for any $n \geq n_\lambda$ the function $\lambda_F(n,\theta)$ will be equal or less than some value λ_{FD} (the designed FFR): $n_\lambda(\lambda_{FD},\theta) = \min\{n : \lambda_F(n,\theta) \leq \lambda_{FD}, \text{ for all } n \geq n_\lambda(\lambda_{FD},\theta)\}$. And finally we choose $n = n_{g\lambda}(\lambda_{FD},\theta) = \max(n_g, n_\lambda)$.

But in fact we do not know θ and we can only get some estimate of this parameter, $\hat{\theta}$, using approval test results. Then, first of all, we should define some part of parameter space Θ_0 in such a way that if $\hat{\theta} \notin \Theta_0$ then redesign of AC should be made. If instead of $n_{g\lambda}(\lambda_{FD},\theta)$ we use $n_{g\lambda}(\lambda_{FD},\hat{\theta})$ then real intensity FFR will be a function of random variable, $\lambda_F(n_{g\lambda}(\lambda_{FD},\hat{\theta}),\theta)$. Let us define $\lambda_F(\hat{\theta},\lambda_{FD},\Theta_0) = \lambda_F(n_{g\lambda}(\lambda_{FD},\hat{\theta}),\theta)$ if $\hat{\theta} \in \Theta_0$ and $\lambda_F(\hat{\theta},\lambda_{FD},\Theta_0) = 0$ if $\hat{\theta} \notin \Theta_0$. Corresponding expected value of FFR $w(\theta,\lambda_{FD},\Theta_0) = E\{\lambda_F(\hat{\theta},\lambda_{FD},\Theta_0)\}$ as function of θ has maximum because for "bad $\hat{\theta} \notin \Theta_0$" we make redesign of airframe but for "very good $\hat{\theta}$" we do not need any inspection. Let us denote by λ^* required FFR, which is defined by specific aviation regulations. By $\lambda^*_{FD}(\Theta_0)$ let us denote the solution of equation $w(\theta,\lambda_{FD},\Theta_0) = \lambda^*$ if the solution of this equation exists for specific Θ_0. If after approval test we see that $\hat{\theta} \in \Theta_0$ then required inspection number $n = n_{g\lambda}(\lambda^*_{FD},\hat{\theta})$.

Now we consider the requirement_B. In this case choice of vector $\vec{t}$ is the choice of p-set function, which in considered case is defined in the following way [1].

Let Z and X be random vectors (r.v.) and suppose that the class is known { P_θ, $\theta \in \Theta$ } to which the probability distribution of the random vector *W=(Z,X)* is assumed to belong. Of the parameter θ, which labels the distribution, it is presumably known only that it lies in a certain set Θ, the parameter space. Let $S_Z(x) = \bigcup_{i=1}^{r} S_{Z,i}(x)$ denote some set of disjoint sets of *z* values as function of *x*. If

$$\sup_\theta \sum_{i=1}^{r} P(Z \in S_{Z,i}(X)) = p$$

then statistical decision function $S_Z(x)$ is p-set function for r.v. *Z* on the base of a vector *x*.

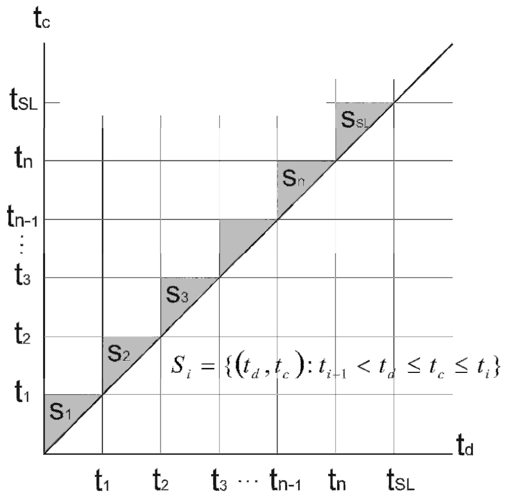

Figure 1. Example of set of sets $S_{Z,i}$, $i = 1,...,n+1$

In our case Z is a vector (T_d, T_c) related with aircraft in operation, X is the estimate of parameter θ, i.e. $x = \hat{\theta}$. This estimate can be obtained using result of fatigue test of airframe of aircraft of the same type in laboratory (i.e. observations of fatigue crack: pairs {(time, fatigue crack size)i, $i = 1,...,k$ }= $\{(tt_i, a_i), i = 1,...,k\}$, where k is number of fatigue crack observations.

Every set $S_{Z,i}$ is some set of points in plain, {(t_d, t_c)}, defined in such a way: $S_{Z,i} = \{(t_d, t_c): t_{i-1} < t_d, t_c \le t_i\}$, $i = 1,...,n+1$, $t_{n+1} = t_{SL}$. Vector of time points of inspection $\vec{t}$ is a function of observation of estimate of parameter θ, i.e. function of $\hat{\theta}$.

