Static and dynamic techniques for non-destructive elastic material properties characterisation

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Abstract

Different non-destructive techniques are developed and adopted for effective and reliable characterization of advanced composite material properties. Static approach using a three-point-bending test and two dynamic methods: impulse excitation and inverse technique based on vibration tests are applied to study the elastic properties of orthotropic laminated composites and aluminium alloys with different carbon nanotubes volume content. An experimental evaluation of the mechanical material properties of modern composites gave the possibility to validate the developed non-destructive techniques, demonstrate their advantages and universality.

1 Introduction

Modern composite materials have promising perspectives for an application in the advanced composite repairs of pipelines. Their technical data could be estimated by using conventional fracture methods [1, 2] or nondestructive technique [3]. In the case of high costs of advanced composites, their experimental testing with conventional fracture methods looks as less effective due to the destructive nature of such experiments. On these reasons different nondestructive techniques are adapted or developed for a characterisation of advanced composite material properties. There are static approach using three-point-bending test and two dynamic methods, namely, impulse excitation method and inverse technique based on vibration tests.

2 Nondestructive techniques for a characterisation of elastic material properties

Three methods, static approach using three-point-bending test [4], impulse excitation method [5] and inverse technique based on vibration tests will be estimated in application to characterize the mechanical material properties of laminated composite specimens and nanocomposite specimens with small geometrical dimensions.

2.1 Static test

A beam bending test is one of the most commonly used types of testing for characterisation of material's mechanical properties. The maximum deflection of the simply supported beam with a rectangular cross section is at $x = l/2$ and is given by [6]

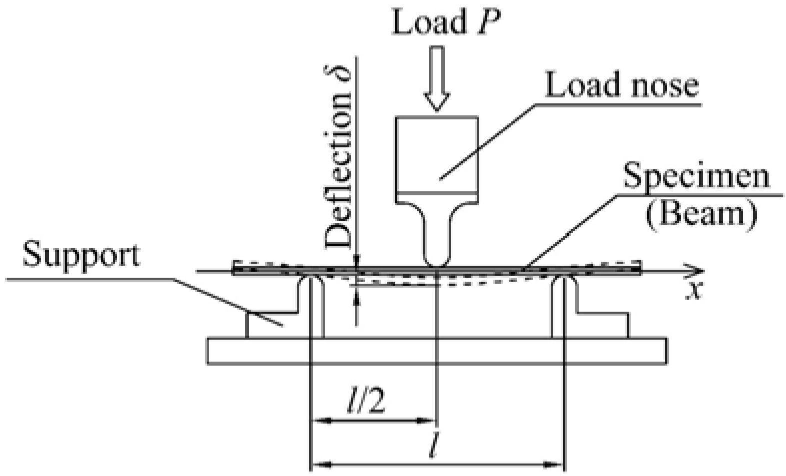


Figure 1: Three-point-bending test.

$$w_{\max} = \frac{Pl^3}{4bh^3E} \equiv \delta \tag{1}$$

where P is the applied load, l is the span length, b and h are the width and thickness of the beam, E is the Young modulus of the beam and δ is the measured centre deflection of the beam.

This expression can be used to determine the modulus of the material in terms of the measured centre deflection, applied load and the geometry of a beam (Fig. 1). Since the main idea of the proposed static test is nondestructive nature, the method has to be slightly modified in such a way to ensure that the deflection δ is lower than the critical deflection at which a fracture might occur. On that reason this approach is only applicable for the elastic behaviour of composite beams so that they can recover at the end of each bending test. This can be obtained usually for strains less than 0.5%. For the need of the present study the equation (1) can be rearranged to the following form

$$\frac{\delta}{Pl} = \frac{1}{4bh^3E} l^2 \tag{2}$$

Performing a set of three-point-bending tests carried out at different values of span l will generate a series of values δ_i/Pl_i, where i denotes the subsequent span lengths. A graph of δ_i/Pl_i plotted against l_i^2 gives a straight line which slope is equal to

$$g = \frac{1}{4bh^3E} \tag{3}$$

from which Young's modulus is calculated as

$$E = \frac{1}{4bh^3g} \tag{4}$$

By another way, the equation (1) permits to derive the longitudinal Young's modulus without subsequent formulas (2)-(4), i.e.

