Portfolio Management Modified by GARCH-Type Models and Moving Average Correlation in the Global Currency Market

Konstantins KOZLOVSKIS, Jurijs LACIS
Faculty of Engineering Economics and Management, Riga Technical University
1/7 Meza Str., Riga, LV-1007, Latvia

ABSTRACT

The development of information and financial technologies offers a wide spectrum of opportunities in Internet trading and its automation in the global financial markets. An investor can use different approaches in his trading. Nowadays, there are lots of information sources about how to trade. Besides, available modern software can help in mathematical model building including econometric models providing wider their practical application. In this paper some GARCH-type models are used as a modification in risk and return evaluation in Markowitz portfolio building approach. Consequently a bibliography of research papers on GARCH-type models and other econometric ones during 30 years since they were introduced by Engle in 1982 would run to hundreds of pages, and it is hard to find the work that is most relevant to practitioners and especially in the field of speculative trading.

In this paper the authors use widely known Markowitz portfolio and modify it by GARCH-type models for operating expected risks and returns in the portfolio traded in the global currency market for speculative purposes. Also moving average correlation is used in the portfolio to predict interdependences between two currency pairs. Such modifications are compared with naive portfolio diversification.

Keywords: global currency market, portfolio management, GARCH-type model, n-period moving correlation

1. INTRODUCTION

Trading opportunities in financial markets are constantly broadened due to development of different factors. Extension of financial services opens new potentialities of Internet trading merging several financial markets under one trading platform. Financial innovations offer new and more interesting financial instruments and products, which allow investors to get round difficulties related to some limitations in classical financial markets, for instance, trading short positions in stock markets (solved by CFD contracts) or new gold trading accounts in which all trades are recalculated in terms of gold (it helps to protect trading accounts against inflation and unpredictable negative fluctuations). Modern information technologies and solutions in the field of software-to-software interface are capable to integrate different information systems with trading platform providing more qualitative analysis of financial data by other software.

Modern trading platforms and achievements in information technologies give unique possibility to automate trading financial instruments. Free competition between brokerage firms leads to increased leverage and splitting trading lots. If a classical trading lot is equal to 1.0, today it is possible to trade smaller lots of 0.1 and even 0.01, for instance, in the global currency market. Of course, this splitting decreases potential profit and risks, but on the other hand, it allows managing complex portfolios with lesser capital.

The Markowitz portfolio is a portfolio, which takes into account the expected return and risk of assets as well as interdependencies between chosen assets through covariation decomposed in correlation and standard deviations. The presence of such variables gives a good opportunity to apply different mathematical approaches to modify a portfolio built by Markowitz approach. In fact, not all mathematical modifications can be easy applied in trading. For example, modifications with copula [11] are effective too, but its implementation is too labor-intensive [10].

In this paper the authors used some econometric models like AR (autoregression) models, GARCH-type models, n-period moving average correlation in portfolio trading in the global currency market implementing automated trading for speculative purposes.

2. METODOLOGY

The main problem is automated trading in the financial markets is programming opportunities and trading strategy complexity. If a trading strategy is too complex, it cannot be programmed or programming can be too complicated to be realized. On the other hand, there should be a possibility to integrate programmed module in trading software. For example, trading software called Metatrader has such possibility. Analyzing opportunities of portfolio trading in the global currency market, the

authors apply relatively simple econometric models to diversify risks of currency pairs using portfolio management. The authors use the 3 types of currency portfolios mentioned below.

2.1. Simple portfolio with equal weights

Despite the sophisticated theoretical models developedin the last 50 years and the advances in methods for estimating the parametersof these models, investors continue to use such simple allocation rules forallocating their wealth across assets (for example, [4]). In this case of currency portfolio capital is distributed between financial instruments in equal proportions calculated as 1/N, where N is number of financial instruments. This portfolio strategy is able to show the forecasting power of used models applied for forecasting expected return without taking into account any dependences between currency pairs. Autoregressive models of time series are representations of a time series by functions of its own lags. The authors use the following autoregression model to predict future returns:

$$p_t = \sum_j a_j p_{t-j} \tag{1}$$

The authors chose the autoregressive process because it is often used to model and predict various types of natural phenomena. Its application is widely distributed in financial data analysis and it is easy to implement in programming software.

