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ROTOR DYNAMICS, VIBRATIONS AND PROBLEMS CONSERNING THEM

ROTORU DINAMIKA, VIBRĀCIJAS UN AR TĀM SAISTĪTĀS PROBLĒMAS

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Anotācija

Mūsdienu tehnikas risinājumu tendence ir palielināt ražību ekvivalenta izmēra iekārtām, kas prasa īpašu agregātu sastāvdaļu kvalitāti. Tādi apgriezieni kā 30,000 min^{-1} mūsdienu iekārtās vairs nav retums, un tie turpina augt, tādēļ rotoru dinamika kļūst arvien aktuālāka problēma. Pateicoties attīstībai informācijas tehnoloģiju jomā, aprēķinus iespējams veikt ikvienai ieinteresētai personai, izmantojot datorus, kas ļauj lietot tādas metodes, kuras bez tiem nebūtu efektīvas. Darbā ir apskatīta sistēmas modelēšana un izplatītākās analīzes metodes, kā arī aktīvās balansēšanas iespēja. Šīs problēmas tālākai risināšanai nepieciešams veikt praktiskos eksperimentus uz esošo modeļu bāzes un iespējams arī veidot savus modeļus.

Introduction

Vibration suppression of rotating machinery is an important engineering problem. Rotating machinery is commonly used in mechanical systems, including machining tools, industrial turbomachinery, marine installations and aircraft gas turbine engines. Vibration caused by mass imbalance is a common problem in rotating machinery. Imbalance occurs if the principal axis of inertia of the rotor is not coincident with its geometric axis. Higher speeds cause much greater centrifugal imbalance forces, and the current trend of rotating equipment toward higher power density clearly leads to higher operational speeds. For example, speeds as high as 30,000 rpm are not rare in current high-speed machining applications. Therefore, vibration control is essential in improving machining surface finish; achieving longer bearing, spindle, and tool life in high-speed machining; and reducing the number of unscheduled shutdowns. A great cost savings for high-speed turbines, compressors, and other turbomachinery used in petrochemical and power generation industries can be realized using vibration control technology.

Dynamic Modeling and Analysis of Rotor Systems

The planar rotor model is the simplest rotor model. Only the motion in the plane, which is perpendicular to the rotating shaft, is considered. The geometric setup of the planar rotor model is shown in Figure 1.

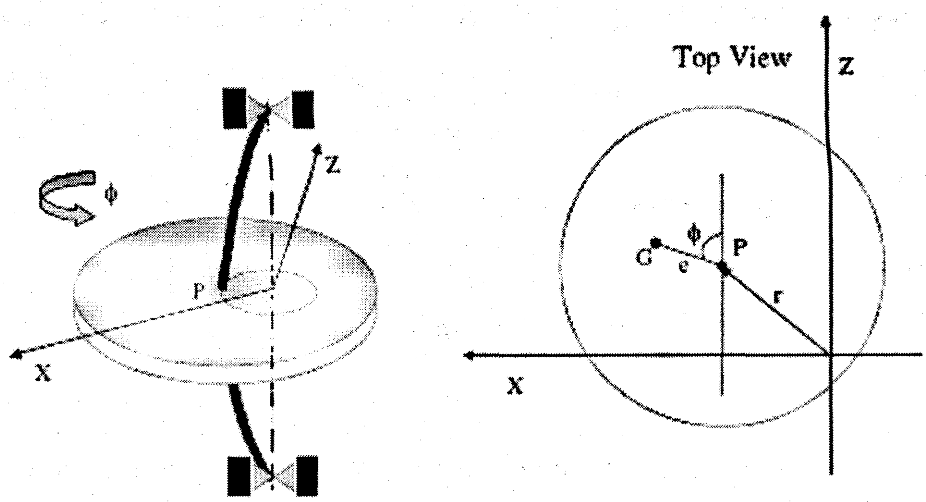

Fig. 1. Planar rotor model

In this model, the imbalance-induced vibration is described by the particle motion of the geometric center of the disk. *P* is the geometric center of the disk, and *G* is the mass center of the disk. The motion is represented by the vector **r**. It is well known that the governing equation of motion is (Childs, 1993) [1]