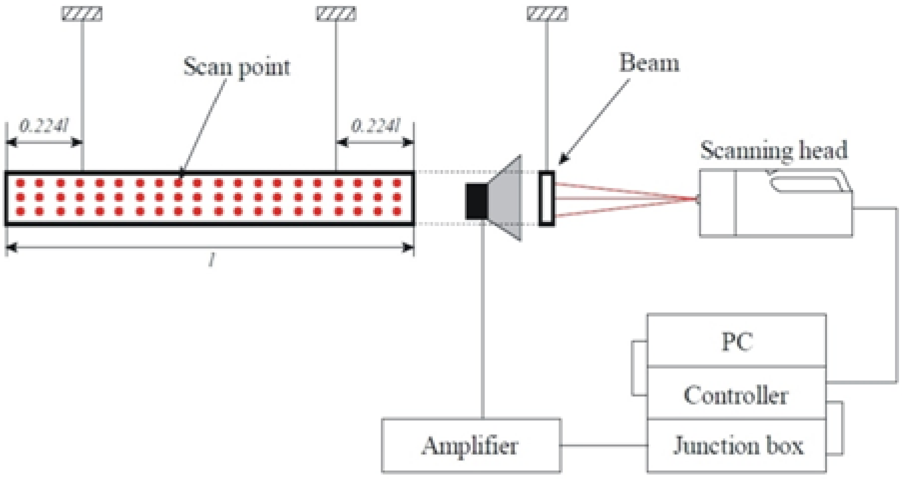


Figure 2: Experimental set-up for a vibration testing.

$$E = \frac{Pl^3}{4bh^3\delta} \tag{5}$$

According to standard rules, this result can be averaged in the form

$$E = \frac{\Delta P l^3}{4bh^3\Delta\delta} \tag{6}$$

where ΔP, $\Delta\delta$ denote respective increases in the linear range [7]. As a result, specification of different span lengths for the beam bending test is unnecessary.

2.2 Impulse excitation method

Vibration test based on the impulse excitation [5] is adopted for the determination of elastic properties of composites beams. This method originally developed for testing of heavy concrete specimens can be applied for lightweight structure providing noncontact vibration excitation and sensing, so that no additional mass will corrupt resonance frequencies (Fig. 2). Beam like specimens used in this method, have specific resonances that are determined by the elastic modulus, material density and geometry. The equation which combines all the above parameters of a specimen is called frequency equation [8]. In order to compute the elastic properties it is necessary to establish dimensions, density and experimental fundamental (the lowest) frequencies in bending and twisting of a beam with free-free boundary conditions. Using the fundamental frequency in bending, the Young's modulus in the longitudinal direction of a beam can be calculated as follows

$$E = 0.9465\frac{\rho f_b^2 l^4}{h^2}T \tag{7}$$

where ρ is the density, f_b is the fundamental frequency in bending, h and l are the thickness and length of the beam, and T is the correction factor which for $l/h \geq 20$ is

$$T = 1 + 6.585\left(\frac{h}{l}\right)^2 \tag{8}$$

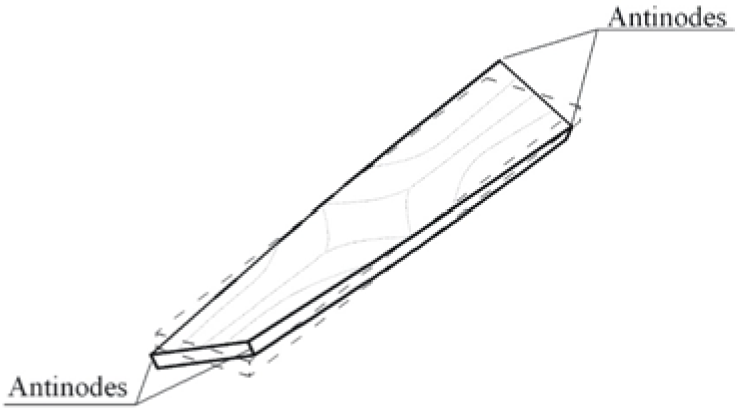


Figure 3: Antinodes location in the beam twisting mode shape.

