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AVERAGING, MERGER AND STABILITY OF LINEAR DYNAMICAL SYSTEMS WITH SMALL MARKOV JUMPS

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Abstract. In this paper it is shown that the asymptotical methods may be succesfully used for exponential mean square stability analysis of the linear dynamical system in $\mathbb{R}^n$ of impulse type which dynamical characteristics are dependent on the step Markov process $\{y(t),\, t \geq 0\}$.

Key words and phrases. linear differential equations, small random perturbations, impulse equations, mean square stability, Lyapunov methods, quadratic Lyapunov functionals, averaging procedure.

Mathematics Subject Classification. 60H10, 60H30.

1 Introduction

Let $\{y_\varepsilon(t),\, t \geq 0\}$ be series of right continuous homogeneous Markov processes [4] on the countable space $\mathbf{Y} \subset \mathbb{R}$ depending on parameter $\varepsilon \in (0,\, 1)$ with weak infinitesimal operators Q_ε defined on any element of the space $\mathbf{V}$ of bounded mappings $v : \mathbf{Y} \to \mathbb{R}$ by the equality

$$Q_\varepsilon v(y) := a(y,\, \varepsilon) \sum_{z \in \mathbf{Y}} [v(z) - v(y)] p(y,\, z,\, \varepsilon), \tag{1}$$

and let us suppose that $0 < a_1 := \inf_{\substack{y \in \mathbf{Y} \\ \varepsilon \in (0,\,1)}} a(y,\, \varepsilon) \leq \sup_{\substack{y \in \mathbf{Y} \\ \varepsilon \in (0,\,1)}} a(y,\, \varepsilon) := a_2 < \infty$. For any fixed $\varepsilon \in (0,\, 1)$ the Markov process with infinitesimal operator (1) is piecewise constant process [4, 7] with switching moments $\{\tau_j^\varepsilon,\ j \in \mathbb{N}\}$. These random variables can be recurrently defined by equalities

$$\tau_0^\varepsilon = 0, \quad \mathbf{P}_y(\tau_j^\varepsilon - \tau_{j-1}^\varepsilon > t) = e^{a(y,\,\varepsilon)t}, \quad j \in \mathbb{N},\ y \in \mathbf{Y},\ t \geq 0.$$

Now we will describe the series of Impulse Dynamical Systems (IDS) in $\mathbb{R}^n$ with parameter $\varepsilon \in (0,\,1)$ this paper deals with. The phase coordinates $x_\varepsilon(t)$ of this systems satisfy:

1) the initial condition

$$x_\varepsilon(0) = x \tag{2}$$

2) the differential equation

$$\frac{dx_\varepsilon}{dt} = A(y(t),\,\varepsilon))\,x_\varepsilon \tag{3}$$

for all $t \in (\tau^\varepsilon_{j-1},\,\tau^\varepsilon_j),\;\; j \in \mathbb{N}$;

3) the conditions of jumps

$$x_\varepsilon(t) = x_\varepsilon(t-0) + B(y(t),\,y(t-0)),\,\varepsilon)x_\varepsilon(t-0) \tag{4}$$

for all $t \in \{\tau^\varepsilon_j,\;\; j \in \mathbb{N}\}$, where the matrices $A(y,\,\varepsilon),\;B(z,\,y,\,\varepsilon)$ are defined as the series

$$A(y,\,\varepsilon) = \sum_{k=1}^{\infty} A_k(y)\,\varepsilon^k,\;\; B(z,\,y,\,\varepsilon) = \sum_{k=1}^{\infty} B_k(z,\,y)\,\varepsilon^k.$$

with matrix coefficients satisfying the inequalities

$$\sup_{y\in\mathbf{Y}} ||A_k(y)|| := \alpha_k < \infty,\;\; \sup_{z,y\in\mathbf{Y}} ||B_k(z,\,y)|| := \beta_k < \infty,\;\; k \in \mathbb{N} \tag{5}$$

and also the series composed of $\alpha_k,\,\beta_k$ are convergent. It is easily to make sure of existence and uniqueness of the above defined process $x_\varepsilon(t)$ for all $t \geq 0$.

