

# 946. Optimization of characteristics of multilayer spherical control joint-hinge stiffness

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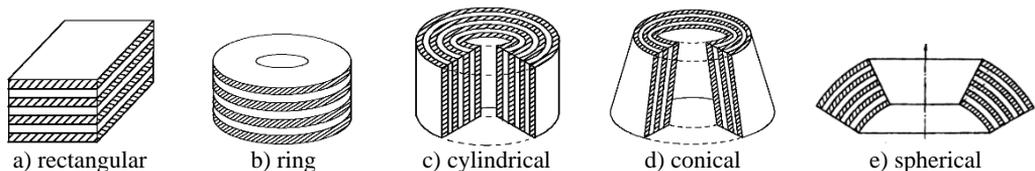
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**Abstract.** In this work the analytical expression is derived on the basis of the variational method for the evaluation of angular stiffness characteristics of the spherical multilayered elastomeric package joint-hinge subjected to loading with hinge moment and hydrostatic pressure on the lateral surfaces. Joint-package consists of alternating metallic and thin elastomeric layers. Metallic plates-layers are assumed to be rigid. It is demonstrated that the hydrostatic pressure can both mitigate and "harden" the hinge angular stiffness characteristics depending on which side surface of the elastomeric layer it is applied. Obtained relationships allow solving problems of optimal design, operation and control program selection for compensating joints of this type.

**Keywords:** elastomers, packaged joint, optimization, hydrostatic pressure, Ritz's method.

## 1. Introduction

Rubber and rubberlike materials (elastomers) are widely used in many sectors of industry [2, 4, 7]. Physical properties of elastomers, as polymeric materials, are qualitatively different from traditional construction materials because of their ability to maintain large elasticity deformations and small volume compressibility under deformation [3, 6]. Reinforced elastomeric structures (laminated elastomeric) consist of a large number of alternating thin layers of rubber and reinforcing layers of other, much more rigid than rubber, material. The connection of elastomer with reinforcing layer is usually done by means of vulcanization or gluing. Multilayer elastomeric structures have a special place: their axial compression stiffness is by several orders greater than the shear stiffness [6, 7, 10]. These structures are used in machine building, shipbuilding, civil engineering, aviation and aerospace due to its unique mechanical properties. Multilayer elastomeric structures successfully replace traditional technical systems, such as bearing, joints, compensating devices, shock-absorbers because of its important advantages: improving of machine dynamics, vibration and noise reduction, low shear and compression stiffness ratio. In practice, the packages of thin layers rubber-metal elements of different shapes are used: flat, cylindrical, conical, and others (Fig. 1). Number of layers may be different (at least three). In many applications of multilayer elastomeric structures their stiffness characteristics – the relationship between external forces imposed on the package and its displacement under load – are of great practical consequence.



**Fig. 1.** Examples of multilayer elastomeric structures

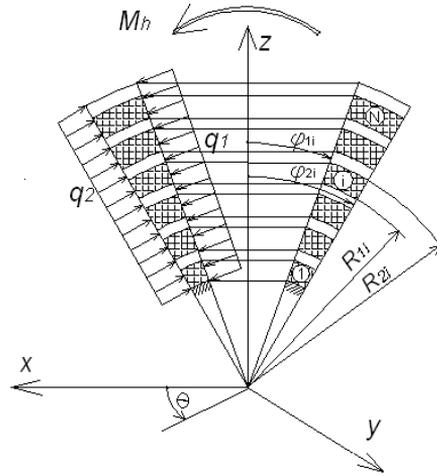
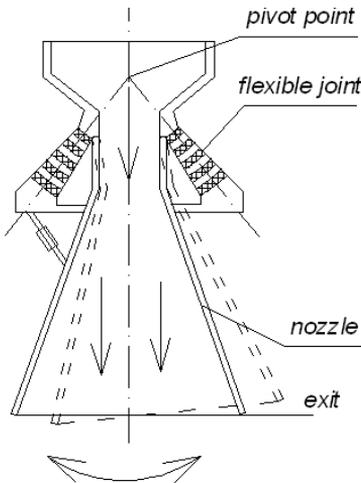
This paper considers a stiffening behavior of spherical multilayer package joint-hinge with perfectly rigid metallic reinforcing layers under influence of hinge moment and hydrostatic pressure exerted on the free side of the package. Strength and stability of elements are regarded as ensured. Flexible laminated spherical hinges are used in rocket vehicle to provide the control