The beam's fundamental frequency in twisting can be used to calculate the in-plane shear modulus as

$$G = 4\rho f_t^2 l^2 R \tag{9}$$

where f_t is the fundamental frequency in twisting and R is the correction factor determined as follows

$$R = \frac{1+\left(\frac{b}{h}\right)^2}{4-2.521\frac{h}{b}\left(1-\frac{1.991}{e^{\pi(b/h)}+1}\right)}\left(1+\frac{0.00851n^2b^2}{l^2}\right)-0.06\left(\frac{nb}{l}\right)^{\frac{3}{2}}\left(\frac{h}{b}-1\right)^2 \tag{10}$$

where n is the number of antinodes – locations that have local maximum displacement in unconstrained beam at the resonant frequency (Fig. 3).

2.3 Inverse technique

The inverse technique applied in this study is shown in Fig. 4 It uses vibration tests and consists of the experimental modal analysis, the numerical model and the material parameters identification procedure developed by applying a non-direct optimisation method based on the planning of the experiments and a response surface method [9]. The first step involves the planning of the investigation depending on the number of measured parameters and experiments. Next, a finite element analysis is applied at the reference points of the experimental design and the different dynamic parameters of the structure are calculated. In the third step, these numerical data are used to determine simple functions using a response surface method. Simultaneously, vibration experiments are carried out to measure the resonance frequencies and corresponding mode shapes of laminated plates. The identification of the material properties is performed in the final step of the method by minimising the error functional, which describes the difference between the experimental and numerical parameters of the structural responses.

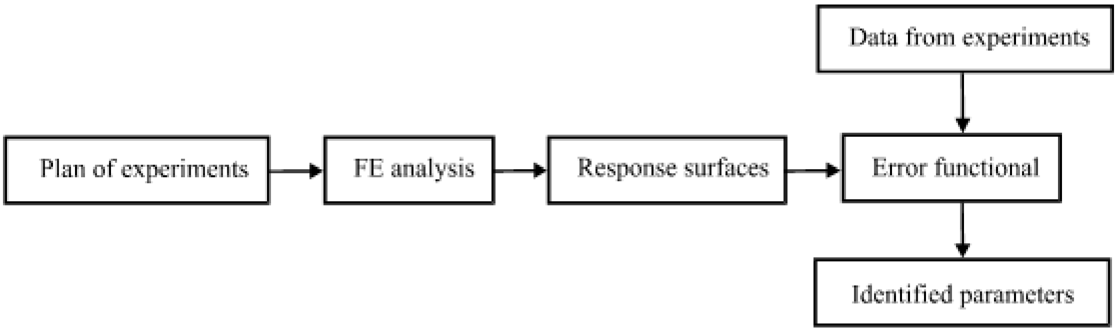


Figure 4: Inverse procedure.

Specimen	Stacking sequence	a mm	b mm	h mm	ρ kg/m^3
Plate 1	$[0]_{16}$	290.0	200.0	2.60	1546.6
Plate 2	$[90]_{16}$	290.0	200.0	2.61	1546.6
Beam 1[1]	$[0]_{16}$	292.0	19.5	2.60	1546.6
Beam 2[1]	$[0]_{16}$	291.0	20.0	2.60	1546.6
Beam 3[1]	$[0]_{16}$	290.0	19.5	2.60	1546.6
Beam 4[1]	$[0]_{16}$	290.0	19.5	2.60	1546.6
Beam 5[2]	$[90]_{16}$	290.0	20.0	2.61	1546.6
Beam 6[2]	$[90]_{16}$	290.0	19.5	2.61	1546.6
Beam 7[2]	$[90]_{16}$	289.0	20.0	2.61	1546.6
Beam 8[2]	$[90]_{16}$	289.0	19.5	2.61	1546.6

[1]cut from the Plate 1 along principal direction x;
[2]cut from the Plate 2 along principal direction y.

Table 2: Geometry, stacking sequence and density of specimens.

3 Characterisation of elastic material properties

An experimental evaluation of the elastic material properties of modern composites is carried out to validate the examined nondestructive techniques and to demonstrate their universality and advantages.

3.1 Laminated composites

Two laminated composite panels were manufactured for the experimental investigation purposes. Carbon/epoxy prepreg (SEAL Texipreg HS 160 RM) was used to manufacture the panels with 16 layers aligned unidirectionally $[0]_{16}$. The panels were cut in such a way, so that it was possible to obtain two plates with different fibres orientation, namely, $[0]_{16}$ and $[90]_{16}$ (Fig. 5), which were used in the inverse technique procedure. Additionally, eight beams were cut out from the panels along their principal directions *x,y* (four beams from each panel) which were used for the auxiliary tests and validation purposes. The dimensions of the specimens were measured carefully and their average thicknesses were estimated. No curvatures at the samples' edges were noticed. The geometry, stacking layers sequence, and density of each test specimen are given in (Table 2).

