

## LATVIAN GDP: THE OPTIMAL TIME SERIES FORECASTING ALGORITHM

BEZRUCKO Aleksandrs, (LV)

**Abstract:** In this work an algorithm is developed for finding optimal time series model for GDP forecasting. Latvian GDP data with quarterly observation frequency is taken as time series. ARMA Analysis of Latvian GDP time series is performed. The set of model has been constructed. In order to check the accuracy of models, different residual tests are performed: autocorrelation, heteroscedasticity and normality of residual distribution. Models are compared in their forecast quality.

**Keywords:** *time series, Gross Domestic Product, ARMA (Autoregressive Moving Average) Analysis, Residual tests, Serial Correlation, Heteroskedasticity*

### 1 Introduction

The analysis and forecast of GDP for any time and any country is important task for economists, policy makers and entrepreneurs. These researches are consisting of many objective and subjective factors. In econometrics forecast, not only statistical methods are used, but a lot of economical and political events must be taken into account.

Working on the paper, different methods of econometrical modelling have been analyzed. For example, analysis methods for German GDP forecast that are described by Lutkepohl in “Applied Time Series Analysis” [1]. Lutkepohl described different ways of ARMA and Residual analysis of time series. In this paper author uses familiar methods of statistical analysis of time series for forecasting Latvian GDP. Computer software enabled the author to perform the search for the best models for certain time series. Based on the analysis of these models, a search algorithm of optimal model is created.

In order to find an optimal model of forecasting Latvian Gross Domestic Product, two different cases of Latvian GDP series with quarterly observation frequency are taken. The first case is Latvian quarterly GDP series in levels (Latvian lats) and second case is the same data in percentage growth. The GDP series are given in Figure 1. The time series length is  $T = 57$ . The time series is taken from the first quarter of year 1996 till the first quarter of year 2009. All searches and forecasts are made using econometrical software EViews 6.0.

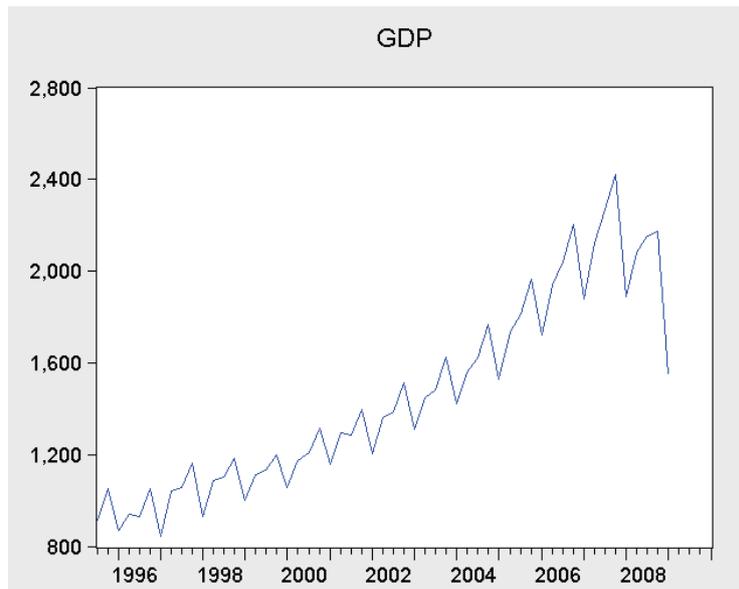


Figure 1 Latvian GDP (Lats) 1995Q1-2009Q1

## 2 Analysis description

### 2.1. The analysis of criteria

At the first stage of choosing the best model, 3 criteria are analyzed: Akaike info, Schwarz and Hannan-Quinn. The best model has minimal values. R-squared statistic is also present. At this stage models with best criteria are taken. ARMA Analysis is done in EViews program language and statistical criteria represent the result of the program (Fig.2).

Table: CRITS Workfile: GDP_AUTOREGRESSION_1:Untitled\									
View	Proc	Object	Print	Name	Edit+/-	CellFmt	Grid+/-	Title	Comments+/-
	A	B	C	D	E				
1	AR / MA	0.000000	1.000000	2.000000	3.000000				
2	0.000000	0.347646	-0.239621	-1.415521	-1.446306				
3	1.000000	-1.378436	-1.841098	-2.093701	-2.105267				
4	2.000000	-1.662691	-2.019881	-2.076773	-2.204613				
5									

Figure 1 ARMA Analysis in EViews 6.0

The R-squared ( $R^2$ ) statistic measures the success of the regression in predicting the values of the dependent variable within the sample. In standard settings,  $R^2$  may be interpreted as the fraction of the variance of the dependent variable explained by the independent variables. The statistic will equal one if the regression fits perfectly, and zero if it fits no better than the simple mean of the dependent variable. It can be negative for a number of reasons. For example, if the regression does not have an intercept or constant, if the regression contains coefficient restrictions, or if the estimation method is two-stage least squares or ARCH.

The Akaike Information Criterion (AIC) is computed as:  $AIC = -2l/T + 2k/T$  where  $l$  is the log likelihood. The AIC is often used in model selection for non-nested alternatives-smaller values of

the AIC are preferred. For example, you can choose the length of a lag distribution by choosing the specification with the lowest value of the AIC.

The Schwarz Criterion (SC) is an alternative to the AIC that imposes a larger penalty for additional coefficients:  $SC = -2l/T + (k \log T)/T$

## 2.2. Residual tests

The second stage is represented by Residual tests: Serial Correlation LM test, Histogram – Normality test, Heteroskedasticity ARCH test and Correlogram Square Residual test. Models have passed the test if P Value is higher than 0,1.

Serial Correlation LM test is an alternative to the Q-statistics for testing serial correlation. The test belongs to the class of asymptotic (large sample) tests known as Lagrange multiplier (LM) tests. Serial Correlation LM test has the higher importance because on this step we are concerning with the possibility that our errors exhibit autocorrelation. LM test check for higher order ARMA errors and is applicable whether or not there are lagged dependent variables.

The null hypothesis of the LM test is that there is no serial correlation up to lag order  $p$ , where  $p$  is a pre-specified integer. The local alternative is ARMA( $r, q$ ) errors, where the number of lag terms  $p = \max(r, q)$ . Note that this alternative includes both AR( $p$ ) and MA( $p$ ) error processes, so that the test may have power against a variety of alternative autocorrelation structures.

The test statistic is computed by an auxiliary regression as follows. First, suppose you have estimated the regression;

$$y_t = X_t \beta + \epsilon_t$$

where  $b$  are the estimated coefficients and  $\epsilon$  are the errors. The test statistic for lag order  $p$  is based on the auxiliary regression for the residuals  $e = y - X\hat{\beta}$ :

$$e_t = X_t \gamma + \left( \sum_{s=1}^p \alpha_s e_{t-s} \right) + v_t$$

Histogram and normality tests are displays a histogram and descriptive statistics of the residuals, including the Jarque-Bera statistic for testing normality. If the residuals are normally distributed, the histogram should be bell-shaped and the Jarque-Bera statistic should not be significant. The Jarque-Bera statistic has a  $\chi^2$  distribution with two degrees of freedom under the null hypothesis of normally distributed errors. [2]

The ARCH test is a Lagrange multiplier (LM) test for autoregressive conditional heteroskedasticity (ARCH) in the residuals. This particular heteroskedasticity specification was motivated by the observation that in many financial time series, the magnitude of residuals appeared to be related to the magnitude of recent residuals. ARCH in itself does not invalidate standard LS inference. However, ignoring ARCH effects may result in loss of efficiency.

The ARCH LM test statistic is computed from an auxiliary test regression. To test the null hypothesis that there is no ARCH up to order  $q$  in the residuals, we run the regression:

$$e_t^2 = \beta_0 + \left( \sum_{s=1}^q \beta_s e_{t-s}^2 \right) + v_t$$

where  $e$  is the residual. This is a regression of the squared residuals on a constant and lagged squared residuals up to order  $q$ . The F-statistic is an omitted variable test for the joint significance

of all lagged squared residuals. The Obs\*R-squared statistic is Engle's LM test statistic, computed as the number of observations times the  $R^2$  from the test regression. The exact finite sample distribution of the F-statistic under  $H_0$  is not known, but the LM test statistic is asymptotically distributed as a  $\chi^2(q)$  under quite general conditions.

Correlogram of squared residuals test displays the autocorrelations and partial autocorrelations of the squared residuals up to any specified number of lags and computes the Ljung-Box Q-statistics for the corresponding lags. The correlograms of the squared residuals can be used to check autoregressive conditional heteroskedasticity (ARCH) in the residuals.

If there is no ARCH in the residuals, the autocorrelations and partial autocorrelations should be zero at all lags and the Q-statistics should not be significant inclusion of ARMA terms. [2]

### 2.3. Out-Of-Sample Forecasting

The final evaluation test is “Out-Of-Sample Forecasting”. At this stage forecasts are compared to real data that we have for the period of the last 3 quarters of 2009.

## 3. Latvian GDP in levels

The analyzed series consist of seasonally unadjusted Latvian quarterly GDP in levels for the period of 1995Q1 – 2009Q1. It is represented in Figure 1. Constructing a model for the logs is more advantageous because the changes in the log series display a more stable variance than the changes in the original series. Time series in logs are shown in Figure 3.

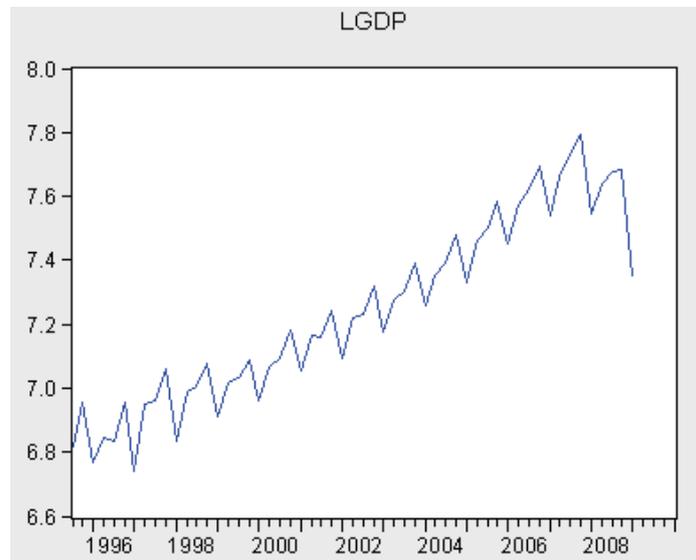


Figure 2 Latvian GDP in logs

Table 1

The analysis of criteria (Levels)

Nr.	LGDP	Akaike	Schwarz	Han-Quinn
1	Ar(1)	-1.326483	-1.290316	-1.312461
2	Ar(2)	-1.487976	-1.451479	-1.473862
3	Ma(1) C	-0.182363	-0.110677	-0.154503
4	Ma(2) C	-0.143123	-0.071437	-0.115263
5	AR(1) MA(1)	-1.878947	-1.806613	-1.850903
6	AR(1) AR(2) MA(1) MA(2)	-2.087592	-1.941604	-2.031137
7	AR(1) AR(2) SAR(4) MA(1)	-3.746621	-3.595105	-3.688722
8	AR(1) AR(2) SAR(4) MA(4)	-4.18207	-4.030555	-4.124172
9	Trend C	-1.914989	-1.843303	-1.887129
10	AR(1) Trend	-1.294331	-1.221997	-1.266287
11	AR(2) SAR(4)	-3.027406	-2.951648	-2.998456
12	AR(1) SAR(4) MA(4)	-3.646746	-3.534175	-3.603589
13	AR(1) AR(2) SAR(4) MA(4) SEAS(1)	-4.224953	-4.035559	-4.15258
14	AR(1) SAR(4) MA(4) D1997Q2	-4.047836	-3.89774	-3.990293

Best models: Nr. 5,7,8,12,13,14. Other models are excluded from further evaluation process. The residual test results are given in Table 2. Models Nr.13 and Nr.14 have undergone all tests. Residuals graph of Model Nr. 13 is given in Figure 4.

Table 2

Residual Test (Levels)

Nr.	Serial Correlation	Histogram	Heteroskedasticity
5	0.69290	0.00000	0.61360
7	0.69350	0.00000	0.79830
8	0.08070	0.40000	0.53920
12	0.61610	0.00003	0.84840
13	0.09250	0.83015	0.87060
14	0.10670	0.40223	0.85250

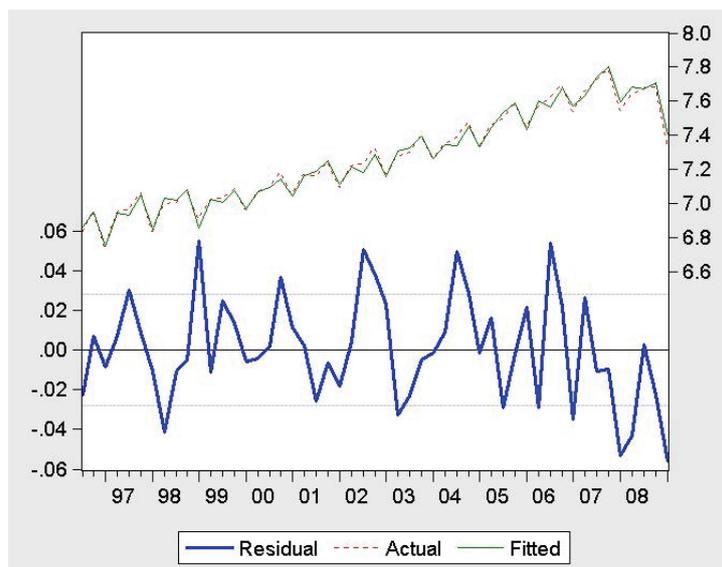


Figure 3 Residuals graph for Model Nr.13

The worst model residuals are given in Figure 5 for comparison.

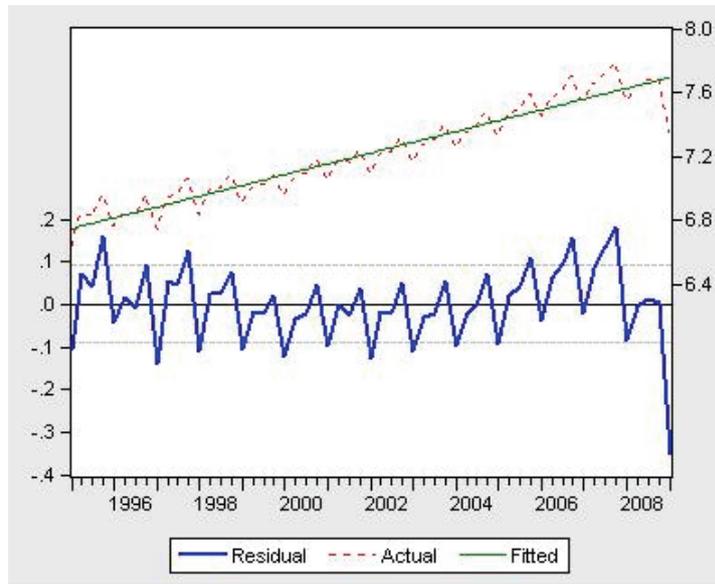


Figure 4 Residuals graph for Model Nr.9

“Out-Of-Sample Forecasting” test (Table 3) shows that the most real forecasts are gained from models Nr.12: AR(1) SAR(4) MA(4). Absolute difference (0.037) is minimal in this case. The best model has passed all Residual tests except Normality. The second and the third results are shown by models Nr. 7 and Nr.13., which have passed almost all residual tests except Nr.7., which also did not pass the Normality test.

Table 3

Out-Of-Sample Forecasting (Levels)

Nr.	Forecast - 1 Step - 09q2			Forecast - 2 Step - 09q3			Forecast - 3 Step - 09q4			Sum %
	LGDP real	LGDPf	Diff	LGDP real	LGDPf	Diff	LGDP real	LGDPf	Diff	
<b>5</b>	0.088	0.435	<b>-0.347</b>	0.031	0.022	<b>0.008</b>	0.034	0.018	<b>0.016</b>	0.371
<b>7</b>	0.088	0.082	<b>0.007</b>	0.031	0.017	<b>0.014</b>	0.034	-0.012	<b>0.045</b>	0.066
<b>8</b>	0.088	0.044	<b>0.044</b>	0.031	0.023	<b>0.007</b>	0.034	-0.020	<b>0.054</b>	0.106
<b>12</b>	0.088	0.103	<b>-0.014</b>	0.031	0.042	<b>-0.012</b>	0.034	0.023	<b>0.011</b>	0.037
<b>13</b>	0.088	0.057	<b>0.031</b>	0.031	0.017	<b>0.014</b>	0.034	-0.021	<b>0.055</b>	0.099
<b>14</b>	0.088	0.050	<b>0.039</b>	0.031	0.013	<b>0.018</b>	0.034	-0.019	<b>0.053</b>	0.109

**LATVIAN GDP IN PERCENTAGE GROWTH**

All evaluations are made with Latvian GDP. The differences and the log difference of time series are shown in Figure 6. The log difference displays a more stable variance than the changes in the original series.

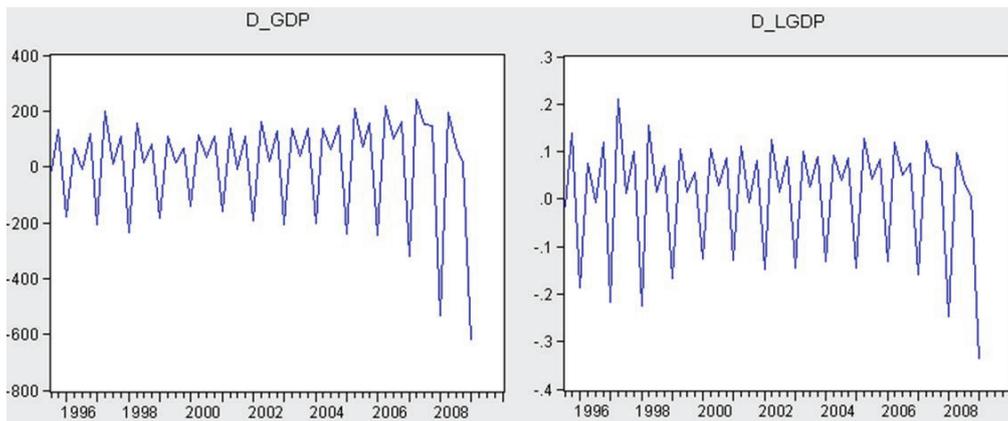


Figure 5 Time series in differences and in log differences

Table 4

The analysis of criteria (Difference)

Nr.	d (LGDP)	Akaike	Schwarz	Han-Quinn
1	Ar(1)	-1.659793	-1.623296	-1.645679
2	Ar(2)	-1.379102	-1.342269	-1.364897
3	Ma(1) C	-1.876758	-1.804424	-1.848714
4	Ma(2) C	-1.309161	-1.236827	-1.281117
5	AR(1) MA(1)	-1.712348	-1.639354	-1.684121
6	AR(1) AR(2) MA(1) MA(2)	-2.033619	-1.886286	-1.976798
7	AR(1) AR(2) SAR(4) MA(1)	-3.728444	-3.575482	-3.670195
8	AR(1) AR(2) SAR(4) MA(4)	-4.197451	-4.044489	-4.139202
9	Trend C	-1.306934	-1.2346	-1.278891
10	AR(1) Trend	-1.636801	-1.563807	-1.608574
11	AR(2) SAR(4)	-3.803588	-3.727107	-3.774463
12	AR(1) SAR(4) MA(4)	-4.191773	-4.078136	-4.148349
13	AR(1) AR(2) SAR(4) MA(4) SEAS(1)	-4.307962	-4.116759	-4.235151
14	AR(1) SAR(4) MA(4) D1997Q2	-4.273152	-4.121636	-4.215253

Models with the best criteria: Nr. 7, 8, 11, 12, 13, 14. Other models are excluded from further evaluation process.

Table 5

Residual Test (Difference)

Nr.	Serial Correlation	Histogram	Heteroskedasticity: ARCH
7	0.6938	0	0.825
8	0.6895	0.393373	0.3453
11	0.8852	0	0.017538
12	0.0503	0.421976	0.6305
13	0.64	0.560093	0.2823
14	0.5554	0.997972	0.7499

Models Nr. 8, 13, 14 have completed all tests. Model Nr. 12 also has good statistic. “Out-Of-Sample” forecasting test (Table 6) shows that the most real forecasts are gained from models Nr.11: AR(2) SAR(4). Absolute difference (0.037) is minimal in this case. This model did not complete the histogram test, but passed all other residual tests. The second result belongs to model Nr.7, which has the same problem with Normality test. Third result is shown by model Nr.14 – this model passed all residual tests.

Table 6

*Out-Of-Sample Forecasting (Difference)*

Nr.	Forecast - 1 Step - 09q2			Forecast - 2 Step -09q3			Forecast - 3 Step -09q4			Sum %
	dLGDP real	dLGDPf	Diff	dLGDP real	dLGDPf	Diff	LGDP real	LGDPf	Diff	
7	0.088	0.084	<b>0.004</b>	0.031	0.022	<b>0.009</b>	0.034	0.004	<b>0.030</b>	0.043
8	0.088	0.034	<b>0.054</b>	0.031	0.019	<b>0.012</b>	0.034	-0.020	<b>0.054</b>	0.120
11	0.088	0.091	<b>-0.003</b>	0.031	0.024	<b>0.007</b>	0.034	0.007	<b>0.027</b>	0.037
12	0.088	0.045	<b>0.043</b>	0.031	0.024	<b>0.007</b>	0.034	-0.020	<b>0.054</b>	0.104
13	0.088	0.022	<b>0.067</b>	0.031	0.009	<b>0.022</b>	0.034	-0.021	<b>0.054</b>	0.143
14	0.088	0.070	<b>0.018</b>	0.031	0.011	<b>0.020</b>	0.034	-0.020	<b>0.054</b>	0.091

#### 4 The search algorithm

The search algorithm is shown in Figure 7. Step-by-step description looks as follows:

Input data: GDP Time series

1. Construction of ARMA models in levels and in differences separately. Starting from this point the model is divided in two branches and the subsequent steps are carried out in parallel for levels and for differences.
2. ARMA Analysis.
3. Performing Residual tests.
4. If during residual tests probability value is less than 10%, the model is excluded from further evaluation. This is not a reliable model.
5. If P Value is higher than 10%, the forecast for specified periods of time is performed.
6. Comparing forecast data with real data. Making an evaluation.
7. The analysis of the results. Two branches of models come back to one point.

Output data: Best model for the GDP Forecasts.

#### Conclusion

In the given paper the author described search algorithm of optimal time series. With a help of statistical modelling the econometric analysis of Latvian GDP is done. Different cases of constructing model are made in Latvian lats (in levels) and in percentage growth (in difference).

Comparison of 1 and 2 cases shows that case in levels and in differences gave approximately the same result – 3.7% deviation from real data in absolute value for forecasts for 3 steps in future. It seems that it is does not matter which way to use. But this is surely not the right path. It is very important to analyze all the results and understand how they are calculated and evaluated. If a model gives the best forecast for one, two or three steps separately – it is does not mean that this model will be best in other cases. Figure 7 shows the algorithm of searching for optimal forecasting model.

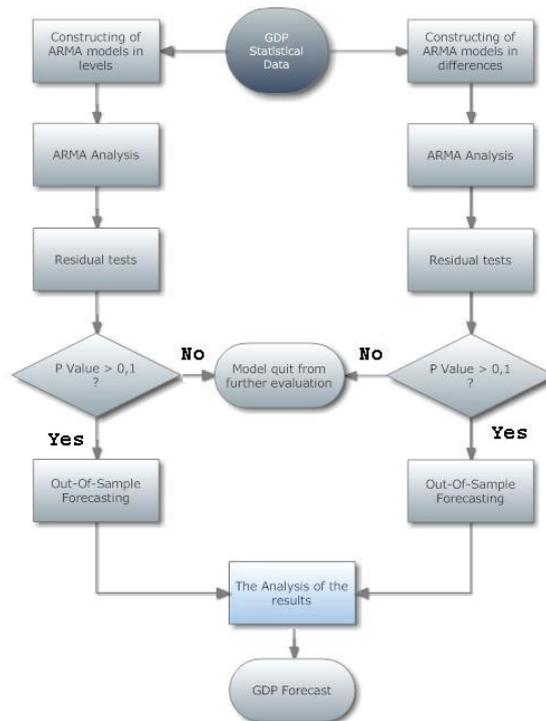


Figure 6 The search algorithm

## References

- [1] LUTKEPOHL H., KRATZIG M. Applied Time Series Econometrics. – Cambridge: Cambridge University press, 2004, 323 pp.
- [2] EViews software documentation: Quantitative Micro Software– <http://www.eviews.com> – Resource is described 2010, 30 of august.
- [3] NOSKO V., Econometrics: introduction to regression analysis of time series. – Moscow: Moscow Institute of Physics and Technology , 2002, 273 pp.
- [4] MOLCANOV I., ARZENOVSKI S. Statistical methods of forecasting. – Rostov-na-Donu: Rostov State University, 2001, 74 pp.

## Current address

**Bezrucko Aleksandrs, Mgr.**

Chair of the theory of probability and the mathematical statistics  
Faculty of Computer Science and Information Technology  
Riga Technical University, 1/4 Meza Street, LV-1048 Riga, Latvia  
e-mail: bezrucko@gmail.com

