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RISK MANAGEMENT OF EQUITY PORTFOLIO CONSTRUCTION ON THE BASIS OF DATA ENVELOPMENT ANALYSIS APPROACH

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Abstract. The research focus of the scientific paper is on the problem of equity portfolio construction. The author recommends applying frontier analysis technique such as Data Envelopment Analysis to the performance measurement of emitters. Using modern computer technologies, the author has calculated efficiency score of twenty Baltic companies which are quoted at NASDAQ OMX Riga and NASDAQ OMX Tallinn stock exchanges on the basis of DEA CCR approach and elaborated proposals for effective asset allocation.

Keywords: Data Envelopment Analysis, Decision Making Units (DMUs) portfolio construction, DEA, performance measurement

Introduction

Actual macroeconomic events are indicative of Eurozone crisis threats. In late October 2011 European leaders obtained an agreement from banks to take 50% loss on the face value of their Greek debt, decreasing its value by 100 billion euro. They also neared agreement on boosting the firepower of the Continent's bailout fund to around €1 trillion to help it protect larger economies like Italy and Spain from the sort of market pressures that pushed Greece to need a rescue. In September 2011 debt rating agency Standard & Poor’s downgraded Italy one level, from A +/A+1 to A/A-1, assessing the prospects for economic growth in Italy in the category of “negative.” The remaining uncertainty in Europe’s recovery and the future of euro impacts activities of all economical subjects. Thus, European companies and potential investors become especially vulnerable.

The construction of profitable and effective equity portfolio is among the most important investment problems. Traditionally the portfolio construction process includes four steps: creation of risk profile, asset allocation, correction of the portfolio structure corresponding to the investor’s requirements and regular control over the portfolio structure to avoid the overweight risk in a particular asset class. Currently risk measurement and asset allocation stages are completed using methods of technical and fundamental analysis, Modern Portfolio Theory by H.Markovitz, Sharpe ratio analysis etc. The main principle of technical analysis is an assumption that the market price of an asset includes information on all influencing factors; the development strategy of the company, balance data and perspectives of development are not estimated. However, fundamental analysis is based on the estimation of financial statements and competitive advantages. The Modern Portfolio Theory is the theory of investment which attempts to maximize portfolio expected return for a given amount of portfolio risk, or equivalently minimize risk for a given level of expected return, by carefully choosing the proportions of various assets. Nevertheless, efforts to translate the theoretical foundation into a viable portfolio construction algorithm have been plagued by technical difficulties stemming from the instability of the original optimization problem with respect to the available data. The results which are obtained on the basis of the mentioned approaches often provide inconsistent conclusions concerning potential investment opportunities and do not provide the possibility to evaluate the enterprise activity of emitters as a process.

Methods of frontier analysis ensure a principally different approach to the problem of equity performance measurement, estimating the performance of production process of each company. They provide an opportunity of complex analysis of company's efficiency level for a certain period of time and comparison of it among investigated objects. The objective of the author's research is to improve and supplement the methodology of risk measurement before the equity portfolio construction on the basis of the Data Envelopment Analysis approach.

In the circumstances of unstable macroeconomic environment and competition, profitability and market capitalization are among the most important indicators of stability and development of companies for the potential investor. Total operating revenue is a measure of the market value of company's production and the demand for it. Market capitalization is a parameter that reflects market value of all of a company's outstanding shares. It is a basic determinant of asset allocation and risk-return parameters. In this connection, the author analyzed the performance of a set of Baltic companies, assuming total operating revenue and market capitalization value as outputs. The objects of the research are Baltic companies which are quoted at NASDAQ OMX Riga and NASDAQ OMX Tallinn stock exchanges; their efficiency level is analyzed using data for the second quarter 2011. Evaluating the performance on the basis of the Data Envelopment Analysis approach, the author included into the set of investigated objects companies that are considered to be liquid at the Baltic stock markets (according to the amount of operations): JSC "Latvijas Balzāms", JSC "Grindeks", JSC "Latvijas gāze", JSC "Liepājas metalurgs", JSC "Latvijas Kuģniecība", JSC "Olainfarm", JSC "Rīgas kuģu būvētava", JSC "SAF Tehnika", JSC "Ventspils nafta", JSC "Valmieras stikla šķiedra", JSC "Arco Vara", JSC "Baltika", JSC "Ekspress Grupp", JSC "Harju Elekter", JSC "Olympic Entertainment Group", JSC "Silvano Fashion Group", JSC "Tallink Grupp", JSC "Tallina Kaubamaja", JSC "Tallina Vesi", JSC "Viisnurk".