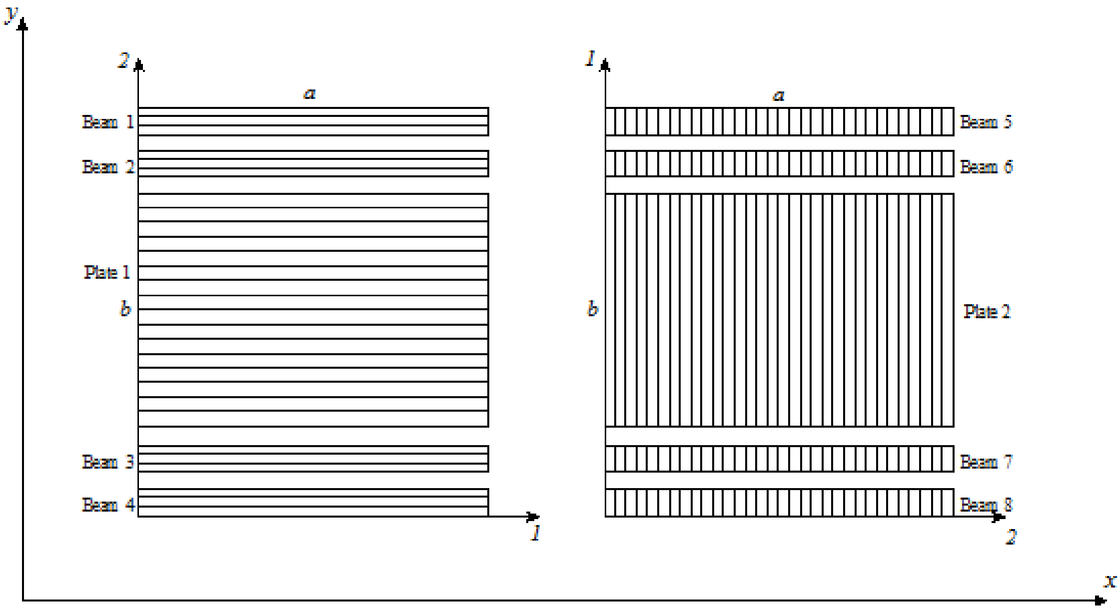


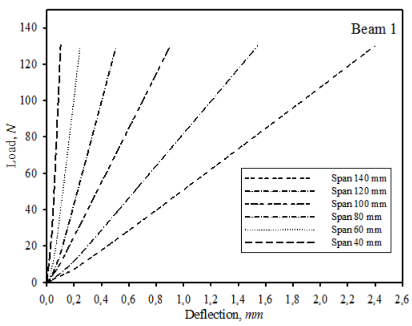
Figure 5: Laminated composite specimens used for non-destructive testing.

3.1.1 Application of static test

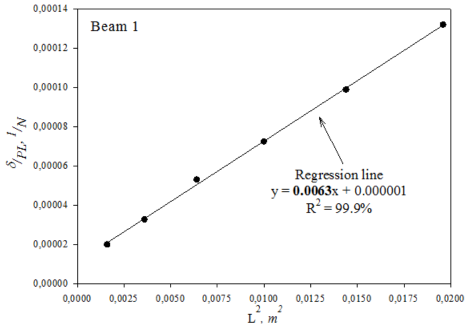
The experimental realisation of the three-point-bending tests (Fig. 1) was done by the application of the ZWICK Z-100 system. It allows applying quasi-static loading for determination of elastic properties of a beam specimen. 10 *kN* load cell is used for the bending load. A beam specimen rests on two supports and the load is applied by a load nose in the middle span. The beam is centred on the supports, with its longer edge perpendicular to the load nose and supports. The loading speed of 0.01 *mm/s* is set for all tests and the load-deflection data are preserved by measurement of the motion of the loading nose relative to the supports. All the beams used in the present study have unidirectional lamination scheme, therefore the Young's modulus E can represents E_1 or E_2 depending on the direction that the beams were cut out from the panels. Beams 1-4 were cut out along *axis 1* and were used for estimation of E_1, whereas Beams 5-8 were cut out along *axis 2* and were used for estimation of E_2.

