DANIELS' EPSILON-SEQUENCE AND MODELLING OF RELIABILITY OF UNIDIRECTIONAL FIBROUS COMPOSITE

Yu. Paramonov

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Institute of Aeronautic, Riga Technical University,
Riga, LV 1019, Latvia
yuri.paramonov@gmail.com

Abstract

This is a new version of the Daniels' sequence (DS) for analyzing the relation of the static strength of unidirectional fiber composite (UFC) component with a fatigue life, a static and a residual tensile strength of UFC itself. The DS allows to explain the existence of fatigue strength, the residual strength and the dependence of static strength on the rate of a tensile test. It explains the structure of these processes of the UFC failure, but to get a numerical result some new hypotheses are needed. New version of the DS allows to simplify a problem of the regression analysis of test data and the prediction of the UFC parameter changes in case of changes of the parameters of its component.

Keywords: Daniels' sequence, strength, fatigue life, composite

1. Introduction

The concept of the DS, with reference to the description of the process of fatigue failure, was first introduced in [1]. Its successful application to describing the relation between the strength of LI and the fatigue life of UFCs is discussed in [2,3]. The more general definition, which can be applied to testing of the UFC specimens both in the fatigue and the tensile strength was considered in [4]. In these papers it was assumed that specimen failure takes place if DS item become more than some critical level which does not depend on the process of loading. But in paper [5] the definition of the Daniels' residual strength was introdused and decreasing of its value was considered as the reason of the failure: the failure of the UFC takes place if the residual strength become lower than the applied load.

Lately, composite materials are widely used in various engineering areas, in particular, aviation. Therefore, the strength and fatigue life of these materials are now a very urgent problem. The first scientific publication on this topic appears to be the Peirce's work [6], which gives an approximation formula for the average strength of the bundle of LIs forming the foundation of the unidirectional fibrous composite. The correctness of the normal approximation of the strength distribution law of the LI parallel system was proved by Daniels [7,8]. His result was refined by Smith [9] already with reference to the series-parallel system (SPS), which was earlier proposed for consideration in [10]. A lot of papers are devoted to the reliability of composite (detail review, for example, is given in [11]) but here we take into account only of the papers connected with the DS.

First, we consider the modified definition of the DS and some other functions, connected with the DS. We present an unified approach to the description of relation between the static strength of LIs (fibers or bundles) and the fatigue life, the fatigue strength and the ultimate static strength of UFC specimens made with the use of such bundles. The definition of the Daniels' epsilon-sequence (DeS) is introduced. The presentation is accompanied by a comparison of calculation results with experimental data.

The modelling of the composite behavior under static and fatigue loading can be made for different purpose. There are different types of structure and component of material, there are different types of definition of composites failure, there are different requirements to the details and precision of description of studied phenomenon. So we consider different versions of the models based on the DS definition.

2. Structure of UFC Specimens for Tension and Fatigue Tests

We consider a composite specimen for the static strength or the fatigue life tests as a series-parallel system: series system, every link of which is a parallel system, or more specifically, a bundle of n_C longitudinal items (fibers or bundles) immersed into a composite matrix (CM). We make an assumption that the CM is a composition of the matrix itself and all the layers with stacking different from the longitudinal one. Here we make an additional assumption also that only LIs carry the longitudinal load but the matrix only redistributes the loads after the failure of some LIs. The case when the failure of the matrix is considered as failure of the specimen also is studied in [12]. It is supposed that the composite is divided into n_L "links" of the same length, l_1. The total length of the composite specimen is equal to $L = n_L\, l_1$. In random K_L links, $1 \le K_L \le n_L$, some defects already exist or can appear, while, in ($n_L - K_L$) links the defects cannot appear. In general case the defected LIs have a different strength distribution function. In some special cases we assume that the strength and fatigue life of the defected LIs are equal to zero. In these cases in flawless links, there are n_C LIs without defects, but in defected links (the links in which there are defected LIs) there are only $N_C^+ = n_C - K_C$ LIs, where K_C ($1 \le K_C \le n_C - 1$) is the random number of missing LIs. The equality $K_C = n_C$ means the failure of both the link and the whole SPS.

