A metamodeling methodology has been proposed for postbuckling simulation of stiffened composite structures with integrated degradation scenarios. The presence of artificial damage between the outer skin and stiffeners has been simulated as softening of the material properties in predetermined regions of the structure. The proposed methodology for the fast design procedure of axially or torsionally loaded stiffened composite structures is based on response surface methodology (RSM) and design and analysis of computer experiments (DACE). Numerical analyses have been parametrically sampled by means of the ANSYS/LS-DYNA probabilistic design toolbox extracting the load-shortening response curves in the preselected domain of interest. These response curves have been simplified using piecewise linear approximation.
identifying the buckling and postbuckling stiffness ratios along with the values of the skin and the stiffener buckling loads. Three stiffened panel designs and a closed box structure with preselected damage scenarios have been elaborated and validated with the tests performed within the COCOMAT project. The resulting design procedure provides a time-effective design tool for preliminary study and for elaboration of the optimum design guidelines for composite stiffened structures with material degradation restraints.

**Keywords:** Metamodeling; postbuckling; stiffened structures; physical validation; structural degradation; design of computer experiments; fast design procedure.

1. Introduction

The design methodology for composite stiffened structures loaded into the postbuckling mode with onset of artificial damage is not yet fully explored. However, there is clear evidence that great potential exists for future increase in the effectiveness of stiffened composite structures by allowing the postbuckling of the skin. Nevertheless, the consequences of material degradation such as debonding between the skin and stiffeners during the service load should be considered to introduce a new generation of design practice. Elaboration of structural design guidelines considering buckling and postbuckling in laminated composites requires a comprehensive study within the design space of selected design variables including preselected damage scenarios. In spite of outstanding advances in computer capacity and speed, the enormous computational cost of complex, high precision scientific simulations makes it impractical to rely exclusively on conventional finite element (FE) simulation codes for the purpose of design optimization. A more appropriate strategy is to utilize approximation analysis models for optimization procedure when highly strained composite stiffened structure would be a design requirement. An alternative would be to approximate the response behavior of composite structures from a set of FE simulations and to build corresponding metamodels, which are more time-efficient in elaboration of the optimum design procedure.

2. Metamodeling Methodology

2.1. *Design of computer experiments and approximation techniques*

Multidisciplinary and multiobjective design and optimization of complex systems requires sophisticated and expensive-to-run computer analysis codes. Thus elaboration of a reliable but time-efficient approximation methodology would be crucial for structural engineering practice. As a good practice one could use mathematical approximations instead of full scale analyses, thus lowering the level of numerical optimization complexity. The metamodels, also referred to as surrogate models, are approximations constructed from response values extracted from FE simulations. A metamodelling methodology is based on the combined use of probabilistic structural analysis, sampling of computer experiments and approximations of the response functions. The sampling data sets may include both computer experiments from
numerical simulations and test data acquired in physical experiments.\(^7\) When FE analyses are used to determine stress/strain responses, the use of classical design of experiments, which requires confirming repeated runs, is not an effective approach. Instead, deterministic computer experiments, sampled according to the space-filling criteria, should be used as a basis for evaluation of parametric/nonparametric approximation functions. Currently there is a wide range of literature concerning different methods for DACE,\(^8\) which include many approaches to space-filling designs. It should be noted that the first space-filling design criterion\(^9\) for numerical experiments was proposed at Riga Technical University by Audze and Eglajs. While the accuracy of a metamodel is directly related to the approximation technique used and to the properties of the problem itself, the types of sampling approaches\(^9\) also have a direct influence on the performance of an approximated model. It is generally accepted that space-filling designs, such as the Latin hypercube designs, are preferable for the building of metamodels. A space-filling design of computer experiments optimized according to the mean squared error uniformity criteria was selected in order to achieve the best performance — minimal prediction error of metamodels.\(^10\)