Six tests were performed for each beam for the span lengths $l_i = 140, 120, 100, 80, 60, 40$ *mm*. The first test of each beam ($l_i = 140$ *mm*) was terminated when the deflection reached the beam thickness $\delta_1 = h$, and its value as well as the applied load P were stored. The deflection equal to the beam thickness assures the strain less than 0.5%. All the subsequent tests of a beam were terminated when the applied load took the value of P (for δ_1). For each span lengths l_i the load-deflection histories were preserved (Fig. 6a) in order to estimate the values of δ_i for calculating δ_i/Pl_i and then plot them on l_i^2. Obtained graphs were approximated by the linear regression lines from which the slopes g were determined (Fig. 6b). Using equation (4), the Young's modulus of each beam was calculated and presented in Table 3.

The application of the three-point-bending test allowed determining only Young's moduli in the principal directions of the panels (E_1, E_2). Moreover, six experiments (as much as span lengths) were required to characterise Young's modulus in one particular direction. Despite of this, the advantage of the method is its nondestructive nature under static load. The fact that any strain gages were attached to the beam samples allows using them for the



(a) load-deflection histories for different span length



(b) regression line after the set of six experiments

Figure 6: Experimental results of the three-point bending tests

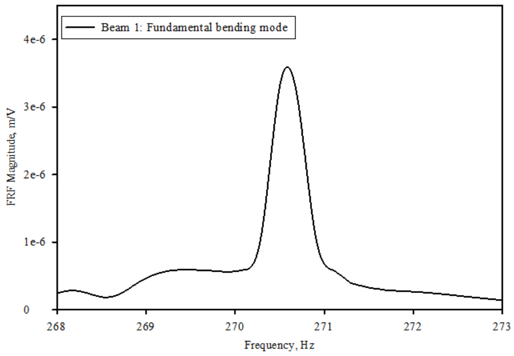
Specimen	Slope	E_1 GPa	E_2 GPa
Beam 1	0.00630	115.78	-
Beam 2	0.00601	118.33	-
Beam 3	0.00621	117.48	-
Beam 4	0.00624	116.90	-
Beam 5	0.0770	-	9.13
Beam 6	0.0795	-	9.18
Beam 7	0.0816	-	8.62
Beam 8	0.0810	-	8.90
Average		117.12	8.95

Table 3: Young's moduli obtained from three-point-bending test.

further investigation. However, systematic errors arise in this approach, especially during the span length measurements. To decrease this effect the tests should be performed for the possible high number of spans in order to assure the high correlation factor (R^2) of the regression line. This provides that the slope is determined with small error. In contrary, the high number of spans will increase the number of experiments and the time needed for the estimation of Young's moduli. Therefore a compromise must be found between the desired accuracy and time spent to perform all experiments.

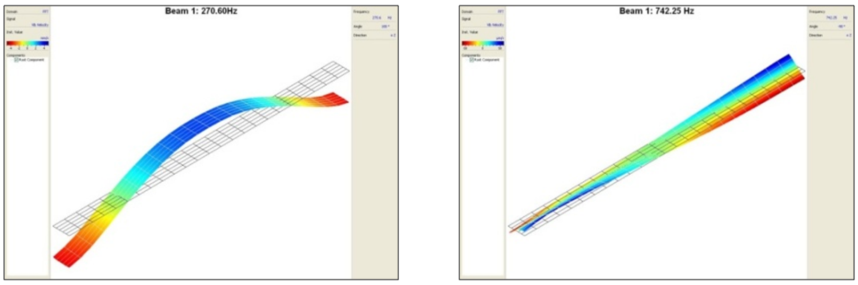
3.1.2 Application of impulse excitation method

In order to estimate the engineering constants of a single lamina used to manufacture the beams specimens, eight beams listed in (Table 2) were tested experimentally to obtain their fundamental bending and twisting frequencies. Depending on direction that the beams were cut out from the panels it was possible to estimate E_1 for the beams cut out along material *axis 1* (Beams 1-4) and E_2 for the beams cut out along material *axis 2* (Beams 5-8). The beams were cut out from the panels in one plane of symmetry $x - y$, therefore the in-plane shear modulus G_{12} can be estimated. The resonant frequencies were determined using the POLYTEC laser vibrometer PSV-400-B in the frequency bandwidth of $0 \ldots 1600$ Hz with the excitation provided by loudspeaker. Free-free boundary conditions were simulated by suspending beams on two thin threads at the locations of nodal lines of beam's bending mode - $0.224l$ from each end (Fig. 2). Such boundaries were used



(a) (b)

Figure 7: Example of frequency response functions of the tested composite beam.