2.2. The Markowitz portfolio modified by GARCH-type models

If a traditional widely known Markowitz portfolio can be expressed as (1),

$$\begin{cases} \mu_p = \sum_j x_j \mu_j \\ \sigma_p^2 = \sum_j \sum_k x_j x_k \rho_{jk} \sigma_j \sigma_k \\ \sum_j x_j = 1 \end{cases} \tag{2}$$

the modified by GARCH-type model in terms of this work can be written as (3).

$$\begin{cases} \mu_p = \sum_j^N x_j \mu_j^{AR(n)} = \hat{\mu}_p \\ \sigma_p^2 = \sum_j^N \sum_k^N x_j x_k \rho_{jk} \sigma_j^{GARCH} \sigma_k^{GARCH} \rightarrow min \\ \sum_j^N x_j = 1 \end{cases} \tag{3}$$

where

μ_p – theexpected portfolio return;

$\hat{\mu}_p$ – theexpected investor's return;

$\mu_j^{AR(n)}$ – theexpected j-th currency pair return estimated by autoregression model AR(n);

σ_p^2 – theexpected portfolio risk;

ρ_{jk} – thecorrelation between j-th and k-th currency pairs;

σ_j^{GARCH} – theexpected currency pair risk estimated by GARCH-type model;

x–theweights.

Markowitz postulates that an investor should maximize expected portfolio return (μ_p) while minimizing portfolio variance of return (σ_p^2) [14]. In the original Markowitz's work the expected return of each asset is a constant for chosen period and based on the previous return data. The same is with the expected risk of an asset. Typical behavior of any financial instruments is shown in the Figure 1. Actually, such dynamics can be described quite well with ARCH/GARCH-type models.

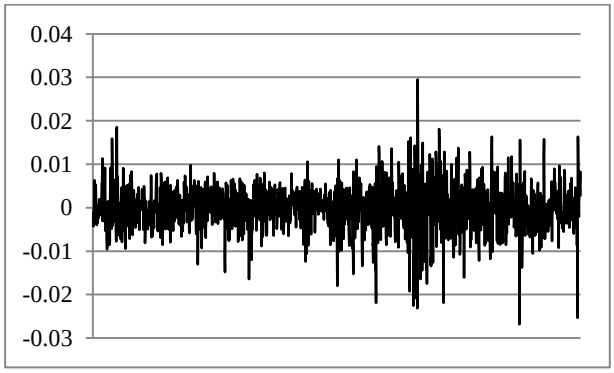

Figure 1 Typical example of security's return (USDJPY)

In 1982 Engle introduced a new class of models called ARCH [6]. An important property of ARCH models is their ability to capture volatility clustering in financial data, i.e. the tendency for large and small swings in returns to be followed by larger and small swings of random direction. Generalized ARCH(p,q) model was first suggested by Bollerslev [5]. ARCH/GARCH effects is widely documented in the financial literature, for instance: for index returns [1], for futures markets[16], for individual stock returns [8], for bond yields and returns of US Treasury Bills [17], etc. In nowadays there are lots of widely used ARCH-type models, the most known of which are EGARCH (introduced by Nelson) [15], ARCH-M (introduced by Engle, Lilien and Robins) [9], TGARCH (introduced by Zakoian) [18], etc. The authors of this paper used the simplest models from the estimated ones.

In the Markowitz portfolio modified by GARCH-type model the expected return of a currency pair can be calculated by conditional mean expressed by any

model(in our case by AR(n)) and the expected portfolio risk can be evaluated by GARCH-type models. Thus, the currency pair's risk will be dependent on news about volatility from the previous period, measured as the lag of the squared residual from the mean equation (the ARCH term) and last period's forecast variance (the GARCH term). Such modification could evaluate expected risks more careful and build more suitable portfolio structure.