Here we assume additionally that the failure of the system can occur only due to failures of links with defects. Inside this link there is some weak micro volume (WMV) the failure of which is a failure of link and the failure of the tested specimen also. In fact we consider a composite as a series WMVs. This paper is devoted really to the fatigue life and tensile strength of one WMV. The relation of the cumulative distribution functions (c.d.f.-s) of these random values r.v.s with the c.d.f.s of the same r.v.s of the whole composite is considered in [13].

3. Basic Daniel's sequence

So we consider the UFC as SPS and we study the reliability of this system. It was usually assumed in the classical theory of reliability that the failure of an element does not influence the operation of the other elements. In this paper, we consider the situation when the failure of an element increases the load on the still workable elements. To describe the loading process model, we use of the Daniels random sequence concept the deterministic version of which was introduced, as it was told already, in [1].

Let in WMV there is n_C LIs and $X_{(1)}, X_{(2)},..., X_{(n_C)}$ be the ordered values of the random strengths of them. Assuming the independence of $X_1, X_2,..., X_{n_C}$, Daniels showed in [7,8] that the random variable

$$R_D = \max(X_{(k)}(n_C - k + 1)/n_C : 1 \le k \le n_C) \quad (1)$$

has an asymptotically normal distribution with the average and standard deviation

$$\mu_D = \max x(1 - F_X(x)) = x^*(1 - F_X(x^*)), \quad \sigma_D = (\mu_D x^* F_X(x^*)/n_C)^{1/2}, \quad (2)$$

which are determined by the c.d.f. $F_X(x)$ of corresponding positive random variable X.

Let us mention three salient features of Daniels' model:

(1) a continuous increase in the external load is assumed, but its rate is not taken into account;

(2) it is assumed that destructions of one by one fibers with a successively growing strength are accumulated;

(3) The strength of a fiber bundle corresponds to the the instant when the load becomes higher than the load-carrying ability of the specimen tested. There after, the destruction, i.e., rupture, of the bundle is presumed.

By "unwrapping" this model in time for specific sample $x_{1:n_C} = (x_1,..., x_{n_C})$ which is a realization of the random vector $X_{1:n_C} = (X_{(1)},...X_{(n_C)})$ and assuming that the process of loading is described by sequence $s^+_{0:\infty} = \{s^+_0, s^+_1, s^+_2,...\}$, we obtain a sequence of local stresses $s_{0:\infty} = \{s_0, s_1, s_2,...\}$ (in the link where the damage develops) described by the equation

$$s_{i+1} = s^+_i / (1 - k(s_i)/n_c), \quad i = 0,1,2,... , \quad (3)$$

where $k(s_i)$ is the number of LIs , the strength of which is lower or equal to s_i, $k(s_i)/n_C$ can be considered as an estimation of the c.d.f. $F_{X,n_C}(s_i)$, obtained by the use of the sample $x_{1:n_C}$.

This sequence can be used for describing both: 1) the fatigue test, if $s^+_i = s^+$, $i = 1,2,3,...$, (here we consider the fatigue test only for describing the fatigue curve and s^+ is some constant; for example, s^+ is the maximum stress of the cyclic load; in general case $s^+_{0:\infty} = \{s^+_0, s^+_1, s^+_2,...\}$ can be defined by specific program) and 2) the tensile static test, if items of sequence $s^+_{0:\infty}$ increase up to infinity. The sequence of local stresses is called Daniels' sequence for a constant external load (DS_CL) for the loading process of the first case and Daniels' sequence for an increasing external load (DS_EL) in the loading process of the second case. In what follows, the concept of DS_CL will be used to describe fatigue tests, while the DS_EL is a tension tests. The realization of these random processes is described by the pair $(s^+_{0:\infty}, x_{1:n_C})$.