of direction of thrust vector by means of movable nozzle. Of all the mechanical deflection types, the movable nozzles are the most efficient to thrust vector control system [9]. Example of an elastomeric spherical hinge application in rocket technology is given in Fig. 2.

Reinforced elastomers have been in use for more than 50 years. There are a large number of publications concerning this structure. The theory of the elastomeric layer is developed [7]; methodology of determination of stress and deformations are proposed; compression, shear and tilting stiffness for round, square and infinite-strip form are determined theoretically [1, 2, 11]. For thin spherical elements there is no theoretical basis and method of stiffness calculation in the case of side follower load taking into account orientation of side surface changing [8, 10].

**2. Problem definition, analytical model and method of analytical solution**

A flexible hinge made up of alternate spherical segments of elastomeric and rigid metal shims is considered. The behavior of elastomeric layers is only taken into account, assuming that the metallic layers geometry makes them perfectly rigid at this scheme of loading. The joint-hinge is loaded with hinge moment  $M_h$  (in  $xoz$  plane) and with simultaneous action of hydrostatic pressure  $q_1$  and  $q_2$  on the free lateral surfaces of elastomeric layers, respectively, on the internal surface ( $\varphi = \varphi_1$ ) and outside one ( $\varphi = \varphi_2$ ). The purpose of this work is to obtain an analytical dependence for the angular stiffness characteristics of package-joint taking into consideration the influence of the pressure  $q_1$  and  $q_2$  on this characteristic. Loading scheme and the geometric parameters are shown in Fig. 3 in section  $\theta = 0$  of spherical coordinates  $r, \varphi, \theta$ .



**Fig. 2.** Example of elastomeric hinge application      **Fig. 3.** Object geometry and loading scheme at  $\theta = 0$

Shear stiffness characteristics of joint-hinge are determined by means of Ritz's variation method using the linear theory of elasticity for an elastomeric material [6, 7]. At first, stiffness characteristic for one elastomeric layer is determined, i.e. joint with only one elastomeric layer is considered, then the package-joint response is studied.

**2.1. Calculation of the stiffness characteristics of a single layer**

The solution is carried out in two stages. Initially only hinge moment  $M_h$  is imposed; loading and deformation scheme (in section  $\theta = 0$ ) is illustrated in Fig. 4. In the considered case the radial displacement  $u$  and strains  $\varepsilon_\varphi$  and  $\varepsilon_\theta$  are equal to zero. Geometrical boundary conditions for the displacement  $w$  and  $v$  (in the meridional and circumferential directions respectively) may be written as: if  $r = R_1, w = 0$  and  $v = 0$ , if  $r = R_2, w = \delta R_2 \cos\theta$  and  $v = -\delta R_2 \cos\phi \sin\theta$ .

Functions, satisfying these conditions and to the expected character of deformation are:

$$w = \delta R_2 \psi(r) \cos \theta, \quad v = -\delta R_2 \psi(r) \cos \phi \sin \theta,$$

$$\psi(r) = \frac{(r^3 - R_1^3) R_2^2}{r^2 (R_2^3 - R_1^3)}.$$

Sought dependence "moment – rotation angle" is defined by means of the Ritz method from the total potential energy functional minimum condition  $\partial \Pi / \partial \delta = 0$  [6]:

$$\Pi = G \int_0^{2\pi} \int_{R_1}^{R_2} \int_{\phi_1}^{\phi_2} 2(\varepsilon_{r\phi}^2 + \varepsilon_{r\theta}^2) r^2 (\sin \phi) d\phi dr d\theta - M_h \delta, \quad (1)$$

$$\varepsilon_{r\phi} = 0.5 \delta R_2 r \frac{d}{dr} \left( \frac{\psi(r)}{r} \right) \cos \theta, \quad \varepsilon_{r\theta} = -0.5 \delta R_2 r \frac{d}{dr} \left( \frac{\psi(r)}{r} \right) \cos \phi \sin \theta.$$