2.3. Markowitz's portfolio modified by GARCH-type models and moving average correlation

In fact, interconnections between financial instruments are not fixed practically in any financial market. The same situation is observed in the global currency market. Too many factors have impact on currency pairs' movements. In long-term perspective the correlation coefficient between two currency pairs is more stable than in short-term perspective [11]. The typical situation in changes in correlation between two currency pairs is shown in the Figure 2 and Figure 3. As it shown the correlation coefficient is widely distributed within interval [-1;1]. The main idea is to use this dynamics of correlation forcing the Markowitz's portfolio.

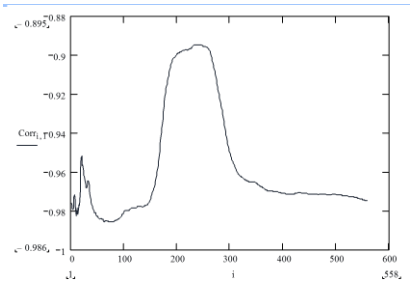

Figure 2 Correlation coefficient in long-term perspective

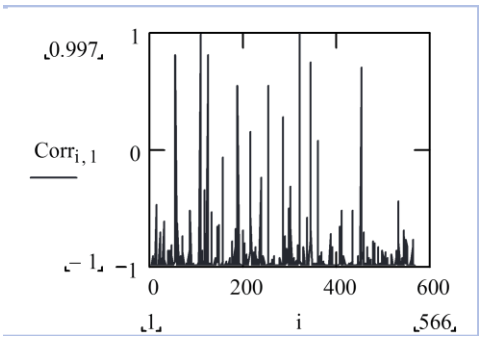

Figure 3 Correlation coefficient in short-term perspective

In 2000 Engle introduced dynamic conditional correlation as a simple class of multivariate GARCH models [7]. To simplify the approach the authors used a simple moving average correlation coefficient which can be calculated as

$$\rho^{MA}{}_{ij,t} = \frac{\sum_{k=1}^{n} \rho_{ij,t-1-k}}{n}, \quad (4)$$

wheren is a period of moving average.

Thus, the most complicated portfolio can be written as

$$\begin{cases} \mu_p = \sum_{j}^{N} x_j \mu_j^{AR(n)} = \hat{\mu}_p \\ \sigma_p^2 = \sum_{j}^{N} \sum_{k}^{N} x_j x_k \rho_{jk}^{MA} \sigma_j^{GARCH} \sigma_k^{GARCH} \rightarrow min \\ \sum_{j}^{N} x_j = 1 \end{cases} \quad (5)$$

Assuming that in the next period the correlation between two currency pairs is equal to average value based on previous periods, such portfolio structure could be able to take into account both volatile variance (GARCH-type component in the portfolio) of currency pairs and their constantly changing interdependence (moving average correlation component in the portfolio).

3. EMPIRICAL RESULTS

The authors analyzed the most liquid currency pairs in the global currency market. According to the statistical information from the Bank for International Settlements, the most tradable currency pairs are USDJPY, USDCHF, USDCAD, GBPUSD, EURUSD, EURJPY, EURGBP, EURCHF, AUDUSD. The average prices were used for each instrument represented by daily data. The time period was chosen from January 1, 2000 to April 1, 2011. Any models used in the paper were estimated on the data of 2004 (total 263 observations) and implemented on the data from 2005 (total 1621 observations).

3.1. Portfolio management with naive diversification

Analyzing the average daily data of 9 currency pairs, the authors found that forecasted average prices can be very simply described by autoregressive process. The Table 1 represents evaluated autoregressions for analyzed currency pairs.