(a) bending (b) twisting

Figure 8: Examples of experimental fundamental modes of tested composite beam

to obtain the both, bending and twisting mode shapes of beams. One vibration test for each of the beams was performed and the FRFs (Fig. 7) were preserved to determine their fundamental bending and twisting frequencies and corresponding mode shapes (Fig. 8). The equations (7) and (9) were then used to calculate E_1, E_2 and G_{12}. The measured resonant frequencies and calculated engineering constants are presented in Table 4.

The proposed method based on impulse excitation appears to be effective approach for a characterisation of engineering constants of laminated composite materials. Only two beams cut along principal directions of panels and one vibration test for each beam are required to determine two longitudinal Young's moduli (E_1, E_2) and in-plane shear modulus (G_{12}). If the beam samples can be provided with a sufficient length-to-width ratio, the method can be recommended for a fast estimation of the material mechanical properties. It is also important to provide a noncontact excitation and vibration sensing for characterisation of lightweight samples in order to avoid any influence of additional mass on the resonant frequencies.

3.1.3 Application of inverse technique

The inverse technique was applied to characterise two laminated composite plates, Plate 1 and Plate 2, to identify the four engineering constants (E_1,E_2,G_{12},υ_{12}) of their constituent layers. The plates were manufactured from the same unidirectional prepreg material (SEAL Texipreg HS 160 RM) but with different layers stacking sequence (Fig. 5). The plan of experiments was built for a vector of 4 design parameters $[x]^T = [x_1, x_2, x_3, x_4] = [E_1, E_2, G_{12}, \upsilon_{12},]$ and 101 experiments. The following borders of domain for the engineering constants were used for both plates:

Specimen	f_b Hz	f_t Hz	E_1 GPa	E_2 GPa	G_{12} GPa	T	R
Beam 1	270.60	742.25	115.28	-	4.53	1.000522	15.68
Beam 2	272.75	741.25	115.52	-	4.74	1.000526	16.45
Beam 3	273.85	742.75	114.86	-	4.50	1.000529	15.69
Beam 4	274.40	741.50	114.99	-	4.49	1.000529	15.69
Beam 5	75.60	741.50	-	8.69	4.67	1.000533	16.34
Beam 6	76.30	742.75	-	8.85	4.47	1.000533	15.57
Beam 7	75.20	740.50	-	8.48	4.63	1.000537	16.34
Beam 8	75.10	744.50	-	8.45	4.46	1.000537	15.57
Average			115.16	8.62	4.56		

Table 4: Young's and in-plane shear moduli obtained from the impulse excitation method.

	Plate 1	Plate 2
E_1, GPa	117.9	116.6
E_2, GPa	8.98	8.94
G_{12}, GPa	4.12	4.30
υ_{12}	0.47	0.34

Table 5: Identified material properties of a single layer of the laminated plates.

$100 \leq E_1 \leq 120\ GPa$
$7 \leq E_2 \leq 10\ GPa$
$3 \leq G_{12} \leq 5\ GPa$
$0.25 \leq \upsilon_{12} \leq 0.50$

Then the FE models of plates were built with mesh density of 30x30 elements and the numerical modal analyses were performed to obtain first ten eigenfrequencies in 101 experimental points. Employing these numerical values, the second order polynomial approximating functions (response surfaces) for all eigenfrequencies were obtained with the correlation coefficients higher than 90%. Independently, the experimental modal analysis of the plates with free-free boundary conditions was performed to preserve first ten resonant frequencies and their corresponding mode shapes. Having acquired experimental and numerical modal parameters, the error functional is built, which is then minimised giving as a result four engineering constants (Table 5). The obtained material properties were then used to calculate the eigenfrequencies of the plates which were compared with the corresponding experimental resonances. The relative errors (residues) between experimental and numerical results were calculated to quantify the accuracy of the identification procedure (Table 6). Additionally, the experimental and numerical mode shapes were compared in order to assure that the relative errors were calculated for the same measured and calculated resonant frequencies.

In general, the average value of residues lower than 1% means that the identification procedure was performed successfully. This is the case for Plate 2 where the average value is 0.55%. Higher value was obtained for the Plate 1 – 1.39%, what indicates that some sources of errors should be studied additionally. Since the plates were tested experimentally in the same conditions, the source of the higher error for the Plate 1 should be sought among the numerical model errors.