The IDS (3)-(4) we will named as *exponentially mean square stable* if there exist such positive numbers M and ρ that $\mathbf{E}_y\,|x_\varepsilon(t+s,\,s,\,x)|^2 \leq M\,e^{-\rho t}|x|^2$ for any $x \in \mathbb{R}^n,\;\, y \in \mathbf{Y}$ and $t \geq s \geq 0$. Due to homogeneity of the Markov process $\{x_\varepsilon(t),\,y(t)\}$ the above inequality is equivalent to the inequality $\mathbf{E}_y\,|x_\varepsilon(t,\,0,\,x)|^2 \leq M\,e^{-\rho t}|x|^2$.

Let us denote by $\mathbf{Q}$ the space of the symmetric $n \times n$ matrix-valued continuous functions $\{q(y),\,y \in \mathbf{Y}\}$ with the subset $\mathbf{K} := \{q \in \mathbf{Q}:\; (q(y)x,\,x) \geq 0, \forall x \in \mathbb{R}^n, \forall y \in \mathbf{Y}\}$ of nonnegative-definite matrices.

The set of inner points of $\mathbf{K}$ can be defined as $\dot{\mathbf{K}} := \{q \in \mathbf{K} : \exists c > 0,\; q \gg cI\}$.

The following theorem was proved in [6].

Theorem. *Equation (1) is exponentially mean square stable if and only if there exist* $q \in \dot{\mathbf{K}}$ *and* $r \in \dot{\mathbf{K}}$ *such that*

$$\mathbf{A}_\varepsilon\,q = -r, \tag{6}$$

where

$$\begin{aligned}(\mathbf{A}_\varepsilon)\,q(y) \;&= A^T(y,\,\varepsilon)q(y) + q(y)\,A(y,\,\varepsilon)\\ &+a(y,\,\varepsilon)\sum_{z\in\mathbf{Y}}[(I + B^T(z,\,y,\,\varepsilon))\,q(z)\,(I + B(z,\,y,\,\varepsilon)) - q(z)]\,p(y,\,z,\,\varepsilon) + Q_\varepsilon\,q(y).\end{aligned} \tag{7}$$

Corollary. *IDS (3)-(4) is exponentially mean square stable if and only if there exists solution* $q \in \dot{\mathbf{K}}$ *of equation (6) with* $r = I$.

Equation (6) will be named *the Lyapunov equation for mean square stability investigation of IDS (3)-(4).*

We will suppose that the infinitesimal generator (1) also can be represented as the uniformly on $y \in \mathbf{Y}$, $\varepsilon \in (0,\, 1)$ convergent series

$$Q_\varepsilon\, v(y) = Q\, v(y) + \sum_{k=1}^{\infty} Q_k\, v(y)\, \varepsilon^k$$

where

$$\begin{aligned} Q\, v(y) &= a(y) \sum_{y \in \mathbf{Y}} [v(z) - v(y)]\, p(y,\, z), \quad (8) \\ Q_k\, v(y) &= \sum_{y \in \mathbf{Y}} [v(z) - v(y)] p_k(y,\, z), \;\; k \in \mathbb{N}, \end{aligned}$$

$p(y,\, z)$ is transition probability of some embedded Markov chains and $p_k(y,\, z),\; k \in \mathbb{N}$ are some positive measures on $\mathbf{Y}$. The operator (8) can be considered [4, 7] as the infinitesimal generator of some homogeneous piece wise constant Markov process $y_0(t)$, $t \geq 0$. Let us assume that this operator has 0 as an isolated simple eigenvalue of multiplicity h, h eigenfunctions with nonintersecting supports $\mathbf{Y}_j,\;\; j = \overline{1, h}$ defined by equalities

$f_j\,(y) = \begin{cases} 1, & \text{for} y \in \mathbf{Y}_j \\ 0, & \text{for} y \in \mathbf{Y}_k, \;\; k \neq j. \end{cases}$ and the remaining part of its spectrum is situated in the half-plane $\mathbf{C}_{-\rho}$ for some positive ρ. The conjugate operator Q^* also [4] has 0 as an isolated eigenvalue of multiplicity h and h invariant measures $\mu_k(y)$ with the same supports $\mathbf{Y}_k$, $k = \overline{1, h}$. If $h = 1$ then [4, 7]the Markov process $y_0(t)$ is uniformly exponentially ergodic. It has unique invariant measure $\mu(y)$.