Table 1 Statistics of AR(n) models

No	Currency pair	Model	R-squared	DW stat
1	USDJPY	AR(3)	0,9759	1,9203
2	USDCHF	AR(3)	0,9835	1,9393
3	USDCAD	AR(4)	0,9940	1,9804
4	GBPUSD	AR(4)	0,9705	1,9722
5	EURUSD	AR(3)	0,9854	1,9391
6	EURJPY	AR(3)	0,9615	1,9458
7	EURGBP	AR(4)	0,9841	2,0104
8	EURCHF	AR(3)	0,9861	1,9464
9	AUDUSD	AR(4)	0,9850	1,9625

In the Table 1 it is shown that autoregression models describe initial data of 2004 very well. Using these

models for forecasting average prices and realizing the corresponding strategy (if forecasted average price in next period is greater than in the previous then long position is open, otherwise, short), return on each currency pair without their weights in portfolio and with them were calculated (see Table 2).

Table 2 Profits using naive portfolio diversification

No	Currency	Profit, pips	
		w/o weights	with weights 0.11
1	USDJPY	-1522	-168
2	USDCHF	1830	201
3	USDCAD	-286	-31
4	GBPUSD	373	41
5	EURUSD	4588	505
6	EURJPY	-830	-91
7	EURGBP	2020	222
8	EURCHF	1511	166
9	AUDUSD	-3490	-384
10	Portfolio	4192	461

Despite of good forecasting power total result is not so good. There are both results: negative and positive. Total result is positive but it cannot be considered as a predicted results based on econometric models. The Figure 4 shows, that the dynamics of cumulative portfolio return resembles behavior of any financial asset, that is total result is dependent on present situation in market and trends. On the hand, the graph eliminates lots of figures of technical analysis which can be interpreted very simply. Thus, cumulative portfolio return can be used as a benchmark for 9 currency pairs. Trading in the trends and using favorable fluctuations in such benchmark, it could be possible to make profit.

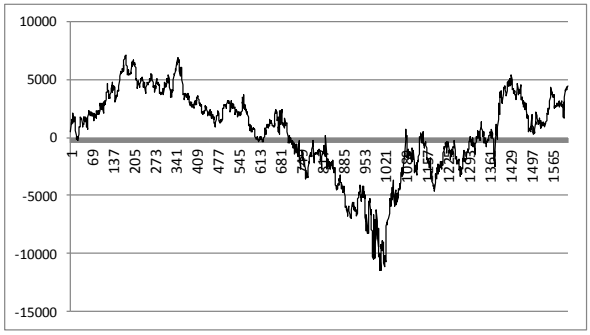

Figure 4 Cumulative portfolio return, pips

Bad implementation of autoregression models can be explained by forecasting average prices. In previous period an investor cannot open position by average price because it is not known yet while trading session is open. Probably, it is worth to forecast average prices based on the lagged values from the past, for instance, by such autoregression model:

$$p_t = \sum_j a_j p_{t-2} \qquad (6)$$

Forecasting average prices for periods t+1 and t+2 using one autoregression model or combined two models, investor could open short or long position in next period by forecasted average price taking into account where the next average price in time period t+2 is forecasted to be.

3.2. Portfolio management forced by GARCH-type models

Implementation of evaluated GARCH-type models in forecasting the variance of currency pair in Markowitz's portfolio showed impossibility to diversify portfolio as it was planned. Using 9 currency pairs with similar expected returns and risks leads to equally weighted portfolio structure. It means that the weights for each currency pair are equal to 0.11 (1/9). To avoid this effect the authors grouped currency pairs in 3 portfolios with 2 currency pairs in each.

Estimated GARCH-type models and profits using the strategy of the second portfolio are shown in the Table 3. For this portfolio the expected investor's return was calculated as 0.1% (average daily return on analyzed currency pairs in 2004).

