

GENERALIZED SMOOTHING SPLINES IN CONVEX SETS ¹

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The talk deals with the following conditional minimization problem

$$\|Tx\|^2 + \|R(Ax - v)\|^2 \longrightarrow \min_{x \in B^{-1}(C)}$$

for linear continuous operators $T : X \rightarrow Y$, $A : X \rightarrow \mathbb{R}^n$ and $B : X \rightarrow \mathbb{R}^m$ in Hilbert spaces X and Y , given diagonal matrix $R = \text{diag}(\sqrt{\rho_i})_{i=1, \dots, n}$ with parameters $\rho_i \geq 0$, $i = 1, \dots, n$, closed convex set $C \subset \mathbb{R}^m$ and vector $v \in \mathbb{R}^n$. This problem generalizes the smoothing problem with weights and the problem on splines in convex sets (see e.g. [1]).

We investigate the existence and obtain a characterization of solutions of this problem in dependence on whether all functionals of operators A and B are linear independent or not. In particular, we consider the cases:

$$\begin{array}{ll} 1) & A = MB, \\ 2) & B = MA, \end{array} \qquad \begin{array}{ll} 3) & A_1 = MB_1, \\ 4) & B_1 = MA_1, \end{array}$$

where M is a matrix with the corresponding size, $A = (A_1, A_2)$ and $B = (B_1, B_2)$.

Special results are proved for $C = \Pi_{i=1}^m [a_i, b_i] \subset \mathbb{R}^m$, not excluding the case when $a_i = b_i$ for some i , which corresponds to interpolating conditions.

This study is closely related to our previous works [2], [3].

REFERENCES

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¹**Acknowledgement** This work was partially supported by the project 2009/0223/1DP/1.1.1.2.0/09/APIA/VIAA/008 of the European Social Fund.