$$m\ddot{r}_x + c\dot{r}_x + kr_x = ma_x\dot{\phi}^2 + ma_z\ddot{\phi},$$

$$m\ddot{r}_z + c\dot{r}_z + kr_z = ma_z\dot{\phi}^2 + ma_x\ddot{\phi}$$

where *m*, *c*, and *k* are the mass, the viscous damping coefficient, and the shaft-stiffness coefficient, respectively. [*aX*, *aZ*] is the vector from *P* to *G*, expressed in the stationary coordinate system. φ is the rotating angle of the rotor. For a constant rotating speed, φ is zero. Although the planar rotor is a very simple rotor model, it can be used to study the basic phenomena in rotor dynamics such as critical speed, the effect of damping, and so on. The planar rotor model is a special case of the Jeffcott model that was first introduced in 1919 [2].

In the Jeffcott model, the rotor was modeled as a rigid disk supported by a massless elastic shaft that was mounted on fixed rigid bearings. This model is also equivalent to a rigid shaft supported by elastic bearings. The major improvement over the simple planar rotor model is that the motion of the rotor is depicted by rigid body motion instead of by particle motion. Although this is a single rigid body model, it can show the basic phenomena in the motion of the rotor, including the forward and backward whirling under imbalance force, critical speeds, the gyroscopic effect, and so on. The fact that the natural frequency is a function of the rotating speed can be predicted by this model. A typical geometric setup of this model is shown in Figure 2.

For a more complicated rotor system, a flexible rotor model was developed. This model allows for the elastic deformation of the rotor during rotation. Certainly, it is more accurate than the rigid rotor model. Breaking down a complex system into many simpler components that are easy to analyze is very common in engineering applications. A complicated rotor system is divided into several kinds of basic elements: rigid disk, bearing (usually linear modeled), flexible shaft segments, couplings, squeeze-film dampers, and so on. The equations of motion for each of these components were developed using the appropriate force-displacement and force-velocity relations and the momentum principles or other equivalent dynamic relations. Then, the system equations were assembled using geometric displacement constraints that guaranteed the connectivity of the components.

