BINARY LAMBDA-SET FUNCTION AND RELIABILITY OF AIRLINE

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ABSTRACT

A definition of binary λ - set function is introduced. It is used for the inspection interval planning in order to limit a probability of fatigue failure rate (FFR) of an airline (AL). A solution of this problem is based on a processing of the result of the acceptance full - scale fatigue test of a new type of an aircraft. Numerical example is given.

1 INTRODUCTION

This paper is really the addition to the previous author paper [1] devoted to the reliability of an aircraft (AC) and presentation [2]. Here we consider the reliability of the process of operation of an airline when after the specified life is reached (retirement time), fatigue failure discovery or fatigue failure takes place a new aircraft is acquired and the operation of airline is continued up to infinity. We consider again the case when for solution of the problem of limitation of fatigue failure rate (FFR) of airline we know the type of distribution function of fatigue life of AC but do know the parameter of this function, for estimation of which we have the result of the acceptance full - scale fatigue test of a new type of an aircraft.

Despite of all the simplicity, the equation $a(t)=\alpha\exp(Qt)$ gives us rather comprehensible description of fatigue crack growth in the interval (t_d,t_c), where (we recall) t_d is the time when a fatigue crack becomes detectable ($a(t_d)=a_d$) and t_c is the time when the crack reaches its critical size ($a(t_c)=a_c$) and fatigue failure takes place. It can be assumed that corresponding random variables $T_d=(\log a_d-\log\alpha)/Q=C_d/Q$ and $T_c=(\log a_c-\log\alpha)/Q=C_c/Q$ have the lognormal distribution because, as it is assumed usually, normal distribution of $\log T_c$ can take place only if either both $\log C_c$ and $\log Q$ are normally distributed or if one of these components is normally distributed while another one is constant. We suppose also, that vector $(X,Y)=(\log(Q),\log(C_c))$ has two dimensional normal distribution with vector-parameter $\theta=(\mu_X,\mu_Y,\sigma_X,\sigma_Y,r)$. It is worth to note, that for the case when a_c and a_d are constants then cdf of C_d is completely defined by the distribution of C_c because $C_d=C_c-\delta$, where $\delta=\log(a_c/a_d)$.

First, we consider solution of the problem for the known distribution parameter and then for the unknown one. Numerical example will be given.

2 PROBABILITY MODEL FOR THE KNOWN θ

Just as in paper [1] for the known θ, there are two decisions: 1) the aircraft is good enough and the operation of this aircraft type can be allowed, 2) the operation of the new type of AC is not allowed. A redesign of AC should be made. In the case of the first decision, the vector $t_{1:n}=(t_1,\ldots,t_n)$, where t_i, $i=1,...,n$, is the time moment of *i*-th inspection, should be defined also. If θ is known the different rules can be offered for the choice of structure of the vector $t_{1:n}$: 1) every interval between the inspections is equal to the constant $t_{SL}/(n+1)$, where t_{SL} is the aircraft specified life (SL) (the retirement time), 2) the probabilities of a failure in every interval are equal to the same value … In this paper we consider the first approach, but really, our considerations can be applied in more general case when the vector $t_{1:n}$ is defined by two parameters, the fixed t_{SL} and the the number of inspections, n, in such a way that probability of failure tends to zero when n tends to infinity.

For the substantiation of the choice of inspection number we should know FFR and the gain of AL as a functions of vector $t_{1:n}$. For this purpose the process of an operation of AL can be considered as an Markov chain (MCh) with $(n+4)$ states. The states $E_1,E_2,\ldots,E_{n+1}$ correspond to an AC operation in the time intervals $[t_0,t_1),[t_1,t_2),\ldots,[t_n,t_{n+1})$, $t_0=0$, $t_{n+1}=t_{SL}$. States E_{n+2}, E_{n+3} and E_{n+4} correspond to the following events : the specified life SL is reached , fatigue failure (FF) or fatigue crack detection (CD) take place. In all these three cases the acquisition of new AC takes place.

	E_1	E_2	E_3	...	E_{n-1}	E_n	E_{n+1}	E_{n+2} (SL)	E_{n+3} (FF)	E_{n+4} (CD)
E_1	0	u_1	0	...	0	0	0	0	q_1	v_1
E_2	0	0	u_2	...	0	0	0	0	q_2	v_2
E_3	0	0	0	...	0	0	0	0	q_3	v_3
...	...	...	...	...	...	...	...	...	...	...
E_{n-1}	0	0	0	...	0	u_{n-1}	0	0	q_{n-1}	v_{n-1}
E_n	0	0	0	...	0	0	u_n	0	q_n	v_n
E_{n+1}	0	0	0	...	0	0	0	u_{n+1}	q_{n+1}	v_{n+1}
E_{n+2} (SL)	1	0	0	...	0	0	0	0	0	0
E_{n+3} (FF)	1	0	0	...	0	0	0	0	0	0
E_{n+4} (CD)	1	0	0	...	0	0	0	0	0	0

Figure 1. Matrix of transition probabilities P_{AL}.

In the corresponding transition probability matrix, P_{AL}, let ν_i be the probability of a crack detection during the inspection number i, let q_i be the probability of the failure in service time interval $(t_{i-1}, t_i]$, and let $u_i = 1 - \nu_i - q_i$ be the probability of successful transition to the next state. In our model we also assume that an aircraft is discarded from a service at t_{SL} even if there are no any crack discovered by inspection at the time moment t_{SL}. This inspection at the end of $(n+1)$-th interval (in state E_{n+1}) does not change the reliability but it is made in order to know the state of an aircraft (whether there is a fatigue crack or there is no fatigue crack). Here it is supposed that fatigue crack is discovered with probability equal to unit if inspection is made in interval (T_d, T_c). It can be shown that

$$u_i = P(T_d > t_i \mid T_d > t_{i-1}),\ q_i = P(t_{i-1} < T_d < T_c < t_i \mid T_d > t_{i-1}),\ \nu_i = 1 - u_i - q_i,\ i = 1, \ldots, n+1. \quad (1)$$

In the three last lines of the matrix P_{AL} there are three units in the first column, corresponding to renewal of an operation of the airline (the AL operation returns to the first interval). All the other entries of this matrix are equal to zero, see Fig.1.

Using the theory of semi-Markov process with rewards and definition of P_{AL} we can get the vector of stationary probabilities, $\pi = (\pi_1, ..., \pi_{n+4})$ which is defined by the equation system

$$\pi P_{AL} = \pi,\ \sum_{i=1}^{n+4} \pi_i = 1 \quad (2)$$

and the airline gain

$$g(n) = \sum_{i=1}^{n+4} \pi_i g_i(n), \quad (3)$$

where

$$g_i(n) = \begin{cases} a_i u_i + b_i q_i + c_i \nu_i, i = 1, \ldots, n+1, \\ d_i, i = n+2, \ldots, n+4, \end{cases}. \quad (4)$$

a_i is the reward defined by the successful transition from one operation interval to the following one and the cost of one inspection; b_i, c_i and d_i correspond to transition to the states E_{n+3} (FF), E_{n+4} (CD) and then to the state E_1 (the "cost" of FF of AC, fatigue crack detection, acquisition of new AC). Let us note that if $a_i = t_i - t_{i-1}$, $b = c = d = 0$ then

$$g(n) = \sum_{i=1}^{n+4} \pi_i g_i(n) = g_t(n) = \sum_{i=1}^{n+1} \pi_i (t_i - t_{i-1}) \quad (5)$$

and $L_j = g_t(n, \theta) / \pi_j$ defines the mean step number of MCh to return to the same state E_j, $\lambda_F(n, \theta) = 1 / L_{n+3} \Delta_t$, where $\Delta_t = t_{SL} / (n+1)$, is the FFR.