The operator (7) can be decomposed in terms of powers of ε

$$\mathbf{A}_\varepsilon = \sum_{k=0}^{\infty} \varepsilon^k\, \mathbf{G}_k \tag{9}$$

where the operators $\mathbf{G}_m \in \mathbf{L}(\mathbf{Q})$, $m \geq 0$ are defined by the formulae

$$(\mathbf{G}_0\, q)(y) := A_0^T\, q(y) + q(y)\, A_0 + (Q\, q)(y), \tag{10}$$

$$\begin{aligned} (\mathbf{G}_1\, q)(y) := {} & A_1^T(y)\, q(y) + q(y)\, A_1(y) + (Q_1 q)(y) \\ & + a(y) \sum_{z \in \mathbf{Y}} [B_1^T(z,\, y)\, q(z) + q(z)\, B_1(z,\, y)]\, p(y,\, z), \end{aligned} \tag{11}$$

$$\begin{aligned} (\mathbf{G}_2\, q)(y) := {} & A_2^T(y)\, q(y) + q(y)\, A_2(y) + (Q_2 q)(y) \\ & + a(y) \sum_{z \in \mathbf{Y}} [B_2^T(z,\, y)\, q(z) + q(z)\, B_2(z,\, y) + B_1^T(z,\, y)\, q(z)\, B_1(z,\, y)]\, p(y,\, z) \\ & + \sum_{z \in \mathbf{Y}} [B_1^T(z,\, y)\, q(z) + q(z)\, B_1(z,\, y)]\, p_1(y,\, z), \end{aligned} \tag{12}$$

$$
\begin{aligned}
(\mathbf{G}_m\, q)(y) \;\; := \;\; & A_m^T(y)\, q(y) + q(y)\, A_m(y) + (Q_m q)(y) + a(y) \sum_{z \in \mathbf{Y}} [B_m^T(z,\, y)\, q(z) + q(z)\, B_m(z,\, y) \\
& + \sum_{k=1}^{m-1} B_{m-k}^T(z,\, y)\, q(z)\, B_k(z,\, y)]\, p(y,\, z) \\
& + [B_1^T(z,\, y)\, q(z) + q(z)\, B_1(z,\, y)]\, p_{m-1}(y,\, z), \\
& + \sum_{z \in \mathbf{Y}} \sum_{t=2}^{m-1} [B_t^T(z,\, y)\, q(z) + q(z)\, B_t(z,\, y) \\
& + \sum_{k=1}^{t-1} B_{t-k}^T(z,\, y)\, q(z)\, B_k(z,\, y)]\, p_{m-t}(y,\, z), \qquad m \geq 3. \qquad (13)
\end{aligned}
$$

Hence like in [7] one can prove the next result

Lemma. *Let $\lambda_0 \notin \sigma(\mathbf{A}_\varepsilon)$ for all sufficiently small $\varepsilon > 0$. Under the above assumptions there exists positive ε_0 such that the solution q^ε of the equation $\mathbf{A}_\varepsilon q^\varepsilon - \lambda_0 q^\varepsilon = f$ for any $f \in \mathbf{Q}$ and $\varepsilon \in (0,\, \varepsilon_0)$ has the form $q^\varepsilon = \sum\limits_{k=-d}^{\infty} \varepsilon^k q_k$ with some $d \in \mathbf{N}$.*

2 Averaging, merger and stability

Let us suppose that the differential equation (3) has the form

$$
\frac{dx_\varepsilon}{dt} = \varepsilon\, A_1(y_\varepsilon(t))\, x_\varepsilon + \varepsilon^2\, A_2(y_\varepsilon(t))\, x_\varepsilon \qquad (14)
$$

for all $t \in (\tau_{j-1}^\varepsilon,\, \tau_j^\varepsilon)$, $j \in \mathbb{N}$, and the conditions of jumps (4) have the form

$$
x_\varepsilon(t) = x_\varepsilon(t-0) + \varepsilon\, B_1(y_\varepsilon(t),\, y_\varepsilon(t-0))\, x_\varepsilon(t-0) + \varepsilon^2\, B_2(y_\varepsilon(t),\, y_\varepsilon(t-0))\, x_\varepsilon(t-0) \qquad (15)
$$