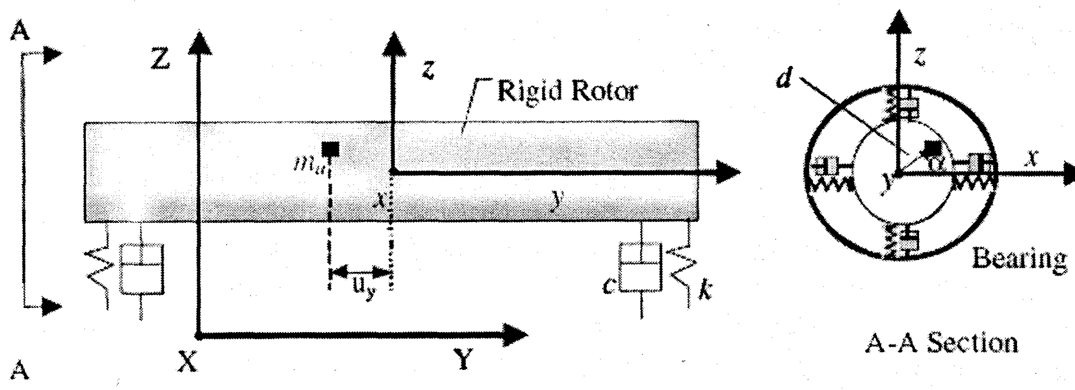

Fig. 2. Geometric Setup of the Rigid Rotor Model

There are two kinds of assembly procedures: the finite element method and the transfer matrix method. Ruhl and Booker (1972) [3] used a finite element model to study the dynamic characteristic of a turbo rotor. In their model, only elastic bending and translational kinetic energy were included, whereas the effects of rotatory inertia, gyroscopic effects, shear deformation, axial torque, axial load, and internal damping were neglected. Dimaragonas (1975) [4] presented a more general model that included rotatory inertia, gyroscopic effects, and internal damping. Gasch (1976) [5] presented a model that was similar to Dimaragonas's but included the effect of distributed eccentricity. At the same time, Nelson and McVaugh (1976) [6] published their model that included rotatory inertia, gyroscopic moments, and axial load. The detailed equations for the elements are expressed in both a fixed and a rotating reference frame. Their work was generalized by Zorzi and Nelson (1977) [7], who included internal damping. Nelson (1980) [8] presented a model that included the shear deformation effects. In general, the governing equation of motion of a flexible rotor can be written as (Lalanne and Ferraris, 1998) [9]

$$M\ddot{q} + C\dot{q} + Kq = f(\phi),$$

where **q** is the generalized coordinate to describe the motion and **M**, **C**, and **K** are coefficient matrices. The dimensions of these matrices are determined by the number of nodes in the model. For a complicated rotor-bearing-foundation system, the system matrices in the governing equations are very large. The computer memory storage requirements and computation time will be large. Therefore, most of the recent research on finite element methods for rotor dynamics has been designed to reduce the order or to reorder the system equations to achieve better computational efficiency while maintaining accuracy. The works of Shiau and Hwang (1989) [10] and of Nelson and Chen (1993) [11] are particularly noteworthy. Their work proposed a modeling procedure using assumed modes to reduce the order of the system matrices. Childs and Graviss (1982) [12] and Chen (1998) [13] used different reordering techniques to increase computational efficiency.

The other important method in rotor dynamics analysis is the transfer matrix method. This method is particularly well suited for "chainlike" structures. It was first used in the area of torsional vibrations. Lund (Lund and Orcutt, 1967; Lund, 1974a, 1974b) [14] presented procedures that use this method for rotor dynamics analysis. The advantage of the transfer matrix method is that it does not require the storage and manipulation of large system arrays. The transfer matrix method uses a marching procedure: it begins with the boundary conditions at one side of the system and successively marches along the structure to the other side. The solution should satisfy all the boundary conditions at all boundary points. The disadvantage of this method is that it is difficult, although not impossible (Kumar and Sankar, 1984) [15], to extend to time domain and nonlinear analysis. Therefore, it is difficult to conduct active balancing controller design using the transfer matrix method.

All methods mentioned above focus on linear systems, which means the system equations are a set of ordinary differential equations that are linearized in the neighborhood of an operating point. Generally, they require that the rotating speed be constant. Only a few analyses have dealt with speed-varying transient rotor dynamics. The earliest paper on the transient response of rotors may be from Lewis (1932) [16]. Using a graphical method, Lewis presented an approximate solution to the problem of running a system that has a single degree of freedom and linear damping through its critical speed from rest at a uniform acceleration. The solution shows that the resonant vibration amplitude is smaller than the corresponding amplitude if the spin speed is held constant at the critical speed. Furthermore, in transient time an apparent shift in the position of the critical speed will occur that is higher than the true critical speed when speed is increasing and lower when speed is decreasing. These effects are commonly observed in reality. This delay is possibly due to the fact that there is not enough time to accumulate energy at critical speed. Shen (1972) [17] used Newton's laws to derive a mathematical formulation for the analysis of both the transient and the steady-state flexible rotor dynamics. Many effects were included in that formulation, but no further numerical examples were given. Childs (1969, 1972) [18] developed a simulation model for general flexible spinning bodies in his two papers. In this development, Childs attempted to separate the rigid body motion and the flexible motion. Although the modal analysis method was proposed as a possible way for the model order reduction, how to apply it was not stated and no examples were given. So far, no further research work on the transient response has followed these two formulations. Recently, Nelson and Meacham (1981) [19] used the component mode synthesis method to conduct transient analysis of rotor-bearing systems under the finite element framework and found that the number of degrees of freedom in the rotor system model is directly proportional to the number of elements (or modes) implicit in the problem. This requires very large computational effort. Subbiah and Rieger (1988) [20] and Subbiah et al. (1988) [20] proposed a methodology that combines finite element and transfer matrix methods to perform the transient dynamic analysis, thereby overcoming the computational difficulties. This approach uses the finite element method to model symmetric shafts and then transforms the system properties to transfer matrix mode. This is a computational technique rather than an analytical tool.