1 Methods of frontier data analysis

The progress of production technology and increase of production volumes have stimulated the development of performance measurement methodology. In the second part of the 20th century there were introduced methods of frontier data analysis that provided a qualitatively different approach to the problem. According to the methodology of methods of frontier data analysis, the efficiency score of investigated DMUs is calculated as a distance from the point that defines the production process of a Decision Making Unit (DMU) to the certain efficiency frontier. Entities that are functioning on the efficiency frontier are considered to be absolutely technically efficient; inefficiency of other DMUs is increasing together with extension of the distance to the efficiency frontier. The value of efficiency score is fluctuating from zero to one.

Methods of frontier analysis may be divided into two groups: parametric (Stochastic Frontier Approach (SFA), Distribution-Free Approach (DFA), Thick Frontier Approach (TFA)) and non-parametric (Data Envelopment Analysis (DEA), Free Disposal Hull (FDH)) methods.
In accordance with parametric approaches, the efficiency frontier is constructed on the basis of econometric modelling, usually in form of Cobb-Douglas (log-linear) production function. Econometric analyses include two error components: an error term that captures inefficiency (u_i) and a random error (v_i). Parametric methods have significant advantages – they provide the possibilities to use panel data, to distinguish the random noise from inefficiency and to calculate the standard error of efficiency measurement results. Nevertheless, the stochastic approaches of performance measurement presume the comparison of investigated DMUs efficiency to the theoretically developed benchmark frontier; therefore the optimal combinations of inputs and outputs sometimes are not achievable practically. The application of parametric methods also requires observance of the restrictions imposed on the distributional assumptions on the inefficiencies and random error.[6]
In contrast to the econometric approaches, non-parametric methods are based on the hypothesis that the efficiency frontier is generated from the empirical results of the most efficient DMUs i.e. benchmarks that „float" on the piecewise linear frontier. The level of technical efficiency of these DMUs is 100%. However, the level of scale efficiency that defines the optimality of output and input proportions may have different values even among absolutely technically efficient DMUs. While mathematical, non-parametric methods require few assumptions when specifying the best-practice frontier, they generally do not account for random errors [7].

2 The CCR DEA Model

The CCR DEA model was developed by Charnes, Cooper and Rhodes in 1978 to evaluate the performance of Decision Making Units (DMUs). To allow for applications to a wide variety of activities, the term DMU might be used to refer to any entity that is to be evaluated in terms of its abilities to convert inputs into outputs. These evaluations can involve governmental agencies and non-profit organizations as well as business firms, hospitals and educational institutions.

The production process might be aimed either at minimization of resources or maximization of production volumes. The orientation of the model should be aimed at controllable variables. Volumes of resources are usually over control of management; therefore only input-oriented model will be examined in the paper.

