Inverse technique for characterisation of elastic and dissipative properties of materials used in a composite repair of pipelines

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Abstract

An inverse technique based on simple vibration tests is developed to characterise the elastic and viscoelastic properties of constituent materials used in the advanced composite repairs of pipelines. In the case of viscoelastic materials, this novel approach allows to preserve the frequency dependence of the storage and loss moduli in a wide range of frequencies. The computational effort is substantially reduced by using an optimisation based on the planning of the experiments and the response surface technique in order to minimize the error functional. The developed inverse technique has been tested on metallic plates and successfully applied to characterise orthotropic material properties of laminated composites and viscoelastic properties of adhesive materials. Good agreement between experimental and numerical results has been obtained.

1 Introduction

Due to great importance to define performance, reliability and safety requirements for advanced composite products and services a considerable effort has been devoted to the study of their mechanical material properties and a lot of different methods have been developed in the last three decades. For many orthotropic sheet materials and sandwich composites a measurement method based on low frequency vibrations [1–10]is not only the simplest approach, it is also the only approach, which does not suffer from grave difficulties of principle, when the results are used to make predictions in the same range of frequencies. There are two other general methods might be used - static measurements [11, 12, 13] and ultrasonics [14, 15, 16], but neither is wholly appropriate for characterisation of mechanical material properties of advanced composites. The difficulties are most obvious, when it comes to the determination of damping "constants", since these would be expected to be frequency and temperature dependent.

An identification of the elastic and damping properties of thin orthotropic plates has been carried out in papers [1, 2, 3, 4, 5]. The simple method for measuring all four elastic constants and at least three of the four damping constants for a flat, thin, orthotropic plate is presented in the paper [1]. These constants are determined from the measurements of low resonant frequencies of thin rectangular plates with free edges. It turns out that the fourth damping constant does not usually have sufficient influence on the low frequency modes for a reliable value to be found by this approach. The paper [2] presents the method for determination of the elastic and damping properties of composite laminates using measured complex eigenfrequencies and mode shapes. These parameters are identified minimising the error function containing the deviations of eigenvalues and responses between experiment and analysis based on hysteretic damping model. Since the damping mainly causes a change of the imaginary part in eigenvalues and eigenvectors, the complex modal parameters are measured. The mixed numerical-experimental technique based on the measurement of vibration data has been developed for identification of the complex moduli of thin orthotropic plates in the papers [3, 4, 5]. The relation between the modal parameters (structural parameters) and the material parameters is obtained using a numerical model of the specimen in combination with the modal strain energy method. Authors give also an overview of the error sources and examine the errors influence on the material properties identified. It is necessary to note that in most investigations the identified material loss factors are low, constant or determined in narrow frequency range.

Only some researchers [6, 7, 8] tried to identify the viscoelastic material properties of sandwich composites using an inverse technique based on vibration tests. The forced steady-state harmonic vibrations and free vibrations have been utilized in the identification procedure in paper [6] to characterise the constitutive parameters of Voigt model and three parameters model of uniaxial viscoelasticity used for a description of viscoelastic materials applied in sandwich beams. These beams consist of a core made of polyvinyl chloride foam and two laminate faces made of glass fibre reinforced polyester. To identify parameters of aluminium honeycomb sandwich panels, an orthotropic Timoshenko beam model has been applied, and the elastic constants and modal damping ratios have been determined in paper [7] minimising the error between experimental and analytical results. The paper [8] proposes an inverse method based on the flexural resonance frequencies and using the sandwich beam theory for the finite element modelling. Like in the case of material characterisation of laminated composites, the examined approaches do not give the possibility to analyse sandwich structures with high damping and to characterise their viscoelastic material properties in wide frequency range, when storage and loss moduli are frequency dependent values.

On this reason the present investigations are focused on the development of new inverse technique based on simple vibration tests to characterise elastic and non-linear viscoelastic mechanical properties of polymeric fillers, adhesives and laminated composites used widely in the advanced composite repair systems to bring an efficiency of damaged section up to the level of undamaged pipeline.