	Plate 1			Plate 2		
Mode (i)	FEM[1]	EXP	R, %	FEM[1]	EXP	R,%
1 (1,1)	78.2	76.0	2.81	76.4	76.3	0.13
2 (0,2)	161.4	158.0	2.11	77.6	78.0	0.52
3 (1,2)	226.5	220.0	2.87	176.5	178.5	1.13
4 (2,0)	280.3	278.1	0.78	210.6	212.2	0.76
5 (2,1)	319.5	316.0	1.10	318.7	321.0	0.72
6 (0,3)	448.4	444.0	0.98	412.7	415.0	0.56
7 (2,2)	457.2	460.0	0.61	519.5	524.0	0.87
8 (1,3)	504.1	500.0	0.81	580.9	578.0	0.50
9 (2,3)	717.6	712.0	0.78	600.0	598.5	0.25
10 (3,0)	767.7	760.0	1.00	666.8	667.5	0.10
	Average		1.39			0.55

[1]FEM analysis for engineering constant obtained from the inverse technique

Table 6: Experimental and numerical resonant frequencies of laminated plates in Hz.

	Three-point-bending	Impulse excitation	Inverse technique[1]
E_1, GPa	117.12	115.16	116.80
E_2, GPa	8.95	8.62	8.85
G_{12}, GPa	-	4.56	4.10
υ_{12}	-	-	0.36

[1]1average value from Plate 1 and Plate 2

Table 7: Estimated engineering constants of constituent layers of laminated composite panels.

The application of the nondestructive inverse technique for the characterisation of laminated composites plates brought satisfactory results of the identified engineering constants of their constituent layers. The advantage of this method is that the samples do not require special shape and time consuming preparation process. The experimental setup with the laser vibrometer and proper consideration of the model errors gave the possibility to improve the final estimation of the desired material properties.

3.1.4 Comparison of the results obtained by different nondestructive techniques

The number of estimated engineering constants depends on the applied method. One vibration test was required to determine four engineering constants using plate samples and the inverse technique procedure. Only two vibration tests were required to estimate three engineering constants using beam samples and the method based on impulse excitation. Two engineering constants were estimated from the static tests carried out on beam samples and as many as six tests were required (for one beam) to estimate the constants. As presented in Table 7, the results are in good agreement which prove their ability for characterisation of composite materials.

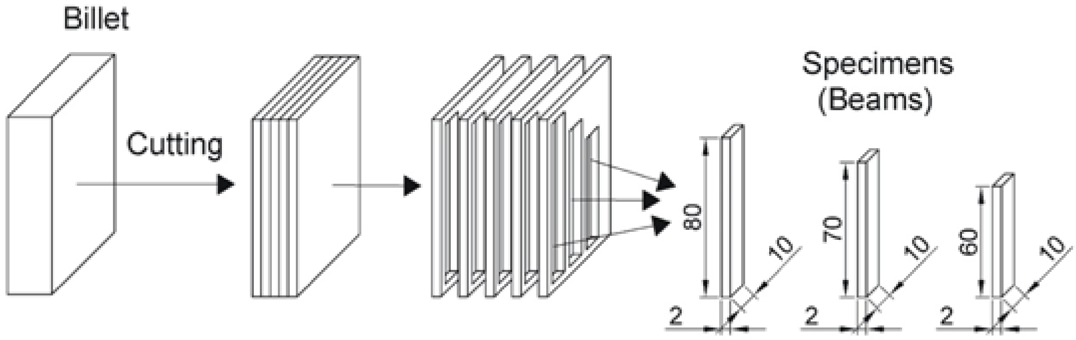


Figure 9: Preparation of beam nanocomposite specimens.

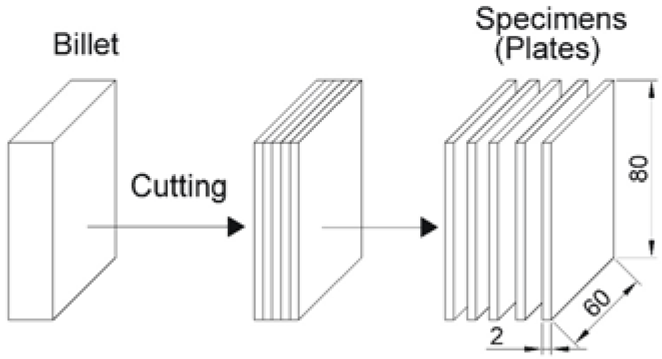


Figure 10: Preparation of plate nanocomposite specimens.