Obtained angular stiffness characteristic is:

$$\delta = \frac{M_h}{\pi G R_2^3 k}, \quad k = \frac{3\alpha^3}{1 - \alpha^3} \left[ (\cos \phi_1 - \cos \phi_2) + \frac{1}{3} (\cos^3 \phi_1 - \cos^3 \phi_2) \right], \quad (2)$$

where:  $\alpha = \frac{R_1}{R_2}$ .

Now the influence of pressure  $q_1$  and  $q_2$ , acting respectively on the free sides of the elastomeric layer at  $\varphi = \varphi_1$  and  $\varphi = \varphi_2$  is determined. First, the effect of the pressure  $q_2$  is only taken into account with simultaneous action of hinge moment  $M_h$ . The lateral surface of the elastomeric layer  $\varphi = \varphi_2$  in an arbitrary section  $\theta$  is considered (Fig. 5).

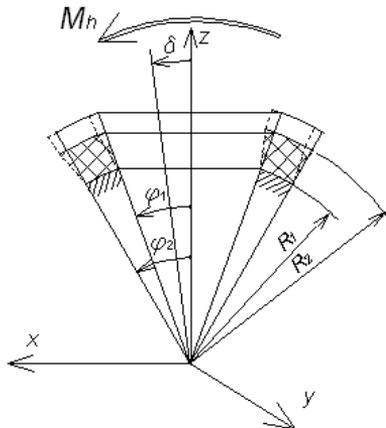


Fig. 4. Loading and deformation scheme at  $\theta = 0$

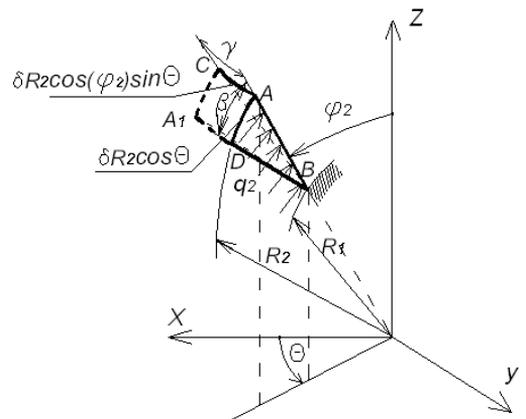


Fig. 5. Deformation scheme in an arbitrary section

For thin elastomeric layers, even at small joint-hinge rotation angles  $\delta$  reconfiguring of the elastomeric layers sides becomes significant. Since the pressure  $q_2$  is follower load, which is always normal to the side surface of the elastomeric layer, significant change of its projections takes place relatively to the undeformed state of the elastomeric layer side. This leads to changing of the joint-hinge loading and must be taken into account in its analytical model. It is assumed that in view of the hinge geometry, its radial and flexural stiffness is significantly greater than the shear stiffness, i.e. radial and bending deformations, which can be induced by

corresponding projections of pressure  $q_2$  on the deformed surface, may be neglected. The elastomeric layer generator  $AB$  under the joint-hinge moment moves meridionally (on  $\varphi$  - direction) at the angle  $\beta = \angle ABD$  and in the circumferential direction  $\theta$  at an angle  $\gamma = \angle ABC$  and takes the position  $A_1B$ . Pressure  $q_2$  remains normal to the deformed side of the elastomeric layer. Relative to the original surface  $\varphi = \varphi_2$  vector  $q_2$  of pressure can be decomposed into radial, circumferential and meridional direction components:

$$q_u = q_2 \cos \gamma \sin(\beta + \phi_2), \quad q_w = q_2 \cos(\gamma) \cos(\beta + \phi_2), \quad q_v = q_2 \sin(\gamma), \quad \sqrt{q_u^2 + q_w^2 + q_v^2} = q_2.$$

From Fig. 5:  $AB = R_2 - R_1$ ,  $AD = \delta R_2 \cos \theta$ ,  $BC = \delta R_2 \cos \phi_2 \sin \theta$ ,  $\tan(\beta) = \frac{\delta \cos(\theta)}{1 - \alpha}$ ,  $\tan(\gamma) = \frac{\delta \cos(\phi_2) \sin(\theta)}{1 - \alpha}$ , where  $\alpha = \frac{R_1}{R_2}$ .