2 Inverse technique

The present inverse technique (Fig. 1) operates on vibration tests and consists of experimental set-up, numerical model and material identification procedure developed with an application of non-direct optimisation methodology based on the planning of experiments and response surface method to decrease considerably computational efforts. In the first stage a plan of experiments is produced in dependence on the number of identified parameters and number of experiments. Then finite element analysis is performed in the reference points of experimental design and dynamic parameters of structure are calculated. In the third stage these numerical data are taken to determine simple functions using the response surface methodology. In parallel vibration experiments are carried out with the purpose to determine natural frequencies and corresponding loss factors of composite structures. An identification of material properties is performed in the final stage minimising the error functional between experimental and numerical parameters of structural responses.

2.1 Experimental analysis

Usually two experimental set-ups are used in the identification procedures based on vibration tests to determine the dynamic characteristics of structures. The first is based on the impulse technique and uses contact measurements by accelerometers (Fig. 2), the second is based on the non-contact laser measurements (Fig. 3). In both cases the PCB impulse hammer or shaker or loadspeaker or piezo-electric actuators glued to the investigated object are applied to produce excitations.

To identify material properties, four rectangular samples have been prepared from aluminium, uni- and multi-directional laminated composite, and sandwich panels. In the present study the eigenfrequencies of homogeneous aluminium and laminated composite plates have been determined using POLYTEC laser vibrometer (Fig. 3) operating on the Doppler principle and measuring back-scattered laser light from a vibrating structure to determine its vibration velocity and displacement. These panels have been suspended using two thin threads to realise free-free boundary conditions and excited with piezo-electric actuators. The experimental set-up used for vibration testing of clamped sandwich panel (Fig. 2) presents the impulse technique, where an excitation is accomplished by PCB impulse hammer with a built-in force transducer. The structural response is detected by three accelerometers located on the panel as presented in Fig. 2 In both experimental set-ups the input and output signals are converted to the frequency domain by fast Fourier transformation in a signal analyser and the frequency response functions are created. After that, these frequency response functions are exported to the modal analysis program, where natural frequencies and corresponding loss factors are calculated.

2.2 Finite element analysis

In the developed inverse technique the finite element method is used for the modelling and dynamic analysis of laminated composite and sandwich panels.

2.2.1 Finite element models

Finite element modelling is based on the first-order shear deformation theory including rotation around the normal. In this case the widely known expressions of displacements have the following form:

$$u = u_0 + z\gamma_x \ , \ v = v_0 + z\gamma_y \ , \ w = w_0 \qquad (1)$$

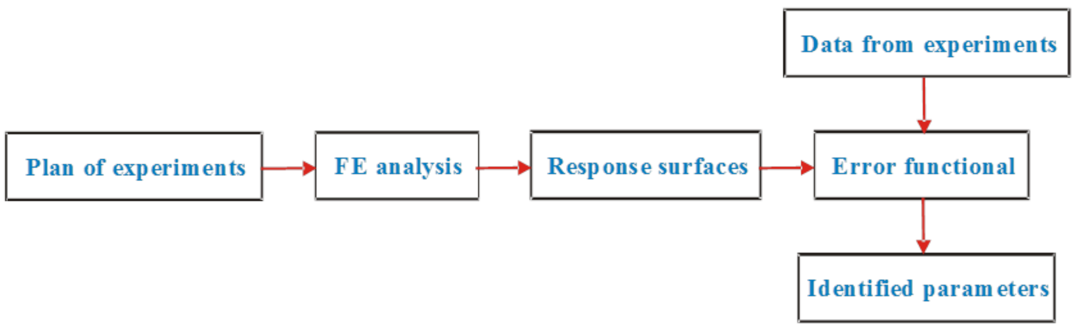

Figure 1: Inverse procedure

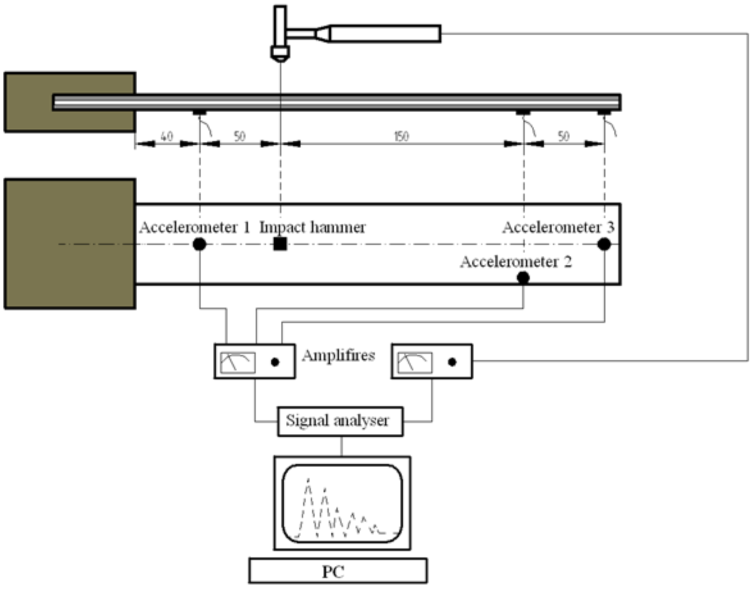

Figure 2: An experimental set-up based on the impulse technique.

where u_0, v_0, w_0 are the displacements in a reference plane, z is the coordinate of the point of interest from a reference plane, γ_x, γ_y are the rotations connected with the transverse shear deformations. For sandwiches this hypothesis is applied separately for each layer. This case corresponds to the broken line model [17] and satisfies to the following displacement continuity conditions between the layers

$$\begin{aligned} u^{(1)} &= u^{(2)}\Big|_{z=z_1} \quad , \quad u^{(2)} = u^{(3)}\Big|_{z=z_2} \\ v^{(1)} &= v^{(2)}\Big|_{z=z_1} \quad , \quad v^{(2)} = v^{(3)}\Big|_{z=z_2} \\ w^{(1)} &= w^{(2)}\Big|_{z=z_1} \quad , \quad w^{(2)} = w^{(3)}\Big|_{z=z_2} \end{aligned} \tag{2}$$

where in the brackets, the numbers of layers are given. At the same time for laminated composite models this hypothesis is used already for the entire laminate [18].

2.2.2 Dynamic analysis

To describe the rheological behaviour of viscoelastic materials, the complex modulus representation [19] is used. Using this model, the constitutive relations will be expressed in the frequency domain for the isotropic material as follows

$$\sigma_0 = E^*(\omega)\varepsilon_0 = E(\omega)\left[1 + \mathrm{i}\eta(\omega)\right]\varepsilon_0, \qquad \eta(\omega) = \frac{E^{''}(\omega)}{E(\omega)} \tag{3}$$

where σ_0 and ε_0 are an amplitude of the harmonically time dependent stress and strain respectively, E^* is the complex modulus of elasticity, E, $E^{''}$ are the real and imaginary parts of the complex modulus of elasticity, η is a loss factor and ω is a frequency. It is necessary to note that the storage and loss moduli in this case are frequency dependent values. For orthotropic materials the complex modulus representation is used by the same way to describe the rheological behaviour of viscoelastic materials.

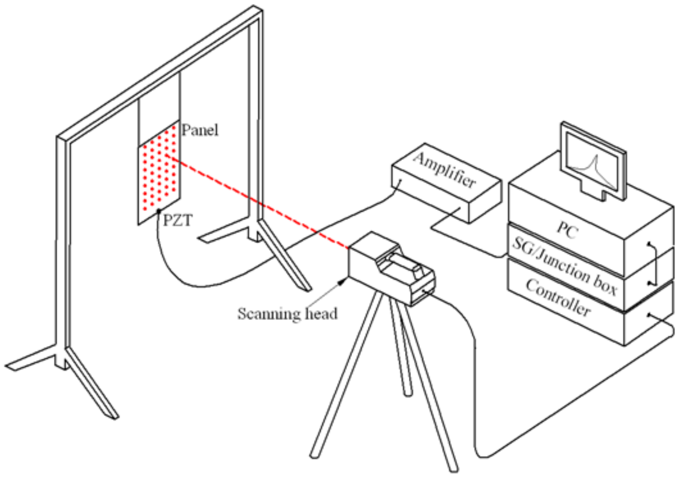

Figure 3: An experimental set-up based on the non-contact laser measurements.

In the method of complex eigenvalues damped eigenfrequencies and corresponding loss factors are determined from the free vibration analysis of a structure

$$\left[K^*(\omega) - \omega^{*2}M\right]\bar{X}^* = 0 \tag{4}$$

where M is the mass matrix of a structure, $K^*(\omega) = K(\omega) + iK''(\omega)$ is the complex stiffness matrix of a structure, $\omega^* = \omega + i\omega''$ is the complex eigenfrequency. The real part ω represents the damped eigenfrequency of a structure and the imaginary part ω'' specifies the rate of decay of the dynamic process. The matrix $K(\omega)$ is determined using the storage moduli $E(\omega)$ and $G(\omega)$, while $K''(\omega)$ is found using the imaginary parts of the complex moduli $E''(\omega) = \eta_E(\omega)E(\omega)$ and $G''(\omega) = \eta_G(\omega)G(\omega)$, where $\eta_E(\omega)$ and $\eta_G(\omega)$ are the material loss factors.