The sampling of experiments may be generalized as a time-consuming procedure without prior definition of the required amount of sample points. Thus design of experiments which could be discontinued and resumed without worsening of the space-filling property would have advantages in engineering practice. Considering this, a sequential design strategy has been used, by arranging and adding new sample points to an already existing design of experiments according to a selected space-filling criterion, thus achieving a good balance of the space-filling quality in the whole design space and a quantitative improvement by added sample points.\(^10,11\) An advantage of the proposed approach is the fine sampling quality even before all experimental runs have been realized. Nonparametric approximations such as multivariate adaptive regression splines (MARSs)\(^12\) or radial basis functions (RBFs)\(^13\) are recognized as most precise for different orders of nonlinearity and problem scale response approximations, though by definition nonparametric approximations do not generate any tangible function. In the present study, the RBFs\(^13\) method uses a series of multiquadric basis functions that are symmetric and centered at each sampling point, and thus this method is most suitable for problems where data are only available at scattered points. On the contrary, the MARS\(^12\) method is based on nonparametric regression by constructing the relation from a set of coefficients and basis functions that are entirely “driven” from the regression data. MARS is particularly suitable for problems with higher input dimensions, where the curse of dimensionality would likely create problems for other techniques. The full-order polynomial approximations are commonly used in engineering practice. However, response parameters are often affected by only some of the input variables, and thus the irrelevant parameters become approximation noise that decreases prediction accuracy. It seems reasonable to use partial polynomials instead of full-order polynomial approximations, as they could be tailored adequately by relevance to the
input variables. There exists an approach to polynomial model building which does not assume a predefined set of basis functions — adaptive basis function construction. This approach allows one to generate polynomials of arbitrary complexity without the requirement to predefine any basis functions or to set the maximal degree of the polynomial. The described approximation techniques have been implemented in an approximation tool and elaborated for simulation of damaged composite stiffened structures.

2.2. Simplification of the load–displacement response

Initially the metamodeling methodology was elaborated for only stiffened panels under axial loading; however, this approach has now been extended for stiffened box structures under axial or torsion loading. It might be generalized that the simplification of the load–displacement curve in order to develop corresponding metamodels is based on numerically obtained load-shortening curves (Fig. 1), where the axial load $P$ ($P_1$ — skin buckling; $P_2$ — stiffener buckling; $P_3$ — collapse load), the stiffness $k$, and the axial shortening $u$ are functions of the design parameters. The simplified load-shortening curve is divided into four-part piecewise linear sections representing the prebuckling, skin and stiffener buckling, and finally the collapse region. Each section occupies a region where the structural stiffness is assumed to be constant and reaches the breakpoint between the two linear curves, which is close to the skin — stiffener buckling load values obtained experimentally. The minimization of the discrepancy criterion between two correlated regions allows one to determine the breakpoints in the load-shortening curves.

$$
\varepsilon_0 = \frac{1}{u_{\text{max}}} \int_0^{u_{\text{max}}} (P(u) - \tilde{P}(u))^2 du. \quad (2.2)
$$

In validation by natural experiments, the numerical postbuckling critical load is more conservative than that obtained in physical tests. Typical load-shortening

![Fig. 1. Typical load-shortening curve of a stiffened panel under compression (a) with corresponding piecewise approximation of load-shortening curves (b).](image-url)
curves of a stiffened panel with and without structural degradation are opposed to their corresponding piecewise approximation of load-shortening curves in Fig. 1. The structural response values \((k_1, P_1, u_1)\) prebuckling, \((k_2, P_2, u_2)\) skin postbuckling, and \((k_3, P_3, u_3)\) stiffener postbuckling region numerical values) have been extracted by the simplification approach proposed and given in detail in Refs. 6 and 7.

3. Sampling of Numerical and Physical Experiments

3.1. Structural configurations

A numerical study has been performed for panel designs with four and five stiffeners\(^{16–19}\) and for one stiffened box structure\(^{20,21}\) as shown in Fig. 2. All carbon-fiber-reinforced plastic stiffened panels\(^{16,17}\) are made out of IM7/8552, while the closed box design is made from 985-GT6-135UD/985-GF3-5H-100 CFRP.\(^{20,21}\) The loading edge and supporting edge of the panel\(^{16,17}\) and stiffened box\(^{20,21}\) structures are fully clamped,\(^{18,19}\) while simply supported boundary conditions have been met at the longitudinal edges of the panel designs (see Fig. 2).