If θ is known we calculate the gain as a function of n, $g(n, \theta)$, and choose the number n_g corresponding to the maximum of the gain :

$$n_g(\theta) = \arg\max_n g(n, \theta). \quad (6)$$

Then we calculate FFR as function of n, $\lambda_F(n, \theta)$, and choose n_λ in such a way that for any $n \geq n_\lambda$ the function $\lambda_F(n, \theta)$ will be equal or less than some value λ:

$$n_\lambda(\lambda, \theta) = \min\{n : \lambda_F(n, \theta) \leq \lambda, \text{ for all } n \geq n_\lambda(\lambda, \theta)\}. \quad (7)$$

And finally

$$n = n_{g\lambda}(\lambda, \theta) = \max(n_g, n_\lambda). \quad (8)$$

3. SOLUTION FOR AN UNKNOWN θ

In [1] the problem of a limitation of fatigue failure probability in an operation of one AC (FFP1) was considered using the definition of binary p-set function

3.1. Binary p-set function

Now let us take into account that we consider the case when the for the estimate of unknown parameter θ , $\hat{\theta}=\hat{\theta}(x_1,...,x_n)$, the result of acceptance test is used and the operation of a new type of aircraft will not take place if the result of the fatigue test in a laboratory is "too bad" (previously, the redesign of the new type of AC should be made). We say that in this case the event $\hat{\theta}\notin\Theta_0$, $\Theta_0\subset\Theta$ takes place (for example, $\hat{\theta}\notin\Theta_0$ if the test fatigue life T_C is lower than some limit; or $n(p,\hat{\theta})$ is too large,…).
Let us define some binary set function

$$S(\hat{\theta},\Theta_0,n)=\begin{cases}\bigcup_{i=1}^{n+1}S_i(n) \ \ if \ \ \hat{\theta}\in\Theta_0, \\ \varnothing, \ \ if \ \ \hat{\theta}\notin\Theta_0\end{cases}, \tag{9}$$

where $S_i(n)=\{(t_d,t_c):t_{i-1}<t_d,t_c\le t_i\}$, $t_i=it_{SL}/(n+1)$, $i=1,...,n+1$; $\varnothing$ is an empty set.
It can be shown that for very wide range of the definition of the set Θ_0 and the requirements to limit FFP1 by the value p^*, where $(1-p^*)$ is a required reliability, there is a preliminary "designed allowed FFP1" , p_{FD}, such that corresponding set function $S(\hat{\theta},\Theta_0,n(p_{FD},\hat{\theta}))$ is *binary p-set function of the level p^* for the vector $Z=(T_d,T_c)$ on the base of the estimate $\hat{\theta}$:*

$$\sup_\theta\sum_{i=1}^{n+1}P(Z\in S_i(n(p_{FD},\hat{\theta}))\bigcap\hat{\theta}\in\Theta_0)=p^* \ . \tag{10}$$

This means that FFP1 will be limited by the value p^* for any unknown $\theta\in\Theta$.

3.2. Binary λ-set function

In similar way, it can be shown that for very wide range of the definition of the set Θ_0 and the requirements to limit FFR of AL by the value λ^*, where λ^* is a required fatigue failure intensity, there is a preliminary "designed allowed FFR" , λ_{FD}, such that corresponding set function $S(\hat{\theta},\Theta_0,n_\lambda(\lambda_{FD},\hat{\theta}))$ is *a binary λ-set function of the level λ^* for the vector $Z=(T_d,T_c)$ on the base of the estimate $\hat{\theta}$:*

$$\sup_\theta E((\lambda(n_\lambda(\lambda_{FD},\hat{\theta}))\,|\,\hat{\theta}\in\Theta_0)*P(\hat{\theta}\in\Theta_0))=\lambda^* \ . \tag{11}$$

This means that FFR will be limited by the value λ^* for any unknown $\theta\in\Theta$.

Let us note, that instead of the words a *binary λ-set function* we should use the words *binary λ_g-set function* if instead of $n_\lambda(\lambda_{FD},\hat{\theta})$ we use $n_{g\lambda}(\lambda_{FD},\hat{\theta})$.
For the requirement of a high reliability the choice of an inspection number will be defined by the limitation of FFR. For very high "cost" of FF of AC it will be defined by the maximum of the gain.