From the above review on rotor dynamics, it is clear that many powerful tools for the linear system and frequency response are available. However, most of these techniques are targeted at the rotor design analysis. For the active vibration control system synthesis, a suitable analytical model is needed that is small in comparison to the overall system equations while still providing the essential dynamic characteristics.

Maslen and Bielk (1992) [21] presented a stability model for flexible rotors with magnetic bearings. Besides the flexible rotor model itself, their model included the dynamics of the magnetic bearing and the sensor-actuator noncollocation. This model can be used for stability analysis and active vibration synthesis. Most recently, an analytical imbalance response of the Jeffcott rotor with constant acceleration was developed by Zhou and Shi (2001a) [22]. The solution quantitatively shows that the motion consists of three parts: the transient vibration at damped natural frequency, the synchronous vibration with the frequency of instantaneous rotating speed, and a suddenly occurring vibration at damped natural frequency. This solution provides physical insight into the imbalance-induced vibration of the rotor during acceleration. It can be used for the synthesis of active vibration control schemes.

For the synthesis of DAVC techniques, most researchers used simplified low-order finite element models of the rotor system. Although the techniques developed can be extended to a high-order system theoretically, the computational load will be heavier and the signal-to-noise ratio will have to be higher. The DAVC techniques can be difficult to implement for the high-order system. Therefore, it is necessary to use a low-order system to approximate the high-order system. Model reduction techniques and the specific impact of the model reduction on the performance of the DAVC schemes require further investigation.

Conclusion

Rotating machinery is widely used in industry. The dynamic analysis and active vibration control of the rotating machinery are important engineering problems for both industry and academia. The major problem faced by the active vibration control scheme is the use of a limited number of actuators to control an infinite number of vibration modes. To design an active control scheme, a reduced-order model should be used and the effect of the spillover of higher vibration modes assessed. Although the available techniques developed for dynamic analysis, rotor imbalance estimation, and active real-time balancing and vibration control can be extended to high-order systems theoretically, the computational load will be heavier and the signal-to-noise ratio of the vibration measurement will have to be higher. Hence, the available techniques could be difficult to implement in high-order systems. Therefore, it is necessary to use a low-order system to approximate the high-order system. The gyroscopic effect caused by the rotating motion and the moment of inertia of the rotating body is a unique dynamic effect in a rotor system and should be considered in model reduction. The specific impact of this model reduction on the performance of the active balancing should also be investigated in the future.

In many active balancing and vibration control methods, the imbalance estimation is coupled with the control strategy. So far, there are no systematic methods available to show the relationship between the estimation and the control strategy. A control action is preferable if it can obtain small imbalance-induced vibration and excite the system to obtain the good imbalance estimation at the same time. Thus, the coupling effect should be investigated by considering the estimation algorithm, the system dynamics, and the control performance. This research can also lay a scientific foundation for the design of an efficient and reliable generic adaptive control system.

It is clear that active balancing can suppress the imbalance- induced vibration. It is also clear that the active balancing can improve product quality and improve the fatigue life of the machine and cutting tools and, hence, reduce the system cost. However, the installation and maintenance of an active vibration system for rotating machinery will increase the system cost. How to assess the active vibration control system from a cost-effective point of view and on a higher process level is not well studied in the literature. We believe this is an interesting and important problem in the active balancing and vibration control of a rotating system.

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