The equation (4) can be written as the non-linear generalised eigenvalue problem

$$K^*(\omega)\bar{X}^* = \lambda^* M\bar{X}^* \tag{5}$$

where $\lambda^* = \omega^{*2}$ is the complex eigenvalue and $\bar{X}^*$ is the complex eigenvector. Solution of the equation (5) starts with a constant frequency ($\omega = \text{const}$). Then at each step the linear generalised eigenvalue problem with $K^*(\omega) = \text{const}$ is solved by the Lanczos method [20], which is programmed in a truncated version, where the generalised eigenvalue problem is transformed into a standard eigenvalue problem with a reduced order symmetric three-diagonal matrix. Orthogonal projection operations are employed with greater economy and elegance using elementary reflection matrices. An iteration process terminates, when the following condition is satisfied

$$\frac{|\omega_{i+1} - \omega_i|}{\omega_i} \times 100\% \leq \xi \tag{6}$$

where ξ is a desired precision and ω_{i+1} is the real part of eigenfrequency of a structure calculated from the linear generalised eigenvalue problem with the storage and loss moduli for the frequency ω_i, which has been obtained from the same equation in the previous step. The modal loss factors of a structure for each vibration mode are determined by the following relation

$$\eta_n = \frac{\lambda_n''}{\lambda_n} \tag{7}$$

This approach gives the possibility to preserve the frequency dependence of viscoelastic materials and to calculate structures with high damping.

2.3 Material identification procedure

The basic idea of material identification procedure developed on vibration tests and non-direct optimisation methodology is that simple mathematical models (response surfaces) are determined only by the finite element solutions in the reference points of the plan of experiments. The identification parameters are obtained minimising the error functional, which describes a difference between the measured and numerically calculated parameters of structural response. A significant reduction in calculations of the identification functional is achieved in this case in comparison with the conventional optimisation methods.

2.3.1 Planning of experiments

Let us consider a criterion for elaboration of the plan of experiments independent on a mathematical model of the designing object or process [21]. The initial information for development of the plan is number of factors n and number of experiments k. The points of experiments in the domain of factors are distributed as regular as possible. For this reason the following criterion is used

$$\Phi = \sum_{i=1}^{k} \sum_{j=i+1}^{k} \frac{1}{l_{ij}^2} \Rightarrow \min \tag{8}$$

where l_{ij} is a distance between the points having numbers i and j $(i \neq j)$. Physically it is equal to the minimum of potential energy of repulsive forces for the points with unity mass if the magnitude of these repulsive forces is inversely proportional to the distance between the points.

For each number of factors n and number of experiments k it is possible to elaborate a plan of experiments, but it needs much computer time. Therefore each plan of experiment is developed only once and it can be used for various designing cases. The plan of experiments is characterised by the matrix of plan B_{ij}, when the domain of factors is determined as $x_j \in [x_j^{\min}, x_j^{\max}]$ and the points of experiments are calculated by the following expression

$$x_j^{(i)} = x_j^{\min} + \frac{1}{k-1}\left(x_j^{\max} - x_j^{\min}\right)\left(B_{ij} - 1\right), \quad i = 1, 2, ..., k \ , \ j = 1, 2, ...n \tag{9}$$

2.3.2 Response surface method

In the present approach a form of the equation of regression is unknown previously. There are two requirements for the equation of regression: accuracy and reliability. Accuracy is characterised as a minimum of standard deviation of the table data from the values given by the equation of regression. Increasing a number of terms in the equation of regression it is possible to obtain a complete agreement between the table data and values given by the equation of regression. However it is necessary to note that prediction in intervals between the table points can be not so good. For an improvement of prediction, it is necessary to decrease a distance between the points of experiments by increasing the number of experiments or by decreasing the domain of factors. Reliability of the equation of regression can be characterised by an affirmation that standard deviations for the table points and for any other points are approximately the same. Obviously the reliability is greater for a smaller number of terms in the equation of regression.