Let $E(T_A)$ be the mean time to absorption (corresponding to the matrix P_{AC}) of aircraft, which begins the service in first interval and the failure probability p_f corresponds to the function $\vec{t}(x)$. Then the FFR and FFP have unambiguous connection: $\lambda = p_f / E(T_A)$.

3. Numerical Example. Comparison of Cases with One and Two Dimensional Unknown Parameters

Suppose we have the following estimate of parameter $\theta = (\mu_X, \mu_Y, \sigma_X, \sigma_Y, r)$: $\hat{\theta}$ =(-8.58688044, 1.9424608, 0.346, 0.0778895, 0.796437) (In [1] the same parameter is used but here instead of $\sigma = 0.155128668$ for more clear demonstration we used $\sigma = 0.346$). It is supposed that all inspection intervals are equal, a_c = 237.8 mm and a_d = 20 mm.

We use the following definitions of component of AL income: for all $i = 1,...,n+1$ $a_i = a(n) = a_0(n) + d_{insp} t_{SL}$, where $a_0(n) = a_{01} t_{SL} / (n+1)$, – is the reward, related to successful transition from one operation interval to the following one, a_{01} defines the reward of operation in one time unit (one hour or one flight); $d_{insp} t_{SL}$ is the cost of one inspection (negative value) which is supposed to be proportional to t_{SL} ; $b_i = b_{01} t_{SL}$ is related to FF (negative value), $c_i = c_{01} a_0(n)$ is the reward related to transitions from any state $E_1,...,E_{n+1}$ to the state E_{n+4} (it is supposed to be proportional to a_0 because it is a part of a_0); $d_i = d_{01} t_{SL}$ is negative reward, the absolute value of which is the cost of new aircraft acquisition after events SL, FF or CD and transition to E_1 takes place. In numerical example we have used the following values: b_{01} = -0.3; d_{insp} = -0.05; a_{01} = 1; c_{01} = 0.1; d_{01} = -0.3. Calculation of $w(\theta, \lambda_{FD}, \Theta_0) = E\{\lambda_F(\hat{\theta}, \lambda_{FD}, \Theta_0)\}$ was made for $(7.2029 \le \mu_X \le 9.9709)$, $(1.3972 \le \mu_Y \le 2.4877)$ for

$t_{SL} = 40000$. The set Θ_0 correspond to the decision to make redesign if estimate of $E(T_c) < 0.3t_{SL}$. Calculation was made for different λ_{FD} assuming that the vector (σ_X, σ_Y, r) is known and it is equal to test estimate (0.155128668, 0.0778895, and 0.796437). It was supposed that this vector is the same for different vectors (μ_X, μ_Y).