At the beginning we will assume that $h = 1$, that is, the Markov process $y_0(t)$ has a unique invariant measure $\mu(dy)$ and will denote by $\hat{y_0}(t)$ the stationary Markov function, corresponding to this measure. This means that for any $t \in \mathbb{R}$ and $A \subset \mathbf{Y}$ one can write $\mathbf{P}(\hat{y_0}(t) \in A) = \mu(A)$. We shall also denote

$$
C_j(y) := A_j(y) + a(y) \sum_{z \in \mathbf{Y}} B_j(z,\, y)\, p(y,\, z),
$$

$$
\overline{A}_j = \sum_{y \in \mathbf{Y}} A_j(y)\, \mu(dy), \;\; \overline{C}_j = \sum_{y \in \mathbf{Y}} C_j(y)\, \mu(dy), \qquad j \;=\; 1, 2,
$$

If $\overline{C}_1 = 0$ one can define [4] the matrix

$$
\begin{aligned}
F \;\; := \;\; & \overline{C}_2 + \sum_{y \in \mathbf{Y}} \sum_{z \in \mathbf{Y}} (\Pi\, C_1)(z)\, p_1(y,\, z) \mu(y) \\
& + \sum_{y \in \mathbf{Y}} \{A_1(y)\, (\Pi\, C_1)(y) + a(y) \sum_{z \in \mathbf{Y}} B_1(z,\, y)\, (\Pi\, C_1)(z)\, p(y,\, z)\}\, \mu(y)
\end{aligned}
$$

and the operator $H \in \mathbf{L}(\mathbf{M}_n(\mathbb{R}))$

$$\begin{aligned} H\,q \;\; := & \;\; \sum_{y\in\mathbf{Y}} \{A_1^T(y)\,q\,(\Pi\,C_1)(y) + (\Pi\,C_1)(y)\,q\,A_1(y) \\ & + a(y) \sum_{z\in\mathbf{Y}} [B_1^T(z,\,y)\,q\,(\Pi\,C_1)(z) + (\Pi\,C_1^T)(z)\,q\,B_1(z,\,y)]\,p(y,\,z)\,\mu(y) \end{aligned}$$

for $q \in \mathbf{M}_n(\mathbb{R})$, where the operator-potential Π.

For the first example let us consider the algorithm described in [6] with $d = 1$. In this case in order to investigate stability of the IDS (14)-(15) on the first step we must deal with the equation

$$Qq(y) = 0. \tag{16}$$

According to our previous assumptions about the operator Q one can conclude that the solution of (16) is an arbitrary symmetric matrix $q(y) \equiv q$. The conjugate equation to (16) has the form

$$Q^*\,p(y) = 0 \tag{17}$$

and its solution can be represented as $p(y) = p\,\mu(y)$ with an arbitrary symmetric matrix p and the invariant measure μ described above.

In the second step we must analyze the possibility of solving the equation

$$Q\,q_0(y) = -I - (\mathbf{G}_1\,q)(y)), \tag{18}$$

that is, due to Fredholm alternative it should be

$$Tr\,\{\sum_{y\in\mathbf{Y}} \{A_1^T(y)\,q + q\,A_1(y) + a(y) \sum_{z\in\mathbf{Y}} [B_1^T(z,\,y)\,q + q\,B_1(z,\,y)]\,p(y,\,z)\}\,\mu(y) + I\}\,p = 0$$

for an arbitrary matrix p. This is equivalent to existence of a symmetric matrix $\overline{q}$ as solution of the equation

$$\overline{C}_1^T\,q + q\,\overline{C}_1 = -I. \tag{19}$$

This equation has positive definite solution if and only if

$$\sigma(\overline{C}_1) \subset \{\mathbf{C} : \Re\lambda < 0\} \tag{20}$$

which is equivalent to asymptotic stability of the ordinary differential equation

$$\frac{d\overline{x}}{dt} = \overline{A}_1\,\overline{x}. \tag{21}$$

If

$$\sigma(\overline{A}_1) \cap \{\mathbf{C} : \Re\lambda > 0\} \neq \emptyset \tag{22}$$

the equation (21) is not asymptotically stable and then quation (19) has a nonpositive defined matrix $\overline{q}$ as its solution. Thus, in both cases (20) and (22) equation (19) has solution $\overline{q}$ and we can find the solution $q_0(y)$ of equation (18). Then the matrix

$$\frac{1}{\varepsilon}\,\overline{q} + q_0(y) \tag{23}$$

allows us to infer on the stability of the IDS (14)-(15). It is clear that if the matrix $\overline{q}$ is positive definite or nonpositive definite then the matrix (23) also has this property for sufficiently small positive ε. Therefore

- *if the averaged equation (21) is asymptotically stable then the IDS (14)-(15) is exponentially mean square stable for sufficiently small positive* ε;
- *if the averaged equation (21) has exponentially growing solutions then equation (14)-(15) also has exponentially mean square growing solutions for sufficiently small positive* ε.