Hinge shear takes place in  $xoz$  plane. To determine the effect of  $q_2$  pressure on the angular stiffness of swivel joint, it is necessary to calculate the moment  $M(q_2)$  in  $xoz$  plane as the results of pressure  $q_2$ , acting on the lateral surface of the deformed elastomeric layer at  $\varphi = \varphi_2$ :

$$M(q_2) = - \iint_{F^*} (q_x r \cos \phi_2 + q_z r \cos \theta \sin \phi_2) dF^*.$$

As soon as  $q_x = q_u \cos \theta + q_v \sin \theta$ ,  $q_z = q_u \sin \theta + q_v \cos \theta$ ,  $dF^* = \frac{r \sin \phi_2 dr d\theta}{\cos \beta \cos \gamma}$ ,

$$M(q_2) = -q_2 \frac{\delta R_2^3}{3} (1 + \alpha + \alpha^2) \cos^2 \phi_2 \sin \phi_2 \int_0^{2\pi} \left( (\sin^2 \theta) \sqrt{1 + \frac{\delta^2 \cos^2 \theta}{(1 - \alpha)^2}} \right) d\theta. \quad (3)$$

The integral (3) is not taken by elementary functions. Let us denote:

$$a^2 = \frac{\delta^2}{(1 - \alpha)^2 \left[ 1 + \frac{\delta^2}{(1 - \alpha)^2} \right]}$$

For the sufficiently thin elastomeric layers  $a^2 \sin^2 \theta < 1$ . Expanding the integrand in (3) into series relatively to  $a^2 \sin^2 \theta$  and leaving the first three members, after integration we obtain:

$$\begin{aligned} I &= \int_0^{2\pi} \left( (\sin^2 \theta) \sqrt{1 + \frac{\delta^2 \cos^2 \theta}{(1 - \alpha)^2}} \right) d\theta \\ &= 4 \sqrt{1 + \frac{\delta^2}{(1 - \alpha)^2}} \cdot \int_0^{\pi/4} \sin^2 \theta \left( 1 - \frac{1}{2} a^2 \sin^2 \theta - \frac{1}{8} a^4 \sin^4 \theta \right) d\theta \\ &= \pi \sqrt{1 + \frac{\delta^2}{(1 - \alpha)^2}} \left( 1 - \frac{3}{8} a^2 - \frac{5}{64} a^4 \right). \end{aligned} \quad (4)$$

From the equations (3) and (4):

$$M(q_2) = -\pi\delta R_2^3 q_2 k_2(\delta),$$

$$k_2(\delta) = \frac{1}{3}(1 + \alpha + \alpha^2) \sqrt{1 + \frac{\delta^2}{(1 - \alpha)^2} \left(1 - \frac{3}{8} \alpha^2 - \frac{5}{64} \alpha^4\right)} \cos^2 \phi_2 \sin \phi_2. \quad (5)$$