The measurement of comparative efficiency is based on the assumption that the performance of each DMU is calculated in comparison to $\boldsymbol{n}$ investigated DMUs. Each DMU consumes varying amounts of $\boldsymbol{m}$ different inputs to produce $\boldsymbol{s}$ different outputs. Specifically, $\boldsymbol{DMU_j}$ consumes amount $\boldsymbol{x_{ij}}$ of input $\boldsymbol{i}$ and produces amount $\boldsymbol{y_{rj}}$ of output $\boldsymbol{r}$. It is necessary to assume that $x_{ij} \geq 0$ and $y_{rj} \geq 0$ and further to assume that each DMU has at least one positive input and one positive output value. Primarily the DEA model was expressed in fractional, i.e. ratio-form. In this form the ratio of outputs to inputs is used to measure the relative efficiency of the $DMU_j = DMU_0$ to be evaluated relative to the ratios of all of the $\boldsymbol{j = 1,2, ..., n}$ DMU_j. The CCR construction can be interpreted as the reduction of the multiple-output/multiple-input situation (for each DMU) to that of a single 'virtual' output and 'virtual' input. For a particular DMU the ratio of this single virtual output to single virtual input provides a measure of efficiency that is a function of the multipliers. In mathematical programming parlance, this ratio, which is to be maximized, forms the objective function for the particular DMU being evaluated. A set of normalizing constraints (one for each DMU) reflects the condition that the virtual output to virtual input ratio of every DMU, including

$DMU_j = DMU_0$, must be less than or equal to unity. [4] The mathematical programming problem may thus be stated as (1):

$$\begin{aligned}
&\max h_0(u,v) = \sum\nolimits_r u_r y_{ro} / \sum\nolimits_i v_i x_{io} \\
&\textit{subject to} \\
&\sum\nolimits_r u_r y_{rj} / \sum\nolimits_i v_i x_{ij} \le 1 \ \textit{for} \ j = 1,\dots,n, \\
&u_r, v_i \ge 0 \ \textit{for all i and r},
\end{aligned} \tag{1}$$

where

h_0 – the function of virtual output and virtual input ratio of DMU_0;
u_r – the output multiplier of DMU_0;
v_i – the input multiplier of DMU_0;
y_{r0} – the output of DMU_0;
x_{i0} – the input of DMU_0;
y_{rj} – outputs of 1,2…n DMUs;
x_{ij} – inputs of 1,2…n DMUs.

The above ratio form yields an infinite number of solutions; if ***(u*, v*)*** is optimal, then ***(αu*, αv*)*** is also optimal for ***α > 0***. However, the transformation developed by Charnes and Cooper (1962) for linear fractional programming selects a representative solution ***(u, v)*** $\sum_{i=1}^{m} v_i x_{io} = 1$ h and yields the equivalent linear programming problem in which the change of variables from ***(u, v)*** to ***(μ, ν)*** is a result of the Charnes-Cooper transformation (2):

$$\begin{aligned}
&\max z = \sum_{r=1}^{s} \mu_r y_{ro} \\
&\textit{subject to} \\
&\sum_{r=1}^{s} \mu_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \\
&\sum_{i=1}^{m} v_i x_{io} = 1 \\
&\mu_r, v_i \ge 0,
\end{aligned} \tag{2}$$

where

z – the CCR input-oriented function of DMU_0 (multiplier form);
μ_r – the output multiplier of DMU_0;
ν_i – the input multiplier of DMU_0;
y_{r0} – the output of DMU_0;
x_{i0} – the input of DMU_0;
y_{rj} – outputs of 1,2…n DMUs;
x_{ij} – inputs of 1,2…n DMUs;
s – number of outputs;
m – number of inputs.

Model that is expressed by (2) can be solved by its dual problem (3):

$$
\begin{aligned}
&\theta^* = \min \theta \\
&\textit{subject to} \\
&\sum_{j=1}^{n} x_{ij}\lambda_j \le \theta x_{io} \qquad i = 1,2,\ldots,m; \\
&\sum_{j=1}^{n} y_{rj}\lambda_j \ge y_{ro} \qquad r = 1,2,\ldots,s; \\
&\lambda_j \ge 0 \qquad\qquad j = 1,2,\ldots,n,
\end{aligned} \tag{3}
$$

where

θ^* – the optimal value of dual variable θ of DMU$_0$;
θ, λ_j – dual variables of DMU$_0$;
y_{r0} – the output of DMU$_0$;
x_{i0} – the input of DMU$_0$;
y_{rj} – outputs of 1,2…n DMUs;
x_{ij} – inputs of 1,2…n DMUs;
s – number of outputs;
m – number of inputs.