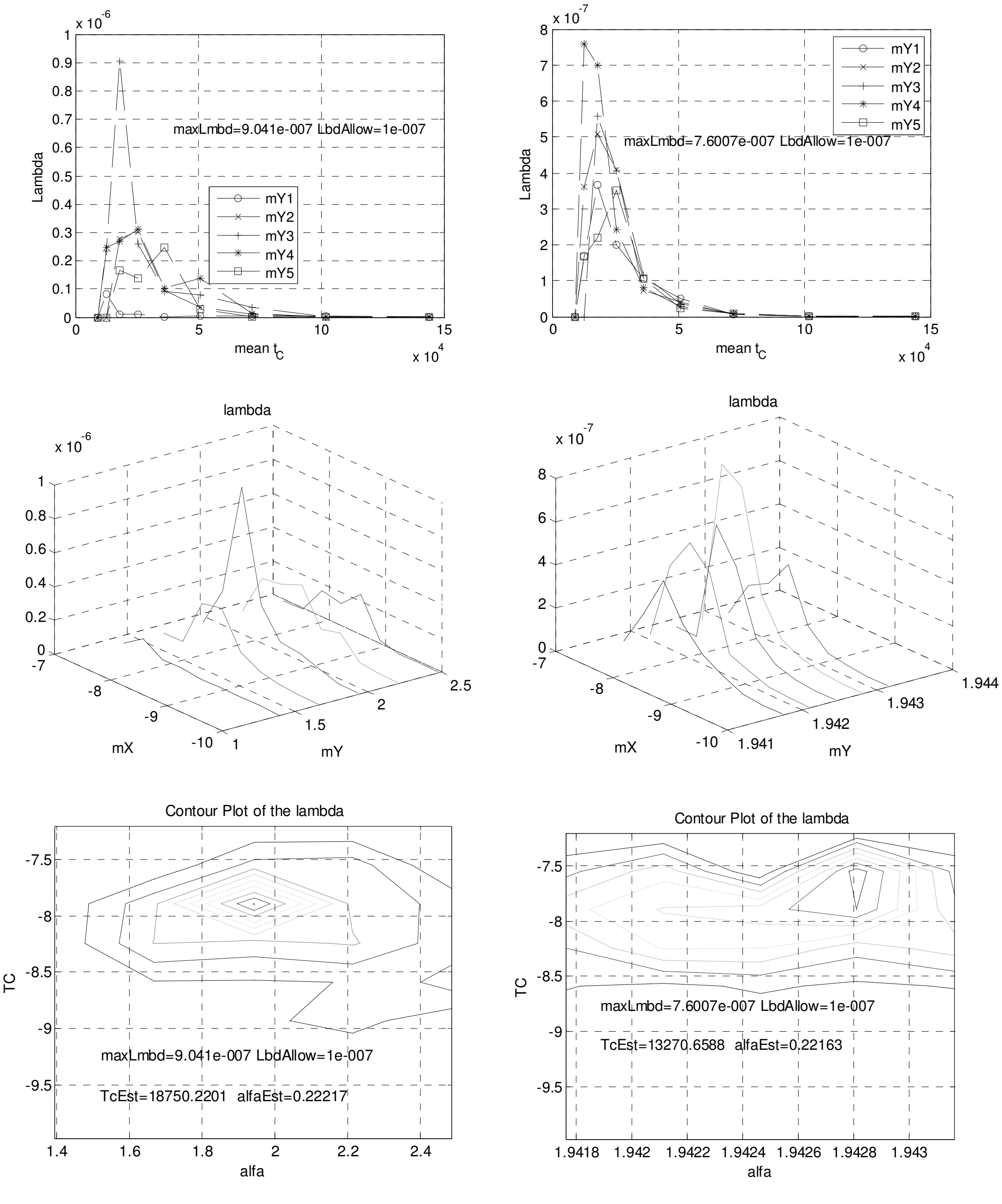

Figure 2. Values of $w(\theta, \lambda_{FD}, \Theta_0)$ as function of $E(T_c)$ for different μ_Y for (σ_X, σ_Y, r) = (0.155128668, 0.0778895, 0.796437) (on the left side) and 0.760*10-6 for (σ_X, σ_Y, r) =(0.346, 0.0001, 0) (on the left side).

On Figure 2 we see $w(\theta, \lambda_{FD}, \Theta_0)$ as function of equivalent mean value of T_C which was calculated as $\exp(\mu_Y - \mu_X)$ for five value of μ_Y $(1.2415 \le \mu_Y \le 2.6435)$ in vicinity of maximum value of $w(\theta, \lambda_{FD}, \Theta_0)$ which is equal to 0.9041*10-6 for (σ_X, σ_Y, r) = (0.155128668, 0.0778895, 0.796437) and 0.760*10-6 for (σ_X, σ_Y, r)=(0.346, 0.0001, 0) (see Table 1).

Table 1.

Lambda allowed	Sy= 0.0778895 C_{XY} =0.796	Sy= 0.0001 C_{XY} =0
0.1*10-6	0.9041*10-6	0.760*10-6

In this case the influence of scatter Y (and of EIFS, α) appears not too significant. But this conclusion is depending on the other components of the problem. Suppose that (σ_X, σ_Y, r) = (0.155128668, 0.0778895, 0.796437) and the value 0.9041*10-6 coincides with required FFR of AL. And now let us suppose that in real test we have got $\hat{\mu}_X$ = -8.5885, $\hat{\mu}_Y$ =1.942460769 then equivalent mean value of T_C $\exp(\mu_Y - \mu_X)$ = 37.4574e+003. If we consider these estimates as real value of parameter the mean value of number of inspection will be equal to 5.5 (see Fig.3).

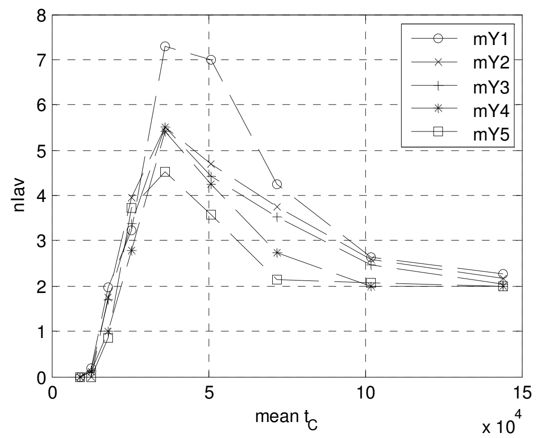

Figure 3. Mean value of inspection number as function of equivalent mean value of $T_C = \exp(\mu_Y - \mu_X)$.

Conclusions

The problem of inspection planning is the choice of the sequence $\{t_1, t_2, ..., t_n, t_{SL}\}$ corresponding to maximum of gain under limitation of AC intensity of fatigue failure. In numerical example the minimax decision, based on observation of some fatigue crack during acceptance full-scale fatigue test of airframe, is considered taking into account two random parameters of fatigue crack growth model. There is some difference in number of inspection to be chosen to provide required reliability and increase the gain of airline service. But this difference depends essentially on the other components of the problem. The method of necessary calculation is provided.

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