3.2 Nanocomposites

Nanocomposites were made by adding CNT to molten aluminum alloy via gravity casting into a mould and, by this way, they are examined in the paper as isotropic materials. Ten nanocomposite billets were prepared using this technology: two billets of pure aluminum alloy LM24 and two billets of the same alloy with different CNT volume content of 0.1, 0.3, 0.5 and 1.0%. Then two types of specimens: beams (Fig. 9) with the dimensions of 80mm or 70mm or 60mm x 10mm x 2mm and plates (Fig. 10) with the dimensions 80mm x 60mm x 2mm were machined from these billets for experimental investigations. It is necessary to note that specimens were taken from 5 different positions from a large billet to study a variation of material properties as a function of billet thickness.

Young moduli obtained by using the impulse excitation method and three-point-bending tests are in very good agreement like it is presented in Fig. 11a. It is seen that both NDT methods show identical dependence of the Young modulus on CNT volume content and sample location in the billet.

Dependence of the Young modulus obtained by using the inverse technique on CNT volume content and sample location in the billet is shown in Fig. 11b. Some difference is observed in the forms of surfaces describing the dependencies of Young modulus between Fig. 11a and 11b. However it is necessary to note that this difference in the absolute values of Young modulus is negligible.

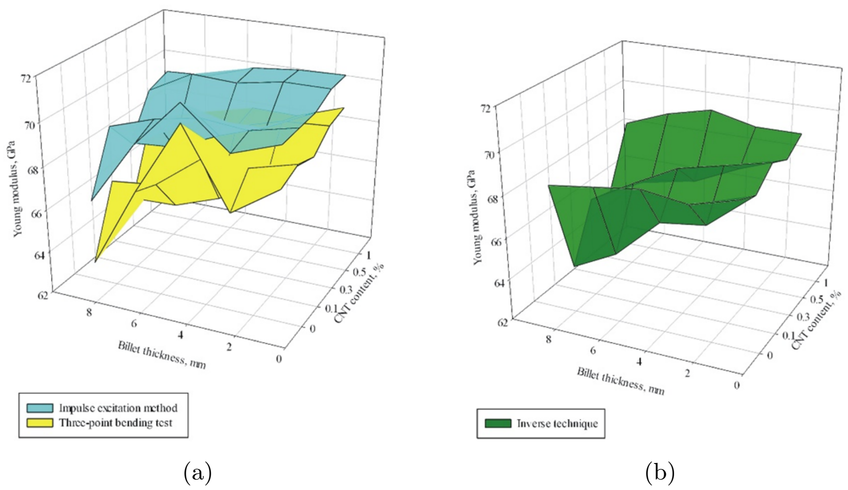


Figure 11: Comparison of Young moduli obtained by different NDT methods

4 Conclusion

The advantages and disadvantages of the applied techniques can be summarised as follows:

1. The application of the three-point-bending test allowed determining only Young's moduli in the principal directions of the orthotropic plates. Despite of this, the advantage of the method is its nondestructive nature under static load. An obstacle of the technique is systematic errors which arise during measurements of the span length. On that reason the tests should be performed for the possible high number of spans so that the correlation factor of the linear regression line approaches 100% and the slope is determined with small error.

2. A simple and effective method for engineering constants identification appears when the vibration method based on impulse excitation principle is applied. The Young's moduli in the material principal directions as well as in-plane shear modulus were estimated applying this method. The advantage of the method over the proposed static approach is little preparation of experimental set-up and short measurement time.

3. Among all proposed methods for the elastic properties characterisation, the approach based on the inverse technique is most suited for the convenient, fast and accurate identification of elastic properties. Four engineering constants of the constituent layers of each laminated composite plates were successfully determined. For that purpose, only one vibration test was needed to acquire resonant frequencies of each plate which were then used in the inverse procedure.

Although the static and impulse excitation methods do not provide information about all engineering constants, they can be used as preliminary tests in order to assess the initial bounds of engineering constants for the identification procedure with application of the inverse technique. They can be also used as known characteristics reducing the number of design parameters and therefore improving the identification results.

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