A set of geometrical design variables (see Table 1), such as the length \(L\) of the composite stiffened structure, the curvature radius \(R\), the height \(h\) of stiffeners, and the stiffener spacing span length \(b\), have been used to elaborate the metamodels. Furthermore, damage criterion variables have been incorporated — degradation length and material degradation/softening ratio. The degradation extent ratio \(D\) multiplied by the panel length \(L\) (Fig. 2) represents the skin–stiffener junction debonding area, while the ratio \(D\%\) represents the structural degradation level by

![Fig. 2. Structural designs of stiffened panels and closed box used in elaboration of the metamodels.](image)

<table>
<thead>
<tr>
<th>Name</th>
<th>Notation</th>
<th>Lower bound</th>
<th>Upper bound</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>(L)</td>
<td>450</td>
<td>750</td>
<td>mm</td>
</tr>
<tr>
<td>Panel inner radius</td>
<td>(R)</td>
<td>800</td>
<td>2000</td>
<td>mm</td>
</tr>
<tr>
<td>Stiffener spacing</td>
<td>(b)</td>
<td>100</td>
<td>200</td>
<td>mm</td>
</tr>
<tr>
<td>Stiffener height</td>
<td>(h)</td>
<td>10</td>
<td>30</td>
<td>mm</td>
</tr>
<tr>
<td>Degradation region ratio</td>
<td>(D)</td>
<td>0.1</td>
<td>1</td>
<td>—</td>
</tr>
<tr>
<td>Degradation (stiffness reduction) ratio</td>
<td>(D%)</td>
<td>0.25</td>
<td>1</td>
<td>—</td>
</tr>
<tr>
<td>Degradation scenarios</td>
<td></td>
<td>1</td>
<td>6</td>
<td>discrete</td>
</tr>
</tbody>
</table>
reduction of stiffness in material elastic properties for both skin and stiffener elements, 
$D\% \times E_x, D\% \times E_y, D\% \times E_z, D\% \times G_{xy}, D\% \times G_{xz}, D\% \times G_{yz}$. The reduction of 
the stiffness ratio parameter $D\%$ was investigated in the range of $1-0.25$, with four 
discrete levels of interest. The reduction ratio $D\%$ equal to 1 means keeping the 
material elastic properties constant; however, the reduction ratio of 0.25 means that 
the elastic properties are reduced to $1/4$ of their original values. A typical load-
shortening response curve in the case with reduced material stiffness properties is 
compared to the undamaged structural response in Fig. 1.

### 3.2. Preselected structural degradation

Multiple delaminations$^{22}$ in various geometry locations, especially through-the-
width debonding,$^{23}$ affect the load-carrying capacity and the design reliability of 
composite structures in the postbuckling. Considering the fact that skin—stiffener 
debonding initiation and growth in the laminates are associated with considerable 
uncertainties, it is a challenge to formulate robust damage mechanisms to be 
implemented as general design practice. An approach to predicting the initiation of 
interlaminar debonding in the skin—stiffener interface based on a global—local 
technique has been elaborated.$^{24}$ The authors$^{24}$ have incorporated a ply damage 
degradation model by reducing the ply mechanical properties once the specified 
failure criterion has been met. Considering constraints of the metamodeling meth-
odology, the structural degradation should be included as a determined rather than a 
random variable. Thus structural degradation locations and configurations should be 
introduced and preselected before training the metamodels. Preselection of the 
degradation scenarios has been made based on industrial certification requirements 
(Fig. 3) and in addition worst case scenario numerical studies.$^{25}$ Moreover, ultrasonic 
C-scan inspection measurement data$^{16,19}$ acquired before the structural tests of 
prefabricated specimens with deboned skin—stiffener junctions [Figs. 3(a)–3(b)] and 
panels after the repeated postbuckling tests [shown in Figs. 3(c)–3(d)] confirmed the 
presence of the debonding in the skin—stiffener regions. Numerical verification has 
been evaluated by assessing the level of required material softening in comparison with 
the design where stiffeners are fully removed. As a result from a comprehensive 
set of benchmark studies six degradation scenarios [Figs. 3(c)(I–VI)] have been 
outlined for implementation in the metamodeling procedure$^{25}$ for both panel and box 
designs.

### 3.3. Validation of the numerical analyses versus physical experiments

Numerical analyses by ANSYS implicit, LS-DYNA explicit, and ABAQUS implicit 
finite element codes have been validated with physical experiments of stiffened 
structures with and without artificial damage.$^{16–21}$ Graphical comparison of the 
physical experiments and numerical analyses response curves is given in Fig. 4. The 
test specimens have been manufactured within the EU FP6 project COCOMAT 
(www.cocomat.de)$^1$ by industrial partners: Aeronnova delivered four stiffener D2
design panels [Fig. 2(a)] for DLR tests,\textsuperscript{16,19} Israel Aircraft Industries manufactured five stiffener D6 design panels [Fig. 2(b)] for Technion tests,\textsuperscript{17,18} and Agusta/Westland made closed box structures [Figs. 2(c)] for Politecnico di Milano (POLIMI) torsion and axial compression tests.\textsuperscript{20,21} Geometrical and laminate stacking configurations along with the testing setup are given in detail in Refs.\textsuperscript{16/C0}.

It should be noted that the closed box structure collapse test has been realized under the combined compression torsion load;\textsuperscript{20,21} the numerical simulation of load-controlled physical tests has not been successful within current research. Therefore pure compression and pure torsion tests have been evaluated and validated numerically only until the skin buckling load. It can be said that overall good agreement (an average of 6\% discrepancy) has been achieved between the numerical results and the physical tests by both implicit and explicit FE codes.\textsuperscript{26} Numerical curves obtained by the explicit LS-DYNA code indicate sudden loss of solution stability due to some dynamic effects that occur in the state near the experimentally obtained collapse load. This has been used as an advantage for the metamodeling procedure (determination of the breakpoint) even if it does not assure extraction of the real collapse numerical value. Implementation of the composite failure criterion is required for ANSYS or ABAQUS implicit solutions, in order to identify the collapse
load level of the structure (Fig. 4). Such an assumption is robust in determining the structural stiffness and critical load levels, but less sensitive for determination of the collapse shortening.

The degradation configurations used in physical tests were composed with a moderate level of artificial damage, and thus a relatively small decrease in the structural response has been observed in physical tests.\textsuperscript{16–21} Therefore graphical validation between the physical and the numerical tests was not illustrative, and thus the preselected worst case scenario with a remarkable decrease in the structural load-carrying capacity was used in further validation of the proposed metamodeling methodology.

4. Building and Validation of Metamodels

A set of 200 sample points for each degradation scenario has been elaborated by extracting postbuckling responses in the stiffened structures. To evaluate selected metamodeling techniques (see Table 2), partial polynomials constructed by the adaptive basis function construction (ABFC)\textsuperscript{14} approach, multivariate adaptive
regression splines (MARSs)\textsuperscript{12} and radial basis functions (RBFs),\textsuperscript{13} and a $\nu$-fold cross-validation\textsuperscript{27} technique with $\nu = 5$ have been used. Here the full data set is partitioned in $\nu$ equally sized subsets, and each of the $\nu$ subsets is retained as validation data for testing of the metamodel, and the remaining $\nu - 1$ subsets are used as training data. The cross-validation process is then repeated $\nu$ times (the folds), with each of the $\nu$ subsets used exactly once as validation data. For the metamodel accuracy measure, the following cross-validated relative error (CVRE) has been used:

$$\text{CVRE} = 100\% \sqrt{\frac{1}{n_t} \sum_{i=1}^{n_t} (y(i) - \hat{y}(i))^2},$$

(4.1)

where $y_i$ is the response value of the $i$th test point, $\hat{y}_i$ the predicted value of the $i$th test point and $n_t$ the number of test points.

Comparing selected metamodeling techniques, an overall approximation cross-validation error can be observed below the 10% margin, thus confirming the methodology efficiency in elaboration of the fast design tool. Moreover, skin and stiffener buckling load values, which are more crucial for the design stage, have low approximation errors close to the 5% level. Parametric and nonparametric approximations are practically equivalent in prediction accuracy, and thus none of them can be regarded as supreme for approximation of the postbuckling phenomena in stiffened structures with degradation. It should be noted that the metamodel accuracy relies on adequacy between the numerical and the physical test results, and hence approximation error estimates should be used considering the safety factor.