This last model is sometimes referred to as the "Farrell model" because it is the one used in Farrell (1957). By virtue of the dual theorem of linear programming we have $z^* = \theta$. Hence either problem may be used. One can solve the dual linear program, to obtain an efficiency score. Setting $\theta = 1$ and $\lambda_k^* = 1$ with $\lambda_k = \lambda_o^*$ and all other $\lambda_k^* = 0$, a solution of dual problem (see Formula 3) always exists. Moreover this solution implies $\theta^* \le 1$. The optimal solution, θ^*, yields an efficiency score for a particular DMU. [3]

The process is repeated for each DMU. i.e., solving the model, expressed by Formula 3, with $(X_o, Y_o) = (X_k, Y_k)$, where (X_k, Y_k) represent vectors with components x_{ik} , y_{rk} and, similarly (X_o, Y_o) has components x_{ok} , y_{ok}. DMUs for which $\theta^* < 1$ are inefficient, while DMUs for which $\theta^* = 1$ are boundary points. Some boundary points may be "weakly efficient" because we have non-zero slacks. This may appear because alternate optima may have non-zero slacks in some solutions, but not in others. However, we can avoid this effect by invoking the following linear program in which the slacks are taken to their maximal values (4).

$$
\begin{aligned}
&\max \sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+ \\
&\textit{subject to} \\
&\sum_{j=1}^{n} x_{ij}\lambda_j + s_i^- = \theta^* x_{io} \qquad i = 1,2,\ldots,m; \\
&\sum_{j=1}^{n} y_{rj}\lambda_j - s_r^+ = y_{ro} \qquad r = 1,2,\ldots,s; \\
&\lambda_j, s_i^-, s_r^+ \ge 0 \;\forall\; i, j, r,
\end{aligned} \tag{4}
$$

where

s_i^- – input slacks;
s_r^+ – output slacks;
θ^* – the optimal value of dual variable θ of DMU$_0$;
λ_j – the dual variable of DMU$_0$;
y_{r0} – the output of DMU$_0$;

x_{i0} – the input of DMU_0;
y_{rj} – outputs of 1,2…n DMUs;
x_{ij} – inputs of 1,2…n DMUs;
s – number of outputs;
m – number of inputs.

It shall be noted that the choices of s_i^- and s_r^+ do not affect the optimal θ^* which is determined from model expressed by (3). These developments lead to the following definitions of DEA efficiency:

DEA Efficiency: The performance of DMU_0 is fully (100%) efficient if and only if both (i) $\theta^* = 1$ and (ii) all slacks $s_i^{-*} = s_r^{+*} = 0$.

Weakly DEA Efficiency: The performance of DMU_0 is weakly efficient if and only if both (i) $\theta^* = 1$ and (ii) $s_i^{-*} \neq 0$ and/or $s_r^{+*} \neq 0$ for some i and r in some alternate optima [1].

The CCR efficiency score is indicative of the overall efficiency level of investigated DMUs. [5]

3 The application of data envelopment analysis approach to the equity portfolio construction

3.1 Methodology of the research

Due to the methodology, the Data Envelopment Analysis approach of comparative performance measurement does not require the specific functional form of the model. Therefore choice of outputs and inputs that are corresponding to the objectives of the research is among significant conditions for the achievement of plausible results.
The problem of keeping profitability is especially topical and important in the circumstances of unstable macroeconomic environment. The market capitalization value reflects the risk-return parameters that are indicative of company's stability and development opportunities. In this connection, there is developed a concept of efficiency measurement of companies which are quoted at the NASDAQ OMX Riga and NASDAQ OMX Tallinn in the research, assuming total operational revenue to be outputs, while equity, operating expenses and finance (interest) expenses are defined as inputs.
The performance evaluation will be completed on the basis of DEA CCR approach that allows calculating overall efficiency score of investigated companies.

