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DIFFUSION APPROXIMATION FOR POISSON TYPE MODEL OF EXCESS RETURNS

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The most popular model for the log of cumulative excess returns Y_t on a portfolio [1] is piecewise constant random process with uniform partition time $\{t_k, k = 0, 1, 2, ...\}$ of the length $t_{k+1} - t_k = h$ and jumps given by difference equation

$$Y_{t_{k+1}} = Y_{t_k} + hc\sigma^2_{t_{k+1}} + \sqrt{h}\sigma_{t_{k+1}} Z_{t_{k+1}} \tag{1}$$

where market uncertainty $\{Z_t\}$ is i.i.d. $N(0, 1)$ sequence, and $\{\sigma_{t_k}\}$ is volatility GARCH(1,1) process

$$\sigma^2_{t_{k+1}} = \sigma^2_{t_k} + h(\omega - \theta\sigma^2_{t_k}) + \frac{Z^2_{t_{k+1}} - 1}{\sqrt{2}}\sigma^2_{t_k}\alpha\sqrt{h} \tag{2}$$

Taking the length of time intervals between observations h more and more finely the author [1] derives the stochastic approximation for $\{Y_t, \sigma_t\}$ as the system of stochastic differential Ito equations. In a difference of the paper [1] we have assumed that

- the market uncertainty $\{Z_t\}$ is the compound Poisson process with switching times $\{t_k\}$ and embedded Markov chain given by difference equation $Z_{t_{k+1}} = \rho Z_{t_k} + \sqrt{1-\rho^2}\xi_{t_{k+1}}$ where $\{\xi_{t_k}\}$ is i.i.d. $N(0, 1)$ sequence;
- two dimensional peace-wise constant process for log of cumulative excess returns Y_t and volatility σ^2 is also compound Poisson process with embedded Markov chain given by equations (1)-(2);
- length of time intervals between jumps is given by formula $\mathbb{P}\{t_{k+1} - t_k > t\} = \exp\left\{-\frac{1}{h}t\right\}$.

This model we may interpret as the impulse dynamical system with fast Markov switching and for stochastic approximation apply limit theorem derived in the paper [2]. The assumption $\rho \neq 0$ leads to more complicated excess returns-volatility relation

$$dY(t) = c\sigma^2(t)dt + \sigma(t)\sqrt{\frac{1+\rho}{1-\rho}}dw_1(t), \tag{3}$$

$$d\sigma^2(t) = \left(\omega - \sigma^2(t)\left(\theta - \alpha^2\frac{\rho^2}{1-\rho^2}\right)\right)dt + \alpha\sigma^2(t)\sqrt{\frac{1+\rho^2}{1-\rho^2}}dw_2(t) \tag{4}$$

which will be duscussed in our presentation.

REFERENCES

[1] D.B. Nelson, ARCH models as diffusion approximation.*J. of Econometrics*, **45**, 1990, 7–38.

[2] Ye. Tsarkov (J.Carkovs), Asymptotic methods for stability analysis of Markov impulse dynamical systems, *Nonlinear dynamics and system theory*, **1**, No. 2, 2002,103 – 115.