The equation of regression can be written in the following form

$$y = \sum_{i=1}^{p} A_i f_i(x_j) \tag{10}$$

where A_i are the coefficients of the equation of regression, $f_i(x_j)$ are the functions from the bank of simple functions $\theta_1, \theta_2, ..., \theta_m$ which are assumed as,

$$\theta_m(x_j) = \prod_{i=1}^{s} x_j^{\xi_{mi}} \tag{11}$$

where ξ_{mi} is a positive or negative integer including zero. Synthesis of the equation from the bank of simple functions is carried out in two stages: selection of perspective functions from the bank and then step-by-step elimination of the selected functions.

On the first stage, all variants are tested with the least square method and the function, which leads to a minimum of the sum of deviations, is chosen for each variant. On the second stage, the elimination is carried out using the standard deviation

$$\sigma_0 = \sqrt{\frac{S}{k-p+1}} \quad , \quad \sigma = \sqrt{\frac{1}{k-1}\sum_{i=1}^{k}\left(y_i - \frac{1}{k}\sum_{j=1}^{k} y_j\right)^2} \tag{12}$$

or correlation coefficient

$$c = \left(1 - \frac{\sigma}{\sigma_0}\right) \times 100\% \tag{13}$$

where k is the number of experimental points, p is the number of selected perspective functions and S is the minimum sum of deviations. It is more convenient to characterise an accuracy of the equation of regression by the correlation coefficient (Fig. 4). If insignificant functions are eliminated from the equation of regression, a reduction of the correlation coefficient is negligible. If in the equation of regression only significant functions are presented, an elimination of one of them leads to important decrease of the correlation coefficient.

2.3.3 Error functional minimisation

The error functional between experimental and numerical parameters of structural responses is written in the case of identification of elastic material properties as follows

$$\Phi(x) = \sum_{i=1}^{N} \frac{\left(f_i^{exp} - f_i^{FEM}\right)^2}{\left(f_i^{exp}\right)^2} \Rightarrow \min \tag{14}$$

At the same time for an identification of viscoelastic material properties it is determined already for each eigenfrequency

$$\Phi_i(x) = \frac{\left(f_i^{exp} - f_i^{FEM}\right)^2}{\left(f_i^{exp}\right)^2} + \frac{\left(\eta_i^{exp} - \eta_i^{FEM}\right)^2}{\left(\eta_i^{exp}\right)^2} \Rightarrow \min \tag{15}$$

To minimise these error functionals, the following constrained non-linear optimisation problems should be solved

$$\begin{aligned} &\min\ \Phi(x),\ H_i(x) \geq 0,\ G_j(x) = 0 \\ &i = 1, 2, ..., I,\ j = 1, 2, ..., J \end{aligned} \tag{16}$$

where I and J are the numbers of inequality and equality constraints. This problem is replaced with the unconstrained minimisation problem in which the constraints are taken into account with the penalty functions. The random search method is used for a solution of the formulated optimisation problem. An application of the curve fitting procedure is required additionally to obtain the frequency dependent viscoelastic material properties.

3 Identification examples and results verification

The developed inverse technique is tested on aluminium panel and applied to characterise orthotropic material properties of uni- and multi-directional laminated composite plate and viscoelastic material properties of 3M damping polymer ISD-112 used as a core material in sandwich panels.

3.1 Aluminium panel

Testing of the developed inverse technique based on vibration tests has been carried out identifying the material properties of homogeneous aluminium 6082-T6 plate with the following dimension $a \times b$=0.3×0.2 m, thickness h=0.002 m and material density ρ=2700 kg/m^3. Free-free boundary conditions have been applied and twelve first eigenfrequencies (Table 8) have been measured by the POLYTEC laser vibrometer (Fig. 3). To describe the isotropic material properties only two material constants are necessary, modulus of elasticity E and shear modulus G. The last is taken instead of Poisson's ratio υ to exclude an application of any scaling technique in the identification process. The boarders of identified parameters are taken in the present analysis as follows E=60-80 GPa and G=22-30 GPa.

The plan of experiments has been produced for 2 design parameters and 38 experiments. Then finite element analysis has been performed in 38 experimental points and 12 first eigenfrequencies have been determined. Employing these numerical values, the approximating functions (response surfaces) for all eigenfrequencies have been obtained. Minimising the error functional (Eq. 14), the elastic material constants have been identified

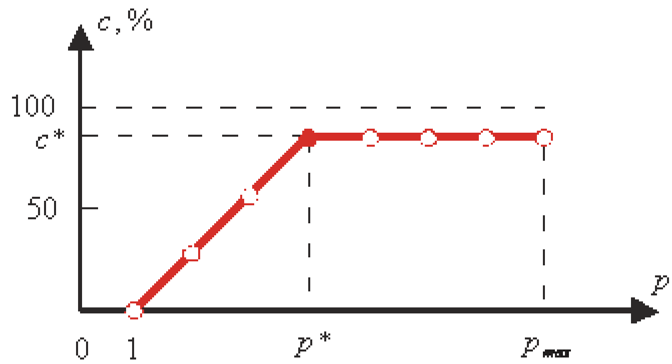

Figure 4: Diagram of elimination for the correlation coefficient.

Mode n	f_n^{EXP}, Hz	f_n^{FEM}, Hz	Δ, %
1	106	107	0.9
2	117	116	0.9
3	247	249	0.8
4	276	274	0.7
5	313	311	0.6
6	368	369	0.3
7	461	464	0.7
8	535	533	0.4
9	664	661	0.5
10	744	736	1.1
11	789	801	1.5
12	806	801	0.6

Table 8: Dynamic characteristics verification for aluminium panel.

Material properties	Identified	Static test	Technical data
E, GPa	69.0	68.9	68.9
G, GPa	25.6	26.3	25.9
υ	0.35	0.31	0.33

Table 9: Material properties verification for aluminium panel.

and presented in Table 9 to compare them with the results of static tension test and technical data presented by producer, where a good correlation is observed. Additionally the identified material properties have been verified comparing the experimentally measured eigenfrequencies with the numerically obtained using the identified elastic constants (Table 8). It is seen from this table that numerical eigenfrequencies are in a good agreement with the experimental results. The difference in terms of residuals is less than 1% in most cases.

3.2 Laminated composite panels

An identification of orthotropic material properties is carried out for the carbon/epoxy IM7/8552 uni- $(0)_{18}$ and multi-directional $(0/90/45/-45)_{S2}$ laminated composite plates with density ρ=1638·kg/m^3 and ρ =1619·kg/m^3 respectively, and following geometrical parameters: a=b=0.2 m, h=0.00225 m (uni-directional plate) and a=b=0.21 m, h=0.002 m (multi-directional plate). The fibre volume content of these laminates is about 60%. Natural eigenfrequencies (Table 10) have been measured by the POLYTEC laser vibrometer (Fig. 3) using free-free boundary conditions for the investigated panels.

To describe the orthotropic material properties of a single layer in the laminated composite plate, five material constants are necessary. There are moduli of elasticity E_1, $E_2 = E_3$, shear moduli $G_{12} = G_{13}$, G_{23} and Poisson's ratios $\upsilon_{12} = \upsilon_{13}$. Since the parameters with so different magnitude, as elastic moduli and Poisson ratio, are taken for identification, some scaling operations should be carried out. In paper [22] the scaling by longitudinal modulus E_1 has been employed. An additional scaling by the first experimental frequency allows a reduction of the number of unknown variables from five

Mode n	Uni-directional			Multi-directional		
	f_n^{EXP},Hz	f_n^{FEM},Hz	Δ,%	f_n^{EXP},Hz	f_n^{FEM},Hz	Δ,%
1	113*	113	0	164*	164	0
2	139*	141	1.4	257*	269	4.7
3	273*	271	0.7	326*	322	1.2
4	-	388	-	421	433	2.9
5	525*	520	1.0	453	450	0.7
6	556*	556	0	-	786	-
7	595*	601	1.0	797	792	0.6
8	-	734	-	848	847	0.1
9	780*	767	1.7	904*	899	0.6
10	899*	888	1.2	1021*	1003	1.8
11	-	977		1325*	1309	1.2
12	1252*	1262	0.8	-	1343	-
13	-	1330	-	1530*	1529	0.1
14	1396*	1380	1.1	-	1661	-
15	-	1521	-	1692*	1668	1.4
16	1574*	1560	0.9	1810	1777	1.8
17	1689	1674	0.9	1878	1906	1.5
18	-	1808	-	-	2040	-
19	-	1885	-	2175*	2162	0.6
20	1897	1888	0.5	2534	2529	0.2

*frequencies have been used in the identification process.

Table 10: Dynamic characteristics verification for laminated composite panels.

to four. The plan of experiments is produced in this case for 4 design parameters and 35 experiments. The boarders for the parameters identified, results of identification and verification by static tests produced according to ASTM guidelines are presented in Table 11, where in the brackets the values obtained from static compression tests are given. In general satisfied agreement of the results is observed. Verification of the identification results by additional dynamic numerical tests (Table 10) shows that the residuals in terms of eigenfrequencies are smaller for uni-directional laminated composite plate even for frequencies have not used in identification. This can be explained by some deviations of layers thickness and angles from the nominal values. It is necessary to note that the number of eigenfrequencies selected for identification have been different for each specimen as well as their combination. A cross validation for all sample points has been performed in such way to achieve a better approximation of the original function and to select the most important (most sensitive to elastic constants) and reliable frequencies.

3.3 Sandwich panel

A sandwich beam with the following dimensions: width B=0.05 m, length L=0.3 m and thickness of layers h_1=0.0012 m, h_2=0.000254 m, h_3=0.0008 m, has been chosen for a characterisation of 3M damping polymer ISD-112 used as a core material. The external layers are made from aluminium 2024-T6: E=64 GPa, υ=0.32, ρ=2695$\cdot$Ns^2/m^4. The clamped boundary conditions are applied from one side of the beam. The structural

Material properties	Identification boarders				Verification	
	Uni-directional		Multi-directional		Identified (average)	Static test
	min	max	min	max		
E_1^0 or E_1, GPa	150		150		153	168 (145)
$E_2 = E_3$, GPa	8.5	10.5	7.5	9.5	8.9	9.1 (8.9)
$G_{12} = G_{13}$, GPa	5.5	6.5	5.0	6.5	5.9	5.6
G_{23}, GPa	4.0	8.0	5.0	10.0	7.2	-
$\upsilon_{12} = \upsilon_{13}$	0.25	0.45	0.25	0.45	0.28	0.33

Table 11: Identification boarders and material properties verification for laminated composite panels.

Mode n	f_n^{EXP},Hz	f_n^{FEM},Hz	Δ,%	η_n^{EXP}	η_n^{FEM}	Δ,%
1	15	16	6.7	0.22	0.20	9.1
2	86	83	3.5	0.32	0.29	9.4
3	227	223	1.8	0.34	0.34	0
4	419	420	0.2	0.38	0.36	5.3
5	663	673	1.5	0.34	0.34	0
6	983	979	0.4	0.28	0.30	7.1
7	1340	1337	0.2	0.24	0.24	0

Table 12: Dynamic characteristics verification for sandwich panel.

dynamic characteristics, eigenfrequencies and corresponding loss factors (Table 12), have been obtained from physical vibration experiment by an impulse technique (Fig. 2).

To describe the viscoelastic isotropic material properties only one material parameter is necessary. This is modulus of elasticity $E^*(\omega) = E(\omega) + iE''(\omega)$. However in this case it is complex value consisting of storage $E(\omega)$ and loss $E''(\omega)$ parts, which are both frequency dependent. As known material parameters, Poisson ratio υ=0.49 and density ρ=1000·Ns^2/m^4 are taken into consideration. The boarders of identified parameters are taken in the present analysis as follows G=0.2-7.2 MPa and η=0-1.5.

The plan of experiments is produced for 2 design parameters and 98 experiments. Then the finite element analysis is performed in 98 experimental points and seven first dynamic characteristics are determined. Employing these numerical values, the approximating functions (response surfaces) for all eigenfrequencies and corresponding loss factors have been obtained with the correlation coefficients higher than 87%. Minimising the error functional (Eq. 15), material properties of 3M viscoelastic damping polymer ISD-112 are found for each eigenfrequency. These values are presented in Fig. 5 by points. Applying the curve fitting procedure, the following shear modulus (MPa) and material loss factor as functions on frequency are obtained in the frequency range f=5-1500 Hz

$$G = 3.195 - 1.397/z + 4.028z^2 \tag{17}$$

where $z = 0.3932 + 0.0004528f$

$$\eta = 1.669 - 0.8355/z - 0.7056/z^2 \tag{18}$$

where $z = 0.1 + 0.0006716f$. These dependencies are presented graphically in Fig. 5 with solid lines and they are used later in the finite element analysis to verify the identified material properties.

Table 12 shows a good correlation between experimental and numerical dynamic characteristics. The largest difference is observed for two first eigenfrequencies and corresponding loss factors.This can be explained by some inaccuracy of experiment, namely by low values of clamping forces at the fixed end of sandwich beam, when measured damping values always higher than calculated numerically due to additional energy dissipation in the clamping device.

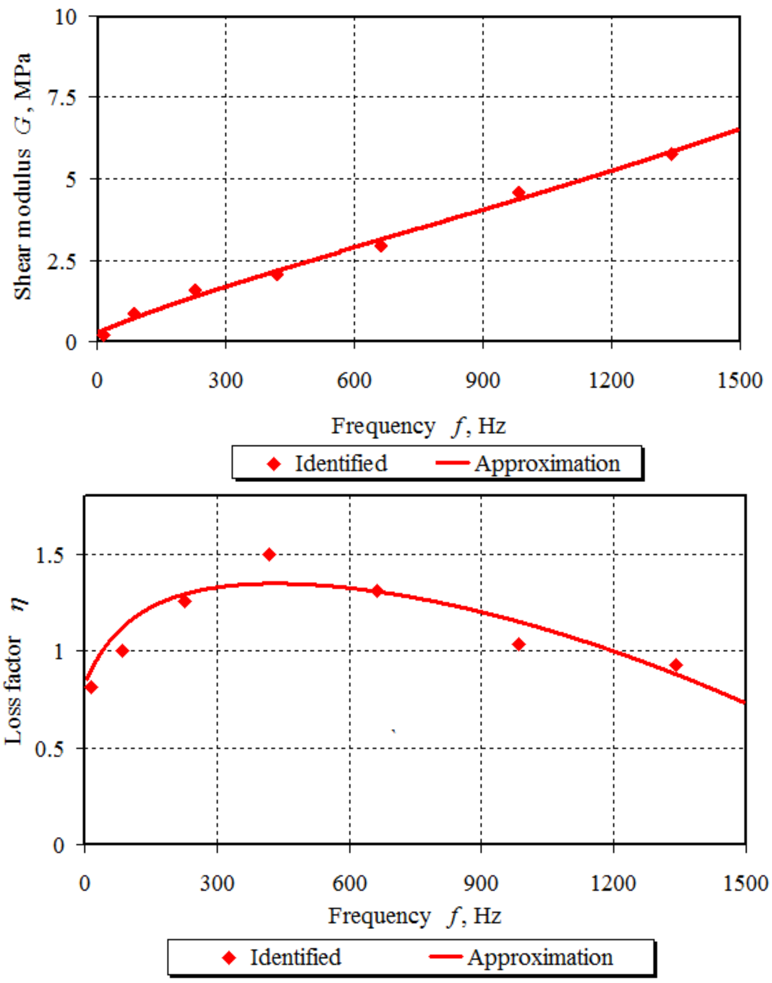

Figure 5: Identified viscoelastic material properties of 3M damping polymer ISD-112.

4 Conclusions

New inverse technique operating on vibration tests has been developed to characterise isotropic and orthotropic, elastic and viscoelastic properties of materials used widely in the advanced composite repairs of pipelines. The optimisation approach based on the planning of experiments and response surface technique to minimise the error functional has been applied in this case to decrease considerably the computational efforts. The present methodology gave the possibility to preserve the frequency dependence for storage and loss moduli of viscoelastic materials in wide frequency range and to analyse structures with high damping. The developed inverse technique has been tested on aluminium plates and successfully applied to characterise orthotropic material properties of laminated composites and viscoelastic material properties of 3M damping polymer ISD-112. Good correlation between experimental and numerical results has been observed in time of verification of identified material properties.

It is important to note that our current approach, like any other inverse approach based on vibration tests, has a non-destructive character and does not require special specimens for testing. The identified mechanical properties of adhesive materials generally reflect all the features of the technological processes used for the advanced composite repair.

The numerical experiments have shown that the accuracy of the developed inverse technique and identified material properties only depends on the accuracy of the physical experiments. The experimental errors mainly appear to be due to badly simulated boundary conditions, an added mass from exciting devices, air damping, and measurement noise.

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