For specific pair $(s^+_{0:\infty}, x_{1:n_C})$ let us define the following functions:

the K-Daniels' function

$$r_{D,K}(k) = x_{(K)}(n_C - k + 1)/n_C , \quad (4)$$

the S-Daniels' function

$$r_{D,s}(s) = s(n_C - k(s) + 1)/n_C , \quad (5)$$

the Daniels' residual function

$$r_{D\max}(x) = \max_{s > x} r_{D,s}(s) . \quad (6)$$

The transition from s_i to s_{i+1} we call a step of DS. It corresponds to the destruction of all LIs with the strength in the interval $(s_i, s_{i+1}]$. So the value

$$n_D = 1 + \max(i : r_{D,S}(s_i) > s^+, \ s_i \in \{s^+_0, s_1, s_2...\}) \quad (7)$$

we call the *DS-fatigue life* . It is *a number of DS steps up to failure: DS residual strength become lower than external load or some specific value.*

And let us define the following values:

the *DS-ultimate tensile strength* by equation

$$r_{D,u} = a^{+}_{n_{D.S}} \tag{8}$$

where $n_{D,S} = 1 + \max(i : r_{D,S}(s_i) > s_i^{+},\ s_i \in \{s_0^{+}, s_1^{+}, s_2^{+} ...\})$;

the DS-fatigue strength

$$s_D = \max(s^{+} : s^{+} \in S^{+}_{N_D=\infty}) \tag{9}$$

where $S^{+}_{n_D=\infty}$ is a set of s^{+} for which there is solution of equation

$$x = s^{+} / (1 - k(x) / n_c)$$

In this case there is such i^{*}, that $s_{i^*+1} = s_{i^*}$. Then $R_{D,n_C}(s_{i^*+1}) = R_{D,n_C}(s_{i^*})$. The changes of processes of s_i and $R_{D,n_C}(s_i)$ will be stopped, some LIs never will be destroyed and n_D will be equal to infinity.

In following we denote random DS by $S_{0:\infty} = \{s_0, S_1, S_2, ...\}$ and use the symbol R instead of r in notations $R_{D,K}(k)$, $R_{D,S}(s)$ … for random versions of corresponding functions if instead of sample $x_{1:n_C}$ we consider random vector $X_{1:n_C}$. For example, random fatigue life is defined by

$$N_D = 1 + \max(i : R_{D,S}(S_i) > s^{+},\ s_i \in \{s_0^{+}, S_1, S_2 ...\}) \tag{7a}$$

The random processes DS_CL and DS_EL and corresponding functions and values are completely determined by the triple $(s^{+}_{0:\infty}, F_X(x), n_C)$.

4. Modeling of fatigue tests

4.1. Daniels' epsilon-sequence

In [12] it is shown that basic DS is very similar to a S-type changes of some physical parameter of composite during fatigue loading. The use of DS-approuch allows to explain the existence of fatigue strength. So it explains the structure of a fatigue phenomenon but numerical results are very pure: the values of N_D are very small, the values of S_D are too large. So there is necessity to make some additional „patches"-assumptions in order to make the considered mathematics useful for numerical description of the fatigue phenomenon.

1) The accumulation of the damages takes a place only in local WMV in which there is some stress concentration which is different in the fatigue and in the tension tests. In simplest case it can be assumed that real **local** external stress is equal to $k_c s^{+}$ in the DS_CL and $k_T s_i^{+}$, $i = 1, 2, ...$, in the DS_EL.

2) The c.d.f. of the local tensile strength does not coincide with the c.d.f. of the strength of LI in a static strength test: the "size" of WMV and adjacent LIs have a specific influence on the local strength. In the following the specific c.d.f. $F_{X_L}(x)$ will be used in the numerical calculation. In the simplest case it can be assumed that $F_{X_L}(x) = F_X(k_F x)$, where k_F is some constant or (in general case) some random variable, or some another hypothesis can be used also.