Graphical validation between the test results and metamodels with and without degradation in the scenario where half the stiffness has been reduced symmetrically [Fig. 3(c) (IV)] in three skin—stiffener junctions is shown in Fig. 5, graphically confirming the metamodeling accuracy versus numerical and test results.

The metamodels have been implemented in the FastDesign tool developed at Riga Technical University in a collaboration between the Institute of Materials and Structures and the Institute of Applied Computer Systems. All of the metamodels in

<table>
<thead>
<tr>
<th></th>
<th>D2 Panel design</th>
<th>D6 Panel design</th>
<th>Closed box design</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_1$</td>
<td>0.41 0.56 0.41</td>
<td>0.58 1.31 1.50</td>
<td>0.41 0.58 1.31</td>
</tr>
<tr>
<td>$k_2$</td>
<td>7.09 9.61 4.04</td>
<td>4.54 4.65 8.03</td>
<td>4.04 4.54 4.65</td>
</tr>
<tr>
<td>$P_1$</td>
<td>4.02 4.45 4.54</td>
<td>4.99 4.93 4.58</td>
<td>4.54 4.99 4.93</td>
</tr>
<tr>
<td>$P_2$</td>
<td>5.65 6.55 5.98</td>
<td>6.92 6.44 6.00</td>
<td>5.98 6.92 6.44</td>
</tr>
<tr>
<td>$P_3$</td>
<td>5.49 6.29 5.77</td>
<td>6.14 6.23 5.90</td>
<td>5.77 6.14 6.23</td>
</tr>
<tr>
<td>$u_1$</td>
<td>4.36 4.79 4.69</td>
<td>5.49 5.05 5.24</td>
<td>4.69 5.49 5.05</td>
</tr>
<tr>
<td>$u_2$</td>
<td>8.37 9.85 7.79</td>
<td>8.98 8.44 8.45</td>
<td>7.79 8.98 8.44</td>
</tr>
<tr>
<td>$u_3$</td>
<td>8.37 9.19 6.94</td>
<td>7.81 7.69 8.64</td>
<td>6.94 7.81 7.69</td>
</tr>
</tbody>
</table>
the FastDesign tool are sparse (also called “partial”) polynomials adaptively generated using the approach of ABFC\textsuperscript{14} implemented in the analysis tool VariReg.\textsuperscript{15} The FastDesign tool (available through www.cocomat.de) is easy to operate by selecting stiffened panel/closed box structure metamodels with corresponding degradation scenarios, and it constructs in a few seconds the load-shortening diagrams, which can be saved/exported for further optimization procedures.

5. Conclusions

To conclude, the metamodels extracted from the load-shortening and the torsion–rotation curves with different levels of structural degradation have been elaborated and implemented within the fast simulation tool for safe and optimal design of curved stiffened structures. In the validation process between the developed metamodels and the numerical simulations, the main response characteristics showed an approximation cross-validation error lower than 10%. At the same time graphical validation confirms that divergence between the FE and physical experiments is within the 10% margin. Thus it may be concluded that metamodels could be treated in a design procedure with the same level of reliability compared to physical experiments. The developed time-efficient design procedure could be used for endorsement of the new generation design scenario where the design limit load could be elevated up to the level of the damaged structure stiffener buckling load.

Acknowledgments

This work is supported by the European Commission, Priority Aeronautics and Space, Contract No. AST3-CT-2003-502723, project COCOMAT (www.cocomat.de). The information in this paper is provided as is and no guarantee or warranty is given that the information is fit for any particular purpose. The user thereof uses the information at his or her sole risk and liability.
The corresponding author would like to thank Ph.D. students E. Eglitis and O. Ozoliņš as well as Dr. G. Jekabsons from Riga Technical University, for their exceptional assistance in the elaboration of the metamodels and in their implementation into the FastDesign tool.

References