Table 3 The comparison of portfolio modified by GARCH

No	Currency	GARCH-type model	Profit, pips	
			w/o weights	with weights
1	USDJPY	AR(3)-EGARCH(1,1)	-1115	-109
	USDCHF	AR(3)-IGARCH(1,1)	2097	1695
	USDCAD	AR(4)-IGARCH(1,1)	-422	-716
2	GBPUSD	AR(4)-IGARCH(1,1)	1197	-1817
	EURUSD	AR(3)-IGARCH(1,1)	3467	3660
	EURJPY	AR(3)-EGARCH(0,1)	-333	1590
3	EURGBP	AR(4)-EGARCH(1,1)	2085	165
	EURCHF	AR(3)-GARCH(1,1)	1556	2587
	AUDUSD	AR(4)-IGARCH(1,1)	-3577	-1414
	Total		4956	5312

Analyzing the dynamics of cumulative portfolio return (see Figure 5) that total behavior is improved. There are no large drawdowns as in the first portfolio.

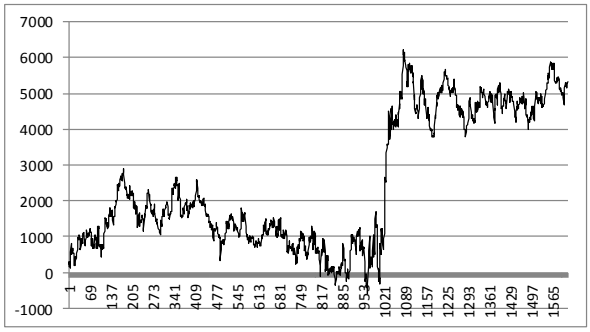

Figure 5 Cumulative portfolio return applying GARCH-type models

Implementation of GARCH-type models in the Markowitz portfolio improved risk management significantly and profitability non-significantly. As it is shown, despite of improvements in the Markowitz portfolio, there is still no possibility for speculative return.

3.3. Portfolio management forced by GARCH-type models and moving correlation

For this portfolio the authors also used the expected investor's return equal to 0.1% and the same GARCH-type models as in the previous portfolio. 3-, 5- and 10-period moving average correlations were calculated between each currency pairs. The total result of applying GARCH-type models and changing-over-time correlation is shown in the Table 4.

Table 4 The impact of moving average correlation on total results

No	Currency	Profit, pips (with weights and using n-period moving average correlation)		
		3-period	5-period	10-period
1	USDJPY	-191	-357	-1115
	USDCHF	1253	1202	2097
	USDCAD	-603	-997	-422
2	GBPUSD	1198	1198	1198
	EURUSD	3468	3468	3468
	EURJPY	-333	-333	-333
3	EURGBP	2085	2085	2085
	EURCHF	1556	1556	1556
	AUDUSD	-3577	-3577	-3577
	Total	4856	4245	4957

Applying GARCH-type models and 3-, 5- and 10-period moving average correlation in the Markowitz portfolio, total result is similar to the previous results. Common dynamics of cumulative portfolio return (see Figure 6) actually repeats the dynamics of the previous portfolio modified only by GARCH-type models.

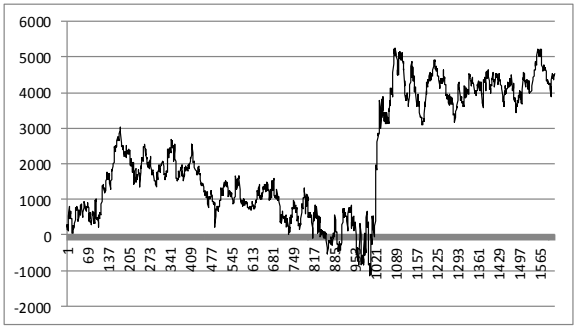

Figure 6 Dynamics of cumulative portfolio return applying GARCH-type models and 10-period moving average correlation

Comparing drawdowns of two portfolios, it seems that the best result belongs to the second portfolio. Using moving correlation in calculations it was noted that values of this correlation is more volatile in USD pairs. In other currency pairs correlation coefficient is more stable or does not have significant impacts on portfolio weights. In any case, in USD pairs the impact of moving average correlation was significant. Probably, this effect can be used in some more specific strategies.

4. CONCLUSIONS

1. Portfolio management methods based on interdependences between currency pairs are more effective than so called naive diversification.
2. Implementation of GARCH-type models in Markowitz's portfolio improves its quality. GARCH-type models can capture clusters in currency pairs' returns and impact on portfolio structure trying to regulate risks (conditional variance) on impact from previous periods. Such modification can decrease total drawdown in cumulative portfolio return.
3. Moving average correlation shows some interesting facts on interdependences of currency pairs. Significant impact is not discovered, but this effect, possibly, could be used in high frequency data, for instance, in 4-hour, 1-hour or even 15-minutes data.
4. Comparing all results, the authors can conclude that despite on positive impact of implemented modifications in the Markowitz portfolio there are no reasons to use this methodology in automated Internet trading for speculative purposes based on average daily data. Probably, some corrections in limiting losses should be made.

5. REFERENCES

[1] V. Akgiray, (1989) Conditional heteroscedasticity in time series of stock returns: evidence and forecasts, **Journal of Business**, 62, 55-80.

[2] C. Alexander, **Market Models**, John Wiley & Sons, 2003.

[3] Bank for International Settlements, December, 2010, Triennial Central Bank Survey/ Report on global Foreign exchange market activity in 2010 http://www.bis.org/publ/rpfxf10t.pdf.

[4] S. Benartzi, R. Thaler. (2001). Naïve Diversification Strategies in Defined Contribution Saving Plans. **American Economic Review** 91:79–98.

[5] T. Bollerslev, (1986) Generalised autoregressive conditional heteroskedasticy. **Journal of Econometrics**, 31, 307-327.

[6] R.F. Engle, (1982) Autoregressive conditional heteroscedasticity with estimates of variance of UK inflation, **Econometrica**, 50, 987-1008.

[7] R.F. Engle, (2000) Dynamic conditional correlation – a simple class of multivariate GARCH models. PDF version retrieved fromhttp://weber.ucsd.edu/~mbacci/engle/391.pdf

[8] R.F. Engle, C. Mustafa, (1992) Implied ARCH models from option prices, **Journal of Econometrics**, 52, 289-311.

[9] R.F. Engle, D. Lilien, R. Robins, (1987) Estimation of time varying risk premiums in the term structure, **Econometrica**, 55, 391-408.

[10] V. Jansons, K. Kozlovskis, N. Lace, (2006). Portfolio Modeling Using the Theory of Copula in Latvian and American Equity Market. **Simulation In Wider Europe**, 19th European Conference on Modelling and Simulation ECMS 2005

[11] K. Kozlovskis, N. Lace, (2006) Challenges of Decision Making in the Latvian Equity Market // The 10th World Multi-Conference on Systemics, Cybernatics and Informatics (WMSCI 2006), 41-44.

[12] K. Kozlovskis, (2007) Moving Slope as a Long-Term Indicator in Foreign Exchange / K. Kozlovskis // International Scientific Dual-Conference Towards Knowledge Based Economy & Enterprise Management: Diagnostics, Strategy, Effectiveness". 12-13 April, Riga, Latvia. Conference Proceedings. – CD. – 6 pages.

[13] H.M. Markowitz, 1952. **Portfolio Selection**. Journal of Finance 7:77–91.

[14] H.M. Markowitz, (1991). "Foundations of Portfolio Theory", **The Journal of Finance**, Volume 46, Issue 2, January, 469 – 477.

[15] D.B. Nelson, (1991) Conditional heteroskedasticity in asset returns: A new approach. **Econometrica**, 59, 347-370.

[16] G.W. Schwert, P.J. Seguin, (1990) Heteroscedasticity in stock returns, **Journal of Finance**, 45, 1129-1155.

[17] A. Weiss, (1984) ARMA models with ARCH errors, **Journal of Time Series Analysis**, 5, 129-143.

[18] J.M. Zakoian, (1994) Threshold heteroscedastic models, **Journal of Economic Dynamics and Control**, 18(5), 931-955.