3) The failure of the LIs does not take place in only one cross section (a plain) of a tested composite specimen. Here we suppose that only some part, $r_F(s)$, of the failure of the LIs is in the same cross section. This part changes in time growing up to unit. In the following numerical example the "mean" value of this function, $r_F(s)$, will be used.

4) During one step of DS, when the stress s_i grows up to s_{i+1} for the failure all LEs with the

strength $s_i < X_L \le s_{i+1}$ some energy is needed the value of which may be is not enough to cause the failure of all these LEs in one fatigue cycle. In simplest case it can be assumed that only part, $\varepsilon_D(s^+)$, of these LEs will be destroyed in one DS-step.

So here for modeling the fatigue test we use the equation

$$s_{i+1} = (k_C s^+ / (1 - rk(s_i)/n_c) - s_i)\varepsilon_D(s^+) + s_i), \quad i = 0,1,2,... \tag{10}$$

instead of equation (3). The sequence (10) we call the Daniels' epsilon-sequence (DeS). The function $\varepsilon_D(s^+)$ will be discussed some later.

The modified Daniels' residual function

$$r_{D\max}(x) = \max_{s>x} r_{D,s}(s)/k_T \, . \tag{6a}$$

From the condition of existence of infinite life : $s_{i+1} = s_i$,- now we get the following definition of DeS fatigue strength $s_D = (1/k_C)\max x(1 - rk(x)/n_C)$.

5) Two types of connections between the number of steps of the DeS and number of fatigue cycles was considered in our previous works [1, 12, 13].

If we are interested only to know the mean fatigue curve then the deterministic version of SeD can be used for corresponding regression analysis.

When the number of steps, *i*, is small, the DeS items, $s_0, s_1, s_2, ...$, and the value of N_D are easily calculated using recurrent formula. For the large *i*, it is possible to assume that *i* is the continuous variable. The value ($s_{i+1} - s_i$) can be considered as derivative, ds/dn, and for $n_C = \infty$ for the initial value s^+ it is possible to obtain an approximate connection of the number of steps needed to reach the critical value, s^*. For $\varepsilon_D = 1$ it can be found, that approximately:

$$N(s, s^*) = \{\arctan((s^* - s_h)/(a_2/a_0)^{1/2}) - \arctan((s_0 - s_h)/(a_2/a_0)^{1/2}))\}/(a_2 a_0)^{1/2},$$

where a_0 and a_2 are the values of the function $h(x) = s/(1 - F(x)) - x$ and its second derivative at $x = s_h$, where s_h is the value of the argument x at which the first derivative is equal to zero (for small x the function $h(x)$ decreases and then goes up; therefore, its expansion into a Taylor's series is used at the minimum point, while disregarding the members with an order of more than 2.). Approximately $N_D(s) = N(s, \infty)$. In [1], some version of this approach was successfully used for the regression analysis of the fatigue data set provided in [14].

The other way it is the use the theory of absorbing Markov chains (MCh) and semi-Markov process (sMP) theory.

The DeS can be limited by of conditions of the two types: 1) some item of DeS become more than some critical value s^* which defines the value of N_D, $N_D < \infty$; 2) there is such i^* that $s_{i^*+1} = s_{i^*}$, $N_D = \infty$.The items of the limited DeS can be considered as the states of some absorbing MCh in which there are two types of corresponding absorbing states. This approach (with $\varepsilon_D = 1$, critical value s^* do not depend on s^+) was successfully used for regression analysis of the data of the fatigue tests of carbon-fibers composites for describing of the fatigue curve. The **c.d.f. of random fatigue strength** was obtained also [3,4]. But for analysis of the data described in [11] the use of the theory of semi-Markov process (sMP) with rewords (as some generalization of MCh) appears to be more appropriate. Details of this analysis with the use of the definition of DeSs are given in next section.

4.2. Absorbing sMP

Every fatigue test of several specimens for description of fatigue curve can be releted with the set of possible limited DeSs : $\{(s_0, s_1, ...)_j,\ j = 1,2,..\}$, where $(s_0, s_1, ...)$ is a realization of a random process $(s_0, S_1, S_2, ...)_j$ for specific load level s^+_{0j}. The items of limited DeS, $s_0, s_1, ...$, we consider as the states of an absorbing sMP. The second type of the sMP must be studied in case when in a program fatigue loading there are load levels corresponding to $N_D = \infty$. Here we consider only loading with the probability of the event $N_D < \infty$ which is very close to the unit.

Different types of matrix of transition probabilities for specific set of states $\{s_0, s_1, ..., s_{N_D}\}$ is considered in [12]. Here we consider the simplest case. We suppose that that only transition to the same and to the next state is possible and these probabilities are functions of a sample $(x_1, x_2, ..., x_{n_C})$ which is a realization of the random vector $(X_1, X_2, ..., X_{n_C})$. We define

$$p_{1,2} = 1 - k(s^+)/n_C, \; p_{1,1} = 1 - p_{1,2},$$

$$p_{i,i+1} = ((k(s_{i+1}) - k(s_i))/n_C)/(1 - k(s_i)/n_C), \; p_{i,i} = 1 - p_{i,i+1}, \; i = 0,1,...,n_D - 1, \; p_{n_D,n_D} = p_{n_D,n_D}. \quad (11)$$

All the other probabilities are equal to 0.

As it was told already, the transition (s_i, s_{i+1}) corresponds to the failure of some part of the LIs with the strength in interval $(s_i, s_{i+1}]$. We assume that time of the transition is proportional to $(k(s_{i+1})/n_c - k(s_i)/n_c)/h(s_i)$, where $h(s_i)$ is proportional to the an area of a hysteresis loop of one fatigue cycle. Let us approximate a strain-stress curve of a composite by an equation

$$\varepsilon(s) = s/E + (s/a)^b$$

where E is the elastic modulus, a, b are some constants, $a >> s$, $b > 1$.

It can be assumed, that the area of hysteresis is proportional to the area between the straight line $\varepsilon_1(s) = \varepsilon(s_{\max})s/s_{\max}$ where $s_{\max}$ is the maximal value of fatigue cycle and the curve $\varepsilon(s)$ that is proportional to the integral $\int_0^{s_{\max}} (\varepsilon_1(s) - \varepsilon(s))ds$ which is proportional to the value $(s_{\max})^{b+1}$. So it can be assumed that the time of transition has the exponential distribution with the parameter

$$\lambda_i = (s^+/R_U)^m n_C/(k(s_{i+1}) - k(s_i))$$

where R_U, m are some model parameters.

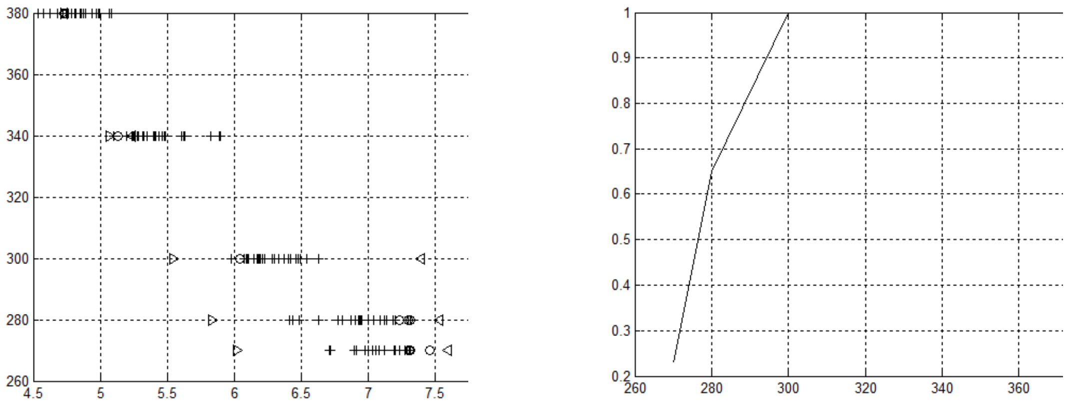
The random time to absorption is equal to the sum of corresponding random variables.

5. Numerical examples

Here we consider the examples of processing of some date in order to show the use of DeS for modeling fatigue strength, residual strength, c.d.f. of static strength of UFC.

Example 1. The estimation of c.d.f of the fatigue strength. The dataset employed in this analysis was kindly given to the authors by W.Q. Meeker, who already studied them [15] and provides the following description of the data: “the data come from 125 specimens analysed in four-point out-of-plane bending tests of carbon eight-harness-satin/epoxy laminate. Both fiber fracture and final specimen fracture occurred simultaneously. Thus, fatigue life is defined as the number of cycles until specimen fracture. The dataset includes 10 right-censored observations (referred to as “runouts” in the fatigue literature)”.

Processing of the data was made inaccordance with the definition (10) but not all parameters was used. In Fig.1a the fatigue life test data (+) (in logarithm scale) and result of Monte Carlo calculations (mean (o) and extreme values (►,◄)) and in Fig. 1b the c.d.f. of fatigue strength are shown. The following model parameters were used : $\theta_0 = 6.44$, $\theta_1 = 0.2$ for lognormal distribution of fatigue life of LIs, $n_C = 100$, $k_C = 1.58$, $\varepsilon = 0.3$, $r = 1$.



a b

Fig. 1. Fatigue curve (a) and c.d.f. of fatigue strength (b).

It can be noted that in accordance with the model calculation the random fatigue strength is in the interval 270-300 MPa but "The dataset includes 10 right-censored observations (referred to as "runouts" in the fatigue literature)". More exactly: there are 2 runouts for the load level 280 MPa and there are 8 runouts for the load level 270 MPa.

Example 2. The estimation of the residual strength. Within the framework of the European research project „OPTIMAT Blades" a series of tests was performed on a glass/epoxy material used in manufacturing of wind turbine rotor blades. Here we consider the processing of the result of the fatigue test the description of which is given in [11]:

1) „To determine the material response under cycling loading, 15 coupons were tested under sinusoidal loading (at a stress ratio equal to 0.1)…. at stress levels… SL1=48.50 MPa, SL2=63.60 MPa and SL3=78.31 MPa" .

2) „ A total of 74 residual strength tests were performed … The three stress levels chosen … for the same ones used for the S-N curve determination exept the high stress level wich was truncated to $5*10^3$ cycles. 25,25 and 24 specimens at each stress level were tested. These in turn, where divided into three groups of approximately 8 coupons, each one cycled for a specific life fraction; 20%, 50% and 80% of the nominal life time".

The considered fatigue models were applied to experimental data set available in [16,17,18].

In Fig.2 the result of calculation of the DeSs (Δ), the residual strength (∇) and fatigue load level(*) (a1, b1 and c1) as the functions of DeS step number, the same and the test of the residual strength (o) as a functions of fatigue cycle number (a2,b2 and c2) are shown. The values of the DeS functions are limited by 200 MPa in order to show all functions in the same scale. In Fig. 3 the fatigue lives are shown: test fatigue lives (+), calculated mean (*) and the extreem values (►,◄) corresponding to two standard deviation from mean value. The test residual strength for preliminary load levels 48.5 MPa, 63.6 MPa and 78.3 MPa for 20%, 50% and 80% of the corresponding nominal lifetime is shown also.

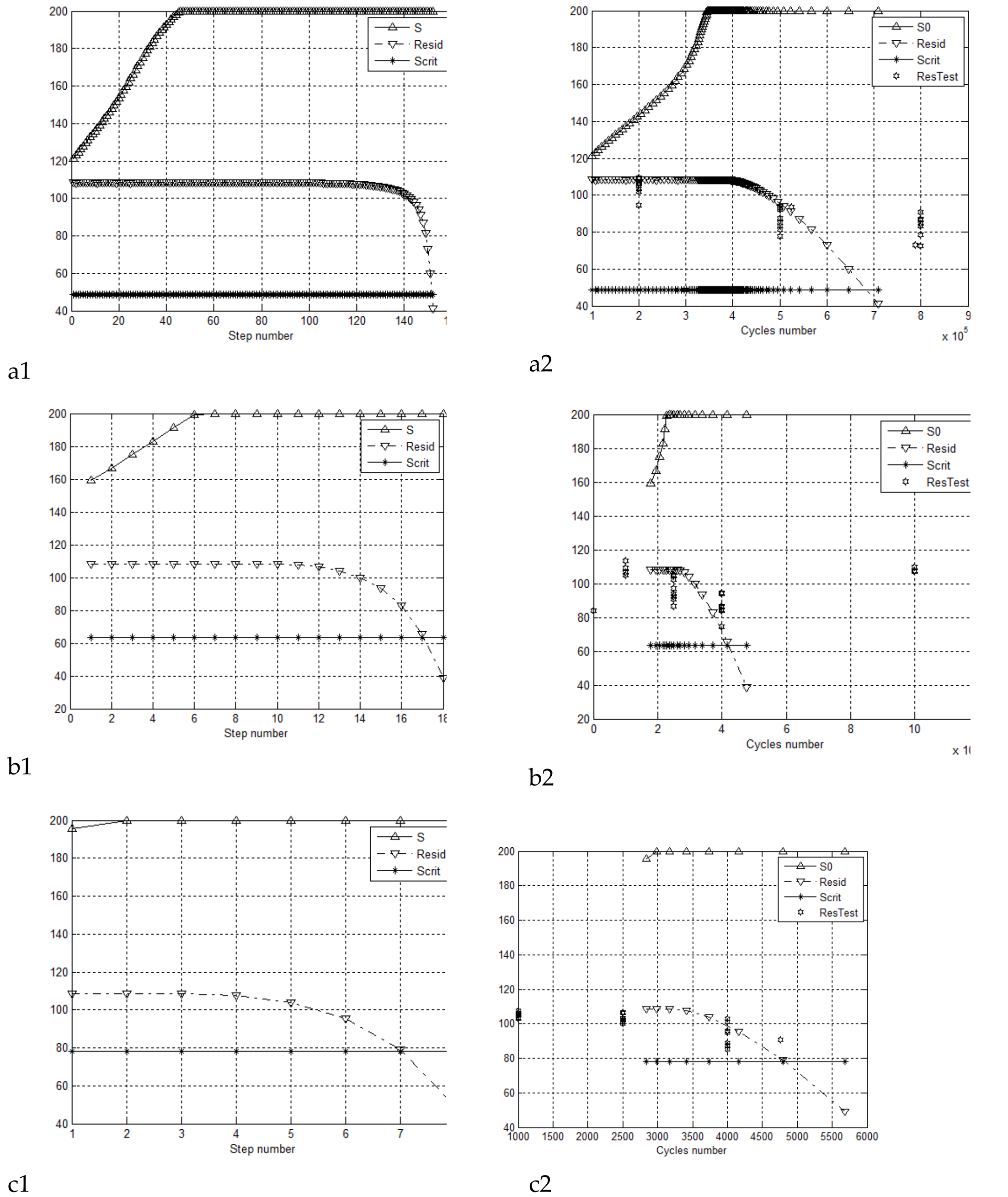


Fig. 2. DeSs (Δ), Daniels' resid. functions (∇), test resid. