MINMAXDM DISTRIBUTION FAMILY FOR TENSILE STRENGTH OF COMPOSITE

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Abstract

Generalization of extended family of weakest-link distributions with application to the composite specimen strength analysis is presented. Composite (specifically, monolayer) specimen for tensile strength is modeled as series system but every "link" of this system is modeled as parallel system. Results of successful attempts of using some specific distribution from this family for fitting of experimental dataset of strength of some carbon fiber reinforced specimens are presented.

1. Introduction

We consider a composite specimen for test of tensile strength as a bundle of n_C longitudinal items (fibers or bundles) immersed into *composite matrix* (CM), which is a composition of the matrix itself and all the layers with stackings different from the longitudinal one. We make very simplified assumption that only longitudinal items (LI) carry the longitudinal load but matrix only redistributes the loads after the failure of some longitudinal items. In fact, therefore, our model is a model of unidirectional (more specifically, monolayer) composite. We divide the composite into n_L parts of the same length l_1 (approximately, this length can be interpreted as the interval in which the load of failed LI is fully transmitted to the adjacent intact LI; the stronger the CM the smaller l_1). The total length of the composite specimens is equal to $l = n_L\, l_1$. We suppose that development of the process of fracture of a specimen takes place in one or in several of these parts ("links"). For simplicity, we call these links as "cross sections" (CS). So using this term we describe the composite as a *series system of CS*. For description of the development of fracture process of the series system it is appropriate to use the ideas on which the extended weakest link distribution family, described in the authors' papers [1-7], is based. Let the process of monotonous tensile loading (i.e. the process of increase of the nominal stress (or mean load of one LI) in the specimen cross section) be described by an ascending (up to infinity) sequence $\{x_1, x_2, ..., x_t, ...\}$, and let $K_{Ci}(t)$, $0 \le K_{Ci} \le n_C$, be the number of failures of LI in *i*-th CS with n_C initial number of LI at the load x_t. Then the strength of *i*-th CS

$$X_i^* = \max(x_t : n_C - K_{Ci}(t) \ge 0), \tag{1}$$

but the ultimate strength of the specimen (which is the sequence of n_L CS) is

$$X = \min_{1\le i\le n_L} X_i^* = \min_{1\le i\le n_L} \max(x_t : n_C - K_{Ci}(t) \ge 0)\,. \tag{2}$$

We consider different versions of cumulative distribution function (cdf) calculation methods and their applications to processing results of test of fiber strands (threads) and strip of them (monolayer) [8].

2. Models of failure of a parallel system with redistribution of load after failure of some LI

Statistical description of the development of the process of fracture of one CS (as loose bundle of LI (fibers or strands)) was initially studied by Daniels [9]. The respective model can be described in a following way. Let $(X_1,...,X_n)$ be random strengths of intact LI in some CS and X_j the *j*-th order statistics in this CS. If there is a uniform distribution of load between n LI, and load increases uninterruptedly, then the ultimate strength of this CS

$$X^* = \max_{1\le j\le n} X_j (n - j + 1)/n\,. \tag{3}$$

We consider the case when $n = n_C - K_C$. Daniels studied the case K_C=0. In the general case for random value of K_C, (technological) failure number, there is a priori distribution $\pi_C = (\pi_1, \pi_2, ..., \pi_{n_C+1})$ (here $\pi_k = P(K_C = k-1)$). Then

$$F_{X^*}(x) = \pi_C \vec{F}(x)\,, \tag{4}$$

where vector column $\vec{F}(x) = (F_1(x), ..., F_{n_{C+1}}(x))'$, $F_k(x)$, $k = 1,...,n_C$, is cdf of X^* if $n = n_C + 1 - k$, $F_{n_C+1}(x)$ is identical with unity (there are no intact LI).

Much broader spectrum of models of the considered process can be developed using the theory of Markov chains. We consider the process of accumulation of failures as an inhomogeneous finite Markov chain (MC) with finite state space $I = \{i_1, i_2, ..., i_{n_C+1}\}$. We say that MC is in state i if $(i-1)$ LI have failed, $i = 1,...,n_C+1$. State i_{n_C+1} is an absorbing state corresponding to the fracture of CS (fracture of all LI in this CS). The process of MC state change and the corresponding process $K_{Ci}(t)$ are described by transition probabilities matrix *P*.

$$P = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{23} & \cdots & p_{1(n_C+1)} \\ 0 & p_{22} & p_{23} & p_{24} & \cdots & p_{2(n_C+1)} \\ 0 & 0 & p_{33} & p_{34} & \cdots & p_{3(n_C+1)} \\ ... & ... & ... & ... & ... & ... \\ 0 & 0 & 0 & 0 & ... & p_{n_C(n_C+1)} \\ 0 & 0 & 0 & 0 & ... & 1 \end{bmatrix} \tag{5.1}$$

At the t-thstep of MC matrix P is a function of *t*, *t*=1,2,...

The cdf of strength of CS is defined on the sequence $\{x_1, x_2, ..., x_t, ...\}$ by equation

$$F_{X^*}(x_t) = \pi_C \Big(\prod_{j=1}^{t} P(j)\Big) u\,, \tag{5.2}$$

where $P(j)$ is the transition matrix for *t*=*j*, column vector $u = (0,...,0,1)'$.

We consider three main versions (hypotheses) of the structure of matrix *P*, denoted as P_a, P_b and P_c. In the simplest version we assume that in one step of MC only failure of one LI can take place. And for the corresponding matrix P_a we define $p_{ii} = 1 - F_C(x_t)$, where

$F_C(x_t) = (F_0(x_t) - F_0(x_{t-1})) / (1 - F_0(x_{t-1}))$ is conditional cdf of strength of a LI, the failure of which did not take place under load x_{t-1}, $F_0(x)$ is the initial cdf of strength of a LI ; $p_{i(i+1)} = 1 - p_{ii}$, $i = 1,...,n_C$, $p_{(n_C+1)(n_C+1)} = 1$, but all the other p_{ij} are equal to zero.

It can be assumed also that the number of failures in one step of MC has binomial distribution. Then for the corresponding matrix P_b we have $p_{i(i+r)} = b(r; p, k) = p^r (1-p)^{k-r} k!/ r!(k-r)!$, $p = F_C(x_t)$, $k = n_c + 1 - i$, $r = 0,...,k$, $i = 1,...,n_C$; and again $p_{(n_C+1)(n_C+1)} = 1$, but all the other p_{ij} are equal to zero.

For both versions of P described by P_a and P_b we suppose a uniform load distribution between intact LI. The third version corresponds to a transverse crack growth in the monolayer. We suppose that the first failure appears in the boundary of CS and all the following failures can appear only in the adjacent LI. The difference between the second and the third version is illustrated in Fig.1. Let now j be ordernumber of LI in a CS (j=1 for the boundary LI). In this case it is easy enough to take into account the stress concentration next to the tip of the crack. Let the redistribution of CS load $x(t)$ between intact LI be defined by a "stress concentration" function $h(j; i, n_C)$. Then in the corresponding P_c matrix $p_{ij} = \prod_{i+1}^{j} F_C(x_{ij}(t)) \prod_{j+1}^{n_C+1} (1 - F_C(x_{ij}(t))$ for $j = i+1,...,n_C$; $p_{i(n_C+1)} = \prod_{i+1}^{n_C+1} F_C(x_{ij}(t))$ for $j = n_C$; $p_{ii} = 1 - \sum_{i+1}^{n_C+1} p_{ij}$, $p_{ij} = 0$ for $j < i$, $i = 1,...,n_C$;

where $x_{ij}(t) = h(j; i, n_C) x(t) n_C / (n_C + 1 - i)$ describes stress in j-th order LI after failure of i-th order LI.

1	0	1	1
0	1	0	1
1	1	1	0
0	0	0	1
1	1	0	1
1	1	1	1

a

0	1	1	1
0	0	1	1
0	1	1	1
0	0	0	1
0	1	1	1
1	1	1	1

b

Fig.1. Failed (0) and surviing (1) longitudinal items (LI) in specimens (under longitudinal load) with six cross sections and four LI; for uniform stress distribution (a) and for the case of transverse crack growth.

3. Models of failure of a series system (chain of links) with damaged items

In the framework of considered problem, there is a special case of n_C=1 (i.e. there is only one fiber, strand or thread). This case was studied in [6]. Below, we remind the main ideas, make the necessary corrections (appropriate for notation of this paper), and provide some generalization. We consider a specimen as a straight binary series system with n_L links of two types. There is a random number of "damaged" links ,K_L, $0 \le K_L \le n_L$, with strength cdf $F_Y(x)$ (we say that they are Y-type links), and there are ($n_L - K_L$) links with strength cdf $F_Z(x)$ (we say they are Z-type links). "Damaged" links appear if stress in LI exceeds *defect initiation stress*. The probability of this event at the load (stress) x is defined by cdf of defect initiation stress $F_K(x)$.

We suppose (see [6]) that the failure process of considered system has two-stages. In the first stage, the process develops along the specimen and damage appear in K_L, $0 \le K_L \le n_L$, links (K_L links of Y-type appear). Then the second stage takes place: the process of accumulation of elementary damages in crosswise direction up to specimen failure. We consider three levels of accuracy of description of the second stage and three corresponding probability models (probability structure). Level A: the development of fracture process takes place in every link (containing or not some initial defects) and the strength of the weakest link defines the strength of the specimen. Level AB: the strength of the link without defects can be (relatively) so high and probability of its fracture before fracture of the damaged link so small that independence of failure probability of any Z-type CS on n_L can be assumed (only the probability that $K_L > 0$ depends on the number of links, n_L). And finally, level B: in addition to the assumption of the level AB it is assumed that the cdf of strength of the critical link does not depend on this number also. Correspondingly we have three probability structures.

A : $X = \min(Y_1, ..., Y_{K_L}, Z_1, ..., Z_{n_L - K_L})$;

AB: $X = \begin{cases} \min(Y_1, ..., Y_{K_L}, Z), & K_L > 0, \\ Z, \quad K_L = 0; \end{cases}$ B : $X = \begin{cases} Y, \ K_L > 0, \\ Z, \ K_L = 0. \end{cases}$.

Two different versions of the first stage can be considered also. First version: (technological) defects appear before the loading and their number does not depend on the subsequent loading. Second version: defects appear during loading (instantly or gradually) and their number depends on the load.

3.1. For "instant fracture" version for structures A, AB, B we have correspondingly

$$F(x) = 1 - (1 - F_Z(x))^{n_L} \sum_{k=0}^{n_L} p_k \delta^k(x), \quad \delta(x) = (1 - F_Y(x))/(1 - F_Z(x)), \tag{6}$$

$$F(x) = 1 - \sum_{k=0}^{n_L} p_k \left(1 - F_Y(x)\right)^k \left(1 - F_Z(x)\right) = 1 - (1 - F_Z(x)) \sum_{k=0}^{n_L} p_k \left(1 - F_Y(x)\right)^k, \tag{7}$$

$$F(x) = p_Y F_Y(x) + (1 - p_Y) F_Z(x), \tag{8}$$

where (in equations (6, 7)) binomial probability mass function (pmf) $p_k = b(k; p_L, n_L) = p_L^k (1 - p_L)^{n_L - k} n_L!/k!(n_L - k)!$ is probability that there is k links of Y-type; $p_Y = 1 - p_0 = 1 - (1 - p_L)^{n_L}$ is the probability that there is at least one link of Y-type (in this case, actually, it is enough to know only p_Y; we should not know two parameters n_L and p_0 separately).

Binomial or Poisson pmf can be used for random number of links of Y-type, K_L. In the latter case equations (6, 7) (approximately, if n_L is sufficiently large) can be written in the following way

$$F(x) = 1 - (1 - F_Z(x))^{n_L} \exp(-\lambda(1 - \delta(x)), \tag{9}$$

$$F(x) = 1 - (1 - F_Z(x)) \exp(-\lambda F_Y(x)), \tag{10}$$

where $\lambda = n_L p_L$ or it is just independent parameter of Poisson pmf. If initiation of the defects depends on the applied load, then it can be assumed that $p_L = F_K(x)$, where $F_K(x)$ is the cdf of defect initiation load.

In the numerical example considered in this paper it was assumed that the strength of defected link S has Weibull distribution; then $Y = \log(S)$ has the smallest extreme value (sev) distribution

$$F_Y(x) = 1 - \exp(-\exp((x - \theta_{0Y})/\theta_{1Y})) \,. \tag{11}$$

And it was assumed also that for link without defects

$$F_Z(x) = 1 - \exp(-\exp((x - \theta_{0Z})/\theta_{1Z})) \tag{12}$$

but for the logarithm of defect initiation stress

$$F_K(x) = 1 - \exp(-\exp((x - \theta_{0K})/\theta_{1K})) \,. \tag{13}$$

In some numerical examples it was considered that if $\theta_{0Z} = C$, but $\theta_{1Z} \to 0$, then

$$F_Z(x) = \begin{cases} 0, & x < C, \\ 1, & x \geq C. \end{cases} \tag{14}$$

3.2. The process of gradual (during loading) accumulation of defects along the chain of n_L links again can be considered as a Markov chain (MC). In this case MC is in state i if there are $(i-1)$ of Y-type links, i=1,...,n_L+1. State i_{n_L+2} is an absorbing state corresponding to the fracture of specimen. The matrix of transition probabilities has the same form as in (5.1) . The initial distribution of K_L is represented now by some row vector $\pi_L = (\pi_{L1}, \pi_{L2}, ..., \pi_{L,n+1,} \pi_{L,n+2})$. In the new approach the number of CS of Y-type and the strength of specimens are random functions of time, $K_L(t)$ and $X(t)$. Now the three main structures we denote by MA, MAB and MB. They have the same description but instead of K_L we should write $K_L(t)$. For example, for the MA we have $X(t) = \min(Y_1, Y_2, ..., Y_{K_L(t)}, Z_1, Z_2, ..., Z_{n_L - K_L(t)})$. In similar way $X(t)$ is defined for the other structures.

Now the ultimate strength of specimen is defined again by equations (2) but it is more convenient to write it in new form:

$$X = x_{T^*} \,, \tag{15}$$

where

$$T^* = \max(t : X(t) > x_t) \,. \tag{16}$$

The cdf of ultimate strength , *X*, is defined again by an equation similar to equation (5.2):

$$F_X(x_t) = \pi_L(\prod_{j=1}^{t} P(j))u \,.$$

Specifying the matrix *P* for probability structures A and AB. The probability that in some element a defect appears at the stress x_t under the condition that it has not appeared at the stress x_{t-1} is

$$b(t) = (F_K(x_t) - F_K(x_{(t-1)}))/(1 - F_K(x_{(t-1)})) \,.$$

Consider the case of *s* defects present. The probability that *r* new defects appear, $0 \leq r \leq k = n - s$, and the total number of defects is equal to *m*=*s*+*r*

$$\tilde{p}_{sm}(t) = (b(t))^r (1 - b(t))^{k-r} k!/r!(k-r)!$$

Conditional probability of Y-type link fracture at the nominal stress x_t

$$q_Y(t) = (F_Y(x_t) - F_Y(x_{(t-1)}))/(1 - F_Y(x_{(t-1)})) \,.$$

Conditional probability of Z-type link fracture at the nominal stress x_t

$$q_Z(t) = (F_Z(x_t) - F_Z(x_{(t-1)}))/(1 - F_Z(x_{(t-1)})) \,.$$

Corresponding probability that none of the links (of both types) fails when there are defects in *m* links for probability structure MA is

$$u_m(t) = (1-q_Y(t))^m (1-q_Z(t))^{n_L - m},$$

and for probability structure MAB

$$u_m(t) = (1-q_Y(t))^m (1-q_Z(t)).$$

The probability of coincidence of these events, which we consider as independent, and the probability of transition from state *i=s+1* to state *j=i+r*

$$p_{ij}(t) = \widetilde{p}_{(i-1)(j-1)}(t) u_{j-1}(t),$$

where $i \le j \le (n+1)$.

It is worth to note that if equation (14) is used and C is large enough (this means that only damaged CS define the strength) then it can be assumed that $q_Z(t)$=0.

Conditional fracture probability (for both probability structure MA and MAB) at state *i*

$$p_{i(n+2)}(t) = 1 - \sum_{j=i}^{n+1} p_{ij}(t).$$

Of course, $p_{ij}(t) = 0$, if $j < i$, and $p_{(n+2)(n+2)}(t) = 1$.

Specifying the matrix *P* for probability structures MB . The corresponding Markov chain has only three states. The first state corresponds to the absence of defective links, the second one means the presence of at least one defective link, and the third, absorbing one, means failure of the specimen. The corresponding probabilities at a *t*-th step are determined by the formulae

$$p_{11}(t) = [1-b(t)]^{n_L}, \quad p_{12}(t) = (1-p_{11}(t))(1-q_Y(t))(1-q_Z), \quad p_{13}(t) = 1 - p_{11}(t) - p_{12}(t),$$

$$p_{21}(t) = 0, \quad p_{22}(t) = (1-q_Y(t))(1-q_Z(t)), \quad p_{23}(t) = 1 - p_{22}(t), \quad p_{31}(t) = p_{32}(t) = 0, \quad p_{33}(t) = 1.$$

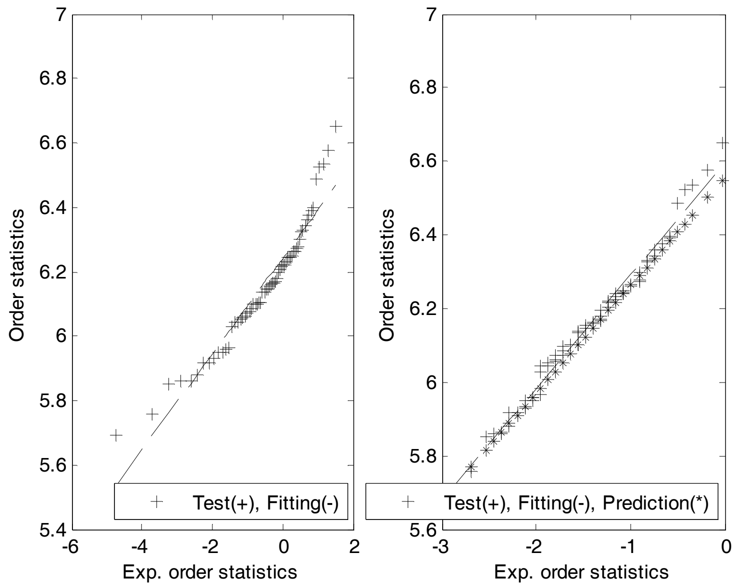
4. MinMaxDM distribution family

Clearly, all the ideas considered in the previous section can be used also for the series system of CS if instead of the word "link" now we use the word CS. Instead of cdf $F_Y(x)$ and $F_Z(x)$, which were defined by (11-12) now we should use cdf of CS strength of Y-type or Z-type correspondingly. For building these cdf in the following numerical examples we again suppose that logarithm of strength of one LI (in one CS) without defect has the smallest extreme value (sev) distribution: $F_0(x) = 1 - \exp(-\exp((x-\theta_{0Z1})/\theta_{1Z1}))$. We use the logarithm scale and in this case the cdf of specimen strength also has location and scale parameters θ_0 and θ_1: $F_X(x) = F_{\overset{0}{X}}((x-\theta_0)/\theta_1))$. Of course it is not the only possible assumption. Different assumptions about the distribution of strength of bundles within the frame of one CS (one "link") , a priori distribution of initial (technological) defects, the influence of length and width of specimens compose a family of the distributions of ultimate composite tensile strength. Taking into account (2) and (3) we denote this family by abbreviation MinMaxD (in memory of Daniels) if the strength $F_{X^*}(x)$ is defined by equation (4) and by abbreviation MinMaxM (because of connection with Markov chain theory), if it is defined by equation (5), and for unified family we suggest an abbreviation MinMaxDM.

5. Processing of test data

In this paper we consider only the application of B-structure to the test data set processing. In [5] there are the test results of both 64 carbon fiber strands with length 20 mm (data_1) and the same number of strips of 10 strands of the same length (data_2) considered. We attempt to obtain statistical description of data_2 using results of processing of data_1. Let x_i be *i*-th order statistic, $i = 1, 2, ..., n$, n is the sample size; $E(X_i)$ is the expected value of ith order statistic, $E(\overset{0}{X}_i)$ is the same but for θ_0=0 and θ_1=1. Then for estimation of θ_0 and θ_1, if all the other parameters are fixed, we have the following linear regression model: $E(X_i)=\theta_0+\theta_1\, E(\overset{0}{X}_i)$. We perform fitting of the data_1 and get linear regression parameter estimates $\widehat{\theta}_0$=6.554 and $\widehat{\theta}_1$=0.1243 assuming that sev distribution holds (here x is logarithm of strength).Then we perform fitting (expected value of "standard" order statistics $E(\overset{0}{X}_i)$ versus order statistics) of the data_2 (+) assuming the same type of distribution (see Fig. 2a). In Fig. 2b we see the fitting of the same data_2 using $E(\overset{0}{X}_i)$ of cdf corresponding to MinMaxMa-Bsev model (for P_a type of matrix P, $F_0(x)$ is sev distribution, structure B (see equation (8) where n_C=5; π_C is a binomial a priori distribution of K_C with p_C=0.01 , $n = n_C$=5; $p_Y = 0.9048$). "Regression prediction"(*), $\hat{x}_i = \hat{\theta}_0 + \hat{\theta}_1 E(\overset{o}{X}_i)$, using estimates $\widehat{\theta}_0$ and $\widehat{\theta}_1$ obtained processing data_1 is shown also. But here we take into account variation of Young's modulus also: Var(E)= 0.03).

Let us make additional explanations. For "fitting" of data_2 we have used parameters, found by processing of the same data. For "Regression prediction" we have used estimates $\widehat{\theta}_0$ and $\widehat{\theta}_1$ obtained processing data_1, which are parameters of component of monolayer (as if we did not get the parameter estimates of data_2 while fitting these data). However it is not PREDICTION but "PREDICTION", because in fact we have used also the estimates of "structure parameters" p_C, n_C and p_Y which was found processing data_2. It would be real prediction if n_C and p_Y are **parameters of technology** and they **are nearly the same for different specimens with the same type of technology and are known in advance .**
Unfortunately, it is only hope, but it is not the fact.



a b

Fig. 2. Fitting (expected value of "standard" order statistics $E(\overset{0}{X}_i)$ versus order statistics) and "prediction" of results of tensile strength test of carbon fiber strip of 10 strands using sev distribution (a) and MinMaxMa-Bsev model (b) (see explanation in text).

The statistic $OSPPt = (\sum_{i=1}^{n}(x_i - \hat{x}_i)^2 / \sum_{i=1}^{n}(x_i - \overline{x})^2)^{1/2}$, where $\overline{x} = \sum_{i=1}^{n} x_i / n$ [4] , as the measure of fitting for Fig.1a is equal to 0.267 (for sev distribution) and as the measure of fitting and

prediction quality for Fig. 1b (for MinMaxMa.sev-B structure model) is equal to 0.161 and 0.192 correspondingly.
Examples of processing data of strength of fibers of different type are given in [6].
Here we consider processing of the test results of carbon reinforced composite specimens $((0^o_6/+-45^o_4/90^o_3)_s$, length : 250 mm, width : 38 mm, thickness : 1.7 mm) which are given in [8]. In Fig. 3a we see fitting of these data (+) using sev distribution (statistics OSPPt=0.2504). In Fig. 3b we see fitting of the same data using MinMaxMa-Bsev model (statistics OSPPt=0.1548). "Prediction" of these data using MinMaxMa-Bsev model (*) and linear regression parameter estimates $\hat{\theta}_0$=6.554 and $\hat{\theta}_1$=0.1243 of data_1 (statistics OSPPt=0.1879) is shown also . This time n_C=50 was used; π_C is a binomial priori distribution of K_C with p_C=0.325 , $n = n_C$=50; $p_Y = 1$.

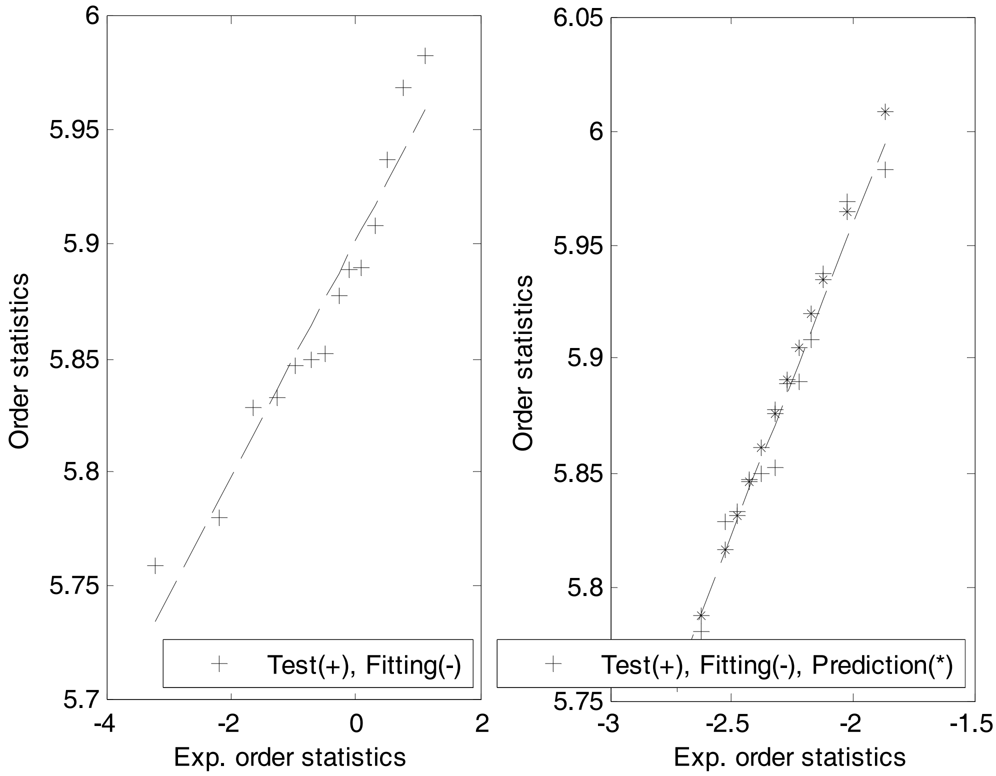


Fig. 3. Fitting (expected value of "standard" order statistics $E(\overset{0}{X}_i)$ versus order statistics) and "prediction" of the tensile strength of carbon reinforced composite specimens test results (+) using sev distribution (a) and MinMaxMa-Bsev model (b) (see explanation in text).

Conclusions

We see that MinMaxMa-Bsev model provides better (than sev distribution) fitting of results of tensile strength test of carbon fiber strip of 10 strands (but only if we assume that in CS there are only 5 strands instead of 10 and taking into account variation of Young's modulus!). It is not surprising, of course, because for MinMaxMa-Bsev we have much more parameters. Nearly the same can be said about processing the specimen data. This time $n_C = 50$ appears much more appropriate. The values $n_C = 5$ and $n_C = 50$ can be interpreted as the numbers of failures of LI which are sufficient to provoke the catastrophic failure of the specimens. Very large value of p_C=0.325 for specimen data set can be explained by the small relative value of ratio of longitudinal layer number to the total number of layers (6/(6+4+3)= 0.4615). There is a temptation to use the coefficient of filling. However there is a large ambiguity of calculation of this value.

As a whole, it seems that MinMaxDM distribution family deserves to be studied much more thoroughly using much more test data. Interpretation of parameters of a corresponding model allows comparison of different composite structures and explanation of some specific features of failure process of composite . For example, the value p_C=0.325 indicates that at least 32.5% of the critical cross section does not carry the longitudinal load.

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