If

$$\sigma(\overline{C}_1) \subset \{\mathbf{C} : \Re\lambda \leq 0\}, \quad \sigma(\overline{C}_1) \cap \{\mathbf{C} : \Re\lambda = 0\} \neq \emptyset \tag{24}$$

then equation (19) has no solutions and therefore $d > 1$ and we must analyze the equation

$$Q\, q(y) = -A_1^T(y)\, q - q\, A_1(y). \tag{25}$$

Due to the assumption (24) the equation

$$\overline{A}_1^T\, \tilde{q} + \tilde{q}\, \overline{A}_1 = 0 \tag{26}$$

has as solution a nonnegative definite matrix $\tilde{q}$ [2]. Then equation (25) must have solution because

$$Tr\,\{(\overline{A}_1^T\, \tilde{q} + \tilde{q}\, \overline{A}_1)\, p\} = 0$$

for any matrix p. Let us assume that the Markov process $\{y(t)\}$ is uniformly exponentially ergodic, that is, its transition probability satisfies the inequality

$$|P(t, y, A) - \mu(A)| \leq e^{-\rho t}$$

uniformly on $A \in \mathfrak{G}, y \in \mathbf{Y}$ for some $\rho > 0$ and all $t \geq 0$. In this case one can define [3] *the potential* Π of Markov process by the equality

$$(\Pi g)(y) := \int_0^\infty \int_{\mathbf{Y}} g(z) P(t, y, dz)\, dt = \int_0^\infty \mathbf{E}_y g(y(t))\, dt \tag{27}$$

for all g satisfying the condition $\int_{\mathbf{Y}} g(z)\mu(dz) = 0$. Next we extend the potential (27) on all $v \in \mathbf{C}(\mathbf{Y})$ by the equality

$$(\Pi v)(y) := \int_0^\infty \{\int_{\mathbf{Y}} v(z) P(t, y, dz) - \int_{\mathbf{Y}} v(z)\mu(dz)\}\, dt. \tag{28}$$

It is clear that $\Pi v \in D(Q)$ and

$$Q\Pi v = -v + \overline{v},$$

where

$$\overline{v} = \int_{\mathbf{Y}} v(z)\mu(dz).$$

By using the definition (28) of the extension potential Π the solution of (25) can be written in the form

$$q_{-1}(y) := (\Pi A_1^T)(y)\,\tilde{q} + \tilde{q}\,(\Pi A_1)(y).$$

This function can also be rewritten in the form [3]

$$q_{-1}(y) = \int_0^\infty \mathbf{E}_y \left\{ (A_1^T(y(t)) - \overline{A}_1^T)\,\tilde{q} + \tilde{q}\,(A_1(y(t)) - \overline{A}_1) \right\} dt.$$

The right part of this formula is a linear continuous operator acting on $\tilde{q}$.

Next we must analyze the equation

$$Q\,q(y) = -I - A_2^T(y)\,\tilde{q} - \tilde{q}\,A_2(y) - A_1^T(y)\,q_{-1}(y) - q_{-1}(y)\,A_1(y). \tag{29}$$

By using the Fredholm alternative one has to verify the orthogonality of the right part of (29) to the matrix measure $p\,\mu(dy)$ for arbitrary matrix p. This equation has a solution if and only if there exists a matrix $\tilde{q}$ which satisfies equation (26) and equation

$$\overline{A}_2^T\,\tilde{q} + \tilde{q}\,\overline{A}_2 + \overline{(\Pi A_1)A_1}^T\,\tilde{q} + \tilde{q}\,\overline{(\Pi A_1)A_1} + \overline{\Pi A_1^T\,\tilde{q}\,A_1} + \overline{A_1^T\,\tilde{q}\,\Pi A_1} = -I, \tag{30}$$

where overline denotes an averaging according to measure $\mu\,(dy)$. Therefore, the following conclusions about stability of (21) can be drawn under the conditions (24):

-if the system (26),(30) has positive definite solution $\tilde{q}$ then equation (21) is exponentially mean square stable for sufficiently small positive ε;

-if the system (26),(30) has nonpositive definite solution $\tilde{q}$ then equation (21) has exponentially mean square growing solutions for sufficiently small positive ε.

In the papers [1, 3] it is proven that under condition $\overline{A}_1 = 0$ the solutions of the system (26),(30) when represented in the form $x(t/\varepsilon^2)$ converge weakly as $\varepsilon \to 0$ to the corresponding solutions of the stochastic equation

$$d\hat{x}(t) = (F + \bar{A}_2)\,\hat{x}(t)\,dt + \sum_{j=1}^{n} D_j\,\hat{x}(t)\,dw_j(t) \tag{31}$$

and if the latter equation is exponentially mean square stable then this property also holds for the system (26),(30). The same result can be obtained using the above analysis of (26),(30) with $\overline{A}_1 = 0$ since then (23) is automatically satisfied and equation (26) is fulfilled for any matrix $\tilde{q}$. Equation (30) is the Lyapunov equation for analysis of mean square stability of equation (31) [5]. It can be easily seen that both equations (31) and (26),(30) have the same asymptotic behaviour as $t \to \infty$. Hence, *under the condition* $\overline{A}_1 = 0$*:*

- if the stochastic approximation of the system (26),(30), given by (31), is asymptotically mean square stable then the system (26),(30) is exponentially mean square stable for sufficiently small positive ε;

- if the stochastic approximation of the system (26),(30), given by (31), has exponentially mean square growing solutions then the system (26),(30) also has exponentially mean square growing solutions for sufficiently small positive ε.

3 Example

Let us analyse the stability of the system of type (14)-(15), described below.
Let us consider the Markov process with two states space $\mathbf{Y} = \{0; 1\}$, defined by the infinitesimal matrix $Q = \begin{pmatrix} -\alpha & \alpha \\ 1-\alpha & \alpha - 1 \end{pmatrix}$ where $\alpha \in (0; 1)$

Then the transition probabilities are $p(0,0) = 0$, $p(1,0) = 1$, $p(0,1) = 1$, $p(1,1) = 0$ and intensities of switching are: $a(0) = \alpha$, $a(0) = -\alpha$. Therefore the invariant measure of this Markov process is given by the equalities $\mu(0) = 1 - \alpha$, $\mu(1) = \alpha$.

Let the above Markov process be switching process for two dimensional MIDS of type (14)-(15), given by the equations:

$$\frac{dx}{dt} = \varepsilon A(y(t))x$$

for all $t \in (\tau_{j-1},\ \tau_j),\ \ j \in \mathbb{N}$;

$$x(t) = (I + B(y(t), y(t-0)))x(t-0)$$

for all $t \in \{\tau_j,\ \ j \in \mathbb{N}\}$,
where $A(y) = Ay = \begin{pmatrix} 0 & 0 \\ 0 & -2\delta \end{pmatrix}$, $B(y) = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_2 \end{pmatrix} zy$

1) In the first step we must deal with the equation (16). The solution is an arbitrary matrix with equal elements $Q = \begin{pmatrix} q & q \\ q & q \end{pmatrix}$. Then we solve the conjugate equation (17). $p(y) = \begin{pmatrix} \frac{(1-\alpha)^2}{\alpha^2} & \frac{1-\alpha}{\alpha} \\ \frac{1-\alpha}{\alpha} & 1 \end{pmatrix} p$

2) In the second step we must find the solution of (19). $C_1(y) = \begin{pmatrix} 0 & 0 \\ 0 & -2\delta \end{pmatrix} y$,
$\bar{C}_1 = \begin{pmatrix} 0 & 0 \\ 0 & -2\delta\alpha \end{pmatrix}$.

$$\begin{pmatrix} 0 & 0 \\ 0 & -2\delta \end{pmatrix} \begin{pmatrix} q_{11} & q_{12} \\ q_{21} & q_2 \end{pmatrix} + \begin{pmatrix} q_{11} & q_{12} \\ q_{21} & q_2 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & -2\delta \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

We obtain, that the solution of this equation do not exists. So we need to continue the algorithm and find the solution of (26)-(30). But from the equation (26):

$$2 \begin{pmatrix} 0 & 0 \\ 0 & -2\delta \end{pmatrix} \begin{pmatrix} q_{11} & q_{12} \\ q_{21} & q_2 \end{pmatrix} = 0$$

follows, that $q_{21} = q_{22} = 0$ of this solution. So the matrix q could not be positive defined and MIDS can not has any stable solution.

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