Total angular hinge stiffness under the simultaneous action of hinge moment  $M_h$  and pressure  $q_2$  is defined from the condition of movable metallic layer equilibrium at  $r = R_2$  in the plane of the hinge bending  $xoz$ :  $M_h - M(q_2) = \delta\pi GR_2^3 k$ . From this expression the value of hinge moment  $M_h$ , which provides a desired angle  $\delta$  of hinge rotation at presence of the pressure  $q_2$ , is defined:

$$M_h = \delta\pi GR_2^3 k + M(q_2) = \delta\pi R_2^3 (kG + k_2(\delta)q_2).$$

If the  $q_1$  pressure on the side surface of an elastomeric layer  $\varphi = \varphi_1$  is imposed, considering the scheme discussed above for  $q_2$  (at  $\varphi = \varphi_2$ ) and taking into consideration the opposite signs of the corresponding direction cosine to the surface  $\varphi = \varphi_1$ , we obtain the equation for the moment  $M_h$ , ensuring predetermined hinge rotation angle  $\delta$ :

$$M(q_1) = \pi\delta R_2^3 q_1 k_1(\delta),$$

$$k_1(\delta) = \frac{1}{3}(1 + \alpha + \alpha^2) \sqrt{1 + \frac{\delta^2}{(1 - \alpha)^2} \left(1 - \frac{3}{8} \alpha^2 - \frac{5}{64} \alpha^4\right)} \cos^2 \phi_1 \sin \phi_1. \quad (6)$$

Consequently, the imposed pressure  $q_2$  leads to increase of control hinge moment for a given rotation angle  $\delta$  providing, i.e. to "mitigation" of angle stiffness; the imposed pressure  $q_1$  at  $\varphi = \varphi_1$  leads to decrease of control joint-hinge moment for a desired rotation angle providing, i.e. "toughen" the angular stiffness. Finally, under the simultaneous action of the pressure  $q_1$  and  $q_2$  on both sides of the elastomeric layer the control joint-hinge moment  $M_h$  is equal to:

$$M_h = \delta\pi R_2^3 [kG + k_2(\delta)q_2 - k_1(\delta)q_1]. \quad (7)$$

## 2.2. Calculation of stiffness characteristics of the package joint-hinge

For estimation of the stiffness characteristics of the packaged-joint consisting of  $N$  elastomeric layers, all of the above calculations are repeated. Deriving of the corresponding equations for each elastomeric layer one should keep in mind, that in a multilayer joint the moments caused by pressure  $q_1$  and  $q_2$  are transferred (and accumulated) from layer to layer – from mobile top layer of the packaged hinge to fixed lower layer. The introduced denotements for arbitrary  $i$ -th layer are:

$$\pi GR_{2i}^3 k^i = k_i, \quad \pi g_1 R_{2i}^3 k_1^i(\delta_i) = k_{i1}, \quad \pi q_2 R_{2i}^3 k_2^i(\delta_i) = k_{i2}. \quad (8)$$

Then from expressions (7) and (8) for total packaged-joint we get a system of equations:

$$\text{for the } N\text{-th upper mobile elastomeric layer: } \delta_N = \frac{M_h}{k_N}, \quad (9a)$$

$$\text{for the } i\text{-th layer: } \delta_i = \left( M_h + \delta_n(k_{N1} - k_{N2}) + \dots + \delta_{i-1}(k_{(i-1)1} - k_{(i-1)2}) \right) \frac{1}{k_i}, \quad (9b)$$

$$\text{for the first (lower fixed) layer: } \delta_1 = \left( M_h + \sum_{j=1}^N \delta_j(k_{j1} - k_{j2}) \right) \frac{1}{k_1}, \quad (9c)$$

$$\text{for the total packaged - joint } \delta = \delta_N + \delta_{N-1} + \delta_{N-2} + \dots + \delta_i + \dots + \delta_1. \quad (9d)$$

System (9) has  $N + 1$  equations and  $N + 1$  desired angular values  $\delta, \delta_1, \delta_2, \dots, \delta_i, \delta_N$  and allows to find: angles of rotation  $\delta_i$  separately for each layer and total angle of packaged joint-hinge  $\delta$  for a given  $M_h, q_1$  and  $q_2$ ; hinge moment  $M_h$ , which provides the desired behavior of package – hinge for the given rotating angle  $\delta$  and pressure  $q_1$  and  $q_2$  value; pressure value  $q_1$  and  $q_2$ , providing the desired behavior mode when hinge moment  $M_h$  and rotation angle  $\delta$  of total hinge package are given.

Introducing the concept of the "middle" layer, from the system (9) for the sufficiently thin elastomeric layers the approximate, but sufficiently accurate analytical expression may be derived for the calculation of the angular packaged-joint stiffness for different combinations of values of  $M_h, q_1, q_2, \delta$ . For this purpose let us assume that all the elastomeric layers work as one "middle" layer, separated in the midsection of pack, the geometry of which is denoted as:  $R_{1m}, R_{2m}, \varphi_{1m}, \varphi_{2m}, k_m, k_{1m}, k_{2m}, \delta_m = \delta/N$  (where  $N$  – the number of elastomeric layers in packaged joint). In this case, from the system (9) for the total angle of rotation of the packaged joint we have:

$$\delta_m = \frac{M_h}{k_m} + \delta_m(k_{m1} - k_{m2}) \frac{N + (N - 1) + \dots + 1}{k_m N}. \quad (10)$$

From the equations (8) - (10) the connection of control hinge moment  $M_h$  with angle of rotation of all package hinge  $\delta$  and pressure  $q_1$  and  $q_2$  is derived:

$$M = \pi \delta R_{2m}^3 \left[ Gk_m + q_2 k_{2m} (\delta_m) \frac{N + 1}{2} - q_1 k_{1m} (\delta_m) \frac{N + 1}{2} \right]. \quad (11)$$

Here we consider small angles of joint rotation  $\delta$ , which provide hinge deformation (angles  $\beta$ ), for which the linear relationships between the angular deformation and shear stresses in the material of the elastomeric layers are still valid. For shear and torsion it is valid for deformation up to 40-50 %. This condition imposes a limit on the number of layers of elastomeric and at a given angle  $\delta$  and a given elastomeric layer geometry.

Let us consider separately the metallic and elastomeric layers. For small angles of rotation  $\delta$  on the surface of the metal layer  $R_{1m} < r < R_{2m}$ , and  $\varphi = \varphi_2$ , we calculate the moment about axis  $y$ :

$$M_y^{met} = - \int_0^{2\pi} \int_{R_{1m}}^{R_{2m}} (xZ_1 + zY_1) r dr d\theta, \quad (12)$$

where  $x = r \sin \phi \cos \theta, z = r \cos \phi$ , the intensity of surface force on the metal layer for surface  $\varphi = \varphi_2$  as the results of pressure  $q_2$  is equal:

$$\begin{aligned} q_r|_{\phi=\varphi_2} &= 0, \quad q_\theta|_{\phi=\varphi_2} = 0, \quad q_\phi|_{\phi=\varphi_2} = -q_2, \\ Z_x|_{\phi=\varphi_2} &= q_r \cos \phi - q_\phi \sin \phi = q_2 \sin \phi_2, \\ Y_x|_{\phi=\varphi_2} &= (q_r \cos \phi + q_\phi \sin \phi) \cos \theta + q_\theta \cos \theta = -q_2 \cos \phi_2 \cos \theta. \end{aligned}$$

Then the equation (12) may be rewritten:

$$M_y^{met} = - \int_0^{2\pi} \int_{R_{1m}}^{R_{2m}} (q_2 \sin \phi_2 r \cos \phi_2 \cos \theta - q_2 \cos \phi_2 \cos \theta r \cos \phi_2) r dr d\theta = 0. \quad (13)$$

As it follows from (13), the presence of pressure does not influence joint shear provided that

the metal layer is completely rigid (not deformable). It should be noted that for large angles of rotation  $\delta$ , which lead to an essentially nonlinear angular deformities in the elastomeric layer packet hinge, this influence is not zero. This is due to the fact that surface loads  $q_r, q_\phi, q_\theta$  on the metal layer at  $\varphi = \varphi_2$  do not change since this metallic layer is not deformed, but arms vary:  $x = r\sin\phi_2\cos\theta + r\delta\cos\phi_2\sin\theta, z = r\cos\phi + r\delta\cos\theta$ .

### 3. Numerical examples and experiment results

The example of theoretical calculation and results of experiment are given below for the multilayer joint-hinge with geometry:  $R_{11} = 390$  mm,  $R_{2N} = 417$  mm,  $h_{elast} = 3$  mm,  $h_{met} = 3$  mm,  $N = 5, \varphi_1 = 47^\circ, \varphi_2 = 57^\circ, \delta = 6^\circ$  and shear modulus of the elastomeric layers  $G = 460$  kPa (Fig. 6).

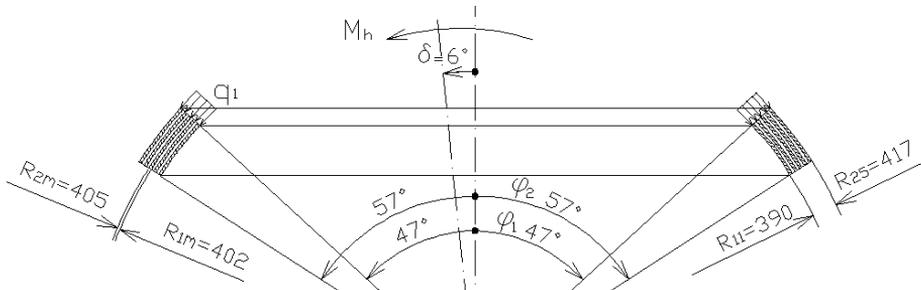
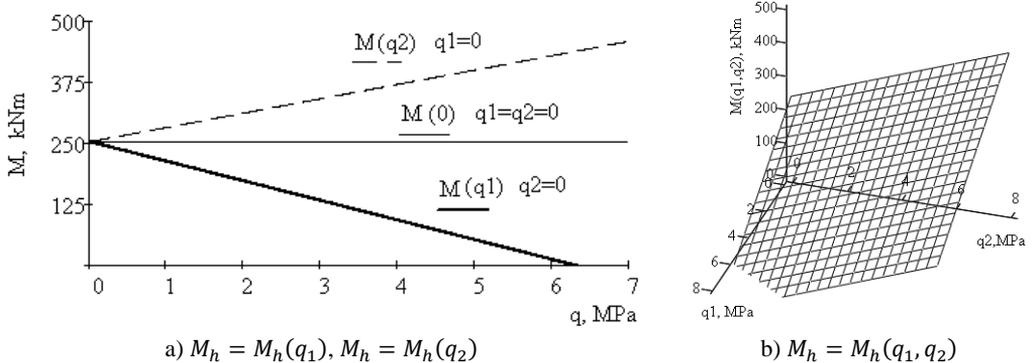


Fig. 6. Testing packaged joint-hinge geometry

Theoretical calculations were performed in accordance with analytical expression (11), where the moment dependence on pressures  $q_1$  and  $q_2$  is linear due to small rotation angle and linear dependence between stress and strain. In Fig. 7 plots of joint-hinge moments  $M_h$  dependence on hydrostatic side pressure  $q_1$  and  $q_2$  are shown for the simultaneous action of hinge moment  $M_h$  and the pressure  $q_1$  or  $q_2$  (separately (Fig. 7a) and together (Fig. 7b)). In Fig. 8 plots of joint-hinge moments  $M_h$  dependence on rotation angle  $\delta$  and the hydrostatic side pressure  $q_1$  and  $q_2$  separately are presented.



a)  $M_h = M_h(q_1), M_h = M_h(q_2)$       b)  $M_h = M_h(q_1, q_2)$   
 Fig. 7. Plots of joint - hinge moment  $M_h$  (kNm) dependence on the side pressures  $q_1$  and  $q_2$  (MPa) for  $\delta = 6^\circ$

Experimental investigation of the stiffness characteristics of this joint were performed at the Moscow Institute of Thermal Technology. Testing was fulfilled under static rotation of joint with fixation of angle (Fig. 9). Because of the complexity of the experiment, only the pressure  $q_1$  was applied to the inner surface side and integral characteristics "moment-rotation" were measured. In Fig. 10 plots of experimental and theoretical joint stiffness characteristics



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