4 NUMERICAL EXAMPLE

The example of the solution of the reliability problem of aircraft fleet is considered in [1, 2]. Here we consider only the problem of reliability of AL.

We use the following definitions of the components of an AL income: for all $i = 1,...,n+1$ $a_i = a_0(n) + d_{insp}t_{SL}$, where $a_0(n) = a_{01}t_{SL} / (n+1)$, - is the reward related to successful transition from one operation interval to the following one, a_{01} defines the reward of operation in one time unit (one hour or one flight); $d_{insp}t_{SL}$ is the cost of one inspection (negative value) which is supposed to be proportional to t_{SL}; $b_i = b_{01}t_{SL}$ is related to FF (negative value), $c_i = c_{01}a_0(n)$ is the reward related to transitions from any state $E_1,...,E_{n+1}$ to the state E_{n+4} (it is supposed to be proportional to a_0 because it is a part of a_0); $d_i = d_{01}t_{SL}$ is negative reward, the absolute value of which is the cost of new aircraft acquisition after events SL, FF or CD and transition to E_1 takes place. In numerical example we have used the following values: $t_{SL} = 40000$, b_{01}=-0.3; d_{insp} = - 0.05; a_{01}=1; c_{01}=0.1; d_{01}=- 0.3.

Suppose we have the following estimate of parameter $\theta = (\mu_X, \mu_Y, \sigma_X, \sigma_Y, r)$: $\hat{\theta}$=(-8.58688044, 1.9424608, 0.155, 0.0778895, 0.796437) (see Fig 2.2 and Table 2.1 in [3]). It was assumed that the set Θ_0 corresponds to the decision to make redesign if the estimate of critical time to failure $t_C = \exp(\hat{\mu}_Y - \hat{\mu}_X)$ is too small: $t_C < 0.3t_{SL}$.

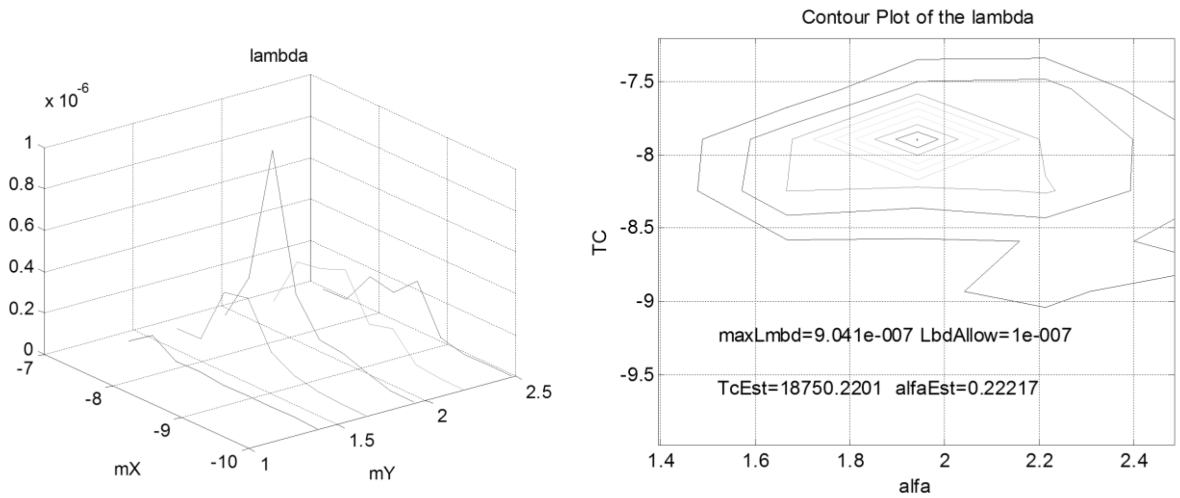

Figure 2. The value of $w(\theta, \lambda_{FD}, \Theta_0)$ for five value of μ_Y $(1.2415 \le \mu_Y \le 2.6435)$ in vicinity of maximum value of $w(\theta, \lambda_{FD}, \Theta_0)$ which is equal to 0.9041*10-6 for (σ_X, σ_Y, r) =(0.155128668, 0.0778895, 0.796437) .

Calculation of $w(\theta, \lambda_{FD}, \Theta_0) = E\{\lambda_F(\hat{\theta}, \lambda_{FD}, \Theta_0)\}$ was made for $(7.2029 \le \mu_X \le 9.9709)$, $(1.3972 \le \mu_Y \le 2.4877)$ assuming that the vector (σ_X, σ_Y, r) is the same for all different vectors (μ_X, μ_Y). It was found that for λ_{FD} =0.1*10-6 the maximum value of $w(\theta, \lambda_{FD}, \Theta_0)$ is equal to 0.9041*10-6 .

Suppose that the value 0.9041*10-6 is required reliability. Then for the known estimate of the parameter the calculation of $n_\lambda(\lambda_{FD}, \hat{\theta})$ for λ_{FD} =0.1*10-6 gives us the required number of inspection. It is equal to 6. For the considered estimate of θ t_C (realization of T_C is equal to

37.4574e+003 so the redesign is not needed. After the necessary calculation of $g(n,\theta)$ it is found $n_g = 4$. So finally, the required number of inspections $n = \max(n_g, n_\lambda)$ is equal to 6.

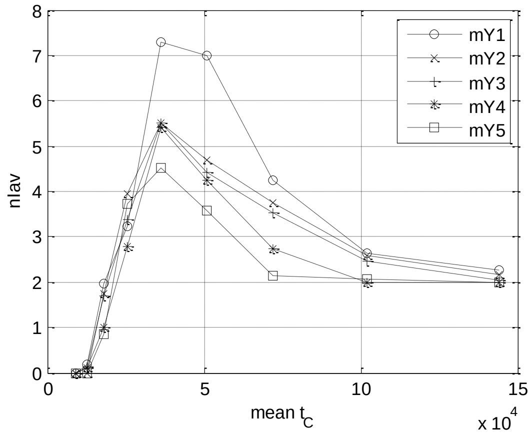

Figure 3. The value of $n_\lambda(\lambda_{FD}, \theta)$ for five value of μ_Y $(1.2415 \le \mu_Y \le 2.6435)$ for λ_{FD} =0.1*10-6 as function of equivalent mean value of T_C which was calculated as $\exp(\mu_Y - \mu_X)$

CONCLUSIONS

The problem of inspection planning on the bases of the result of acceptance full-scale fatigue test of AC structure is the choice of the sequence $\{t_1, t_2, ..., t_n, t_{SL}\}$ providing the limitation of FFR of AL if some requirements to the result of acceptance full-scale fatigue test are met. If these requirements are not met the redesign of the new type of an aircraft should be made.

The definition of binary λ-set function is introduced for description of corresponding mathematical procedure, based on the observation of some fatigue crack during the acceptance full-scale fatigue test of aircraft structure. In general case the the desire to increase the gain of airline service can be taken into account but under condition that required reliability is already provided. The limitation of FFR is provided for **any unknown** parameter of the fatigue crack model. The method of necessary calculation is provided.

REFERENCES

1. Paramonov Yu, Tretjakov S, Hauka M. Fatigue-prone aircraft fleet reliability based on the use of a p-set function. RT&A, #01(36) (Vol. 10) 2015, pp.40-49.
2. Hauka M, Tretjakov S, Paramonov Yu (2014). Minimax inspection program for reliability of aircraft fleet and airline. In: *Proceedings for 8th IMA International Conference on Modelling in Industrial Maintenance and Reliability (MIMAR)*, Oxford, Institut of Mathematics and its Applications, 10 – 12 July 2014, pp. 120-124
3. Paramonov Yu., Kuznetsov A., Kleinhofs M. (2011). *Reliabilty of fatigue-prone airframes and composite materials*. Riga: RTU. (*http://gnedenko-forum.org/library/Paramonov/*Reliability_Paramonov.pdf)