3.2 Efficiency measurement results of Baltic companies on the basis of CCR DEA approach

The application of the DEA approach requires the determination of assumptions, concerning orientation measures of the model and the concept of returns to scale (RTS). The production process may be aimed either at minimization of resources (input-oriented) or maximization of production volumes (output-oriented). It is emphasized in the international researches that the orientation of the model should be aimed at controllable variables. Usually volumes of resources are considered to be over control of management, therefore there is applied the assumption of input orientation in the research. Since the constant returns to scale CRS approach represents the total (overall) efficiency level, CCR DEA model is considered to be the basic concept of the research.[2]

The results of companies' performance evaluation on the basis of CCR input-oriented model, assuming total operating revenue and market capitalization values as outputs, are represented in Figure 1.

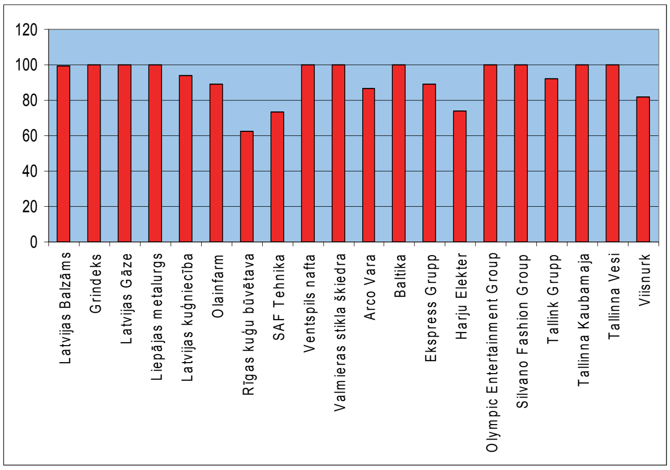

Fig. 1. DEA CCR efficiency score of Baltic stock exchange quoted companies, (%)

According to the obtained results, investigated companies might be separated into three groups. The first group includes 100% DEA CCR efficient companies: JSC "Latvijas Balzāms", JSC "Grindeks", JSC "Latvijas gāze", JSC "Liepājas metalurgs", JSC "Ventspils nafta", JSC "Valmieras stikla šķiedra", JSC "Baltika", JSC "Olympic Entertainment Group", JSC "Silvano Fashion Group", JSC "Tallina Kaubamaja", JSC "Tallina Vesi". The above-mentioned companies have demonstrated the best result, operating on the efficiency frontier at the observation period. High efficiency level of these emitters is indicative of their ability to maximize the volume of outputs using minimal volumes of inputs and to ensure optimal proportions of output and inputs in the process of production, thus of both 100% technical and scale efficiency in comparison to the set of investigated objects. For example, the state-owned company JSC "Latvijas gāze" ensuring 375.9 million euro market capitalization value and 278.9 million euro total revenue, is operating using only equity capital and having no interest expenses. Despite of high volatility of the share price, JSC "Liepājas metalurgs" has the total revenue value 183 million euro at the second quarter 2011. According to the latest company's announcement, JSC "Liepājas metalurgs" is investing into the equipment modernization project; the commercial pledge of 72.19 million lats is guaranteed by the Ministry of Finance of the Republic of Latvia. Due to this fact, potential investors might expect the reduction of company's operational costs. JSC "Olympic Entertainment Group" and JSC "Tallina Kaubamaja" are among the leading companies on the Tallinn stock exchange according to their output values. The enterprise activity of JSC "Olympic Entertainment Group" is oriented at casino and hotel business segments, ensuring the total revenue of 60.8 million euro by the end of the second quarter 2011. JSC "Tallina Kaubamaja" is the largest department store in Estonia that is listed since 1996 on the Tallinn stock exchange. This fact makes securities of the emitter attractive for potential investors.

The second group consists of companies that are having the performance above the 80% level: JSC "Latvijas Kuģniecība", JSC "Olainfarm", JSC "Arco Vara", JSC "Ekspress Grupp", JSC "Tallink Grupp", JSC "Viisnurk". JSC "Olainfarm" is one of the most rapidly growing Baltic companies. According to the latest company's announcement, preliminary sales results of JSC "Olainfarm" for September 2011 show that sales have increased by 102% compared to the same period last year and have reached 3.71 million lats (5.28 million euro). The most rapid sales increase has been experienced in Canada, where sales have increased 442 times, in Ukraine they increased nearly 4 times, in Belarus, a country heavily hit by its currency crisis, the sales have grown by 71%, in Russia by 63%. Main sale markets of AS “Olainfarm” during September 2011 were Ukraine, Russia, Belarus and Latvia. Nevertheless, the company has lower production volumes, higher finance expenses (228 thousand euro) than its nearest competitor JSC “Grindeks”, having the 88.89% performance level. Having the highest market capitalization value 467.6 million euro and total revenue of 400.8 million euro, JSC “Tallink Grupp” is only 92.31% DEA CCR efficient. Among possible reasons are high finance (26.7 million euro) and operational expenses (391.5 million euro). This fact is indicative of inefficient organization of company’s operational activity.

The third group includes companies that are having the efficiency below the 80% level: JSC “Rīgas kuģu būvētava”, JSC “SAF Tehnika”, JSC “Harju Elekter”.

According to the JSC “SAF Tehnika” interim report data in August 2011, the company’s non-audited net sales for 12 months of the financial year 2010/11 were 10.9 million LVL (15.5 million EUR) representing a year-on-year increase of 7%. Sales in the Asia Pacific, Middle East and Africa region formed the largest sales proportion (37%) comprising 4.05 million LVL (5.76 million EUR) although it was by 32% less than in previous financial year 2009/10. The net profit of JSC “SAF Tehnika” for the 12 months of financial year 2010/11 was 780 thousand LVL (1.1 million EUR) representing 52% of the net profit of previous financial year 2009/10. JSC “SAF Tehnika’s” non-audited net sales for the fourth quarter of financial year 2010/11 were 1.99 million LVL (2.84 million EUR), representing 53% of the fourth quarter of the previous financial year. [8] Reporting quarter was the weakest in this financial year unlike from last financial year 2009/10 when the fourth quarter was the best. This information had negative impact on the share price of JSC “SAF Tehnika” that decreased by 27.27% since January 2011.
Operating with loss in both 2010 and 2011 financial years (572.6 thousand euro in the second quarter 2011), JSC “Rīgas kuģu būvētava” has the lowest performance level among all investigated companies. Nevertheless, on October 19th 2011 JSC „Rīgas kuģu būvētava” received an official announcement from SJSC «Черноморнефтегаз» tenders trade commission about the approval of the proposed price by JSC „Rīgas kuģu būvētava. The proposal of JSC „Rīgas kuģu būvētava „to delivery gas platform 35.11.4. for USD 399 800 000 was considered the most economically beneficial and was accepted as a result of evaluation. [8] This corporative event caused the increase of equity price by 77.6%, demonstrating that shares of JSC „Rīgas kuģu būvētava” are a good investment opportunity for a risk-tolerant investor.

Conclusions

The scientific paper is devoted to the equity portfolio construction problem. The most important stages of this process are choice of potential assets, risk evaluation and asset allocation. Traditionally potential investors are using methods of technical and fundamental analysis, Modern Portfolio Theory for this purpose. However, the results which are obtained on the basis of the mentioned approaches often provide inconsistent conclusions concerning investment opportunities and do not provide the possibility to evaluate the enterprise activity of emitters as a process.
The methodology of Data Envelopment Analysis is considered to be a sophisticated tool for performance measurement that allows the investigation of complex production processes among a set of Decision Making Units (DMUs).

The author has implemented the DEA CCR approach, analyzing efficiency scores of a set of companies which are quoted at NASDAQ OMX Riga and Tallinn. According to the results, emitters might be divided into three groups: 100% DEA CCR efficient, having the performance above the 80% level and having the performance below the 80% level. Equities of fully CCR DEA efficient companies could be included into the portfolio with conservative investment strategy. Companies which are included into the second and the third group, have lower level of performance. However, securities of these emitters might be attractive for investors with higher level of risk tolerance.

To sum up, the author recommends using the Data Envelopment Analysis approach methodology as an additional tool for analysis of investment opportunities and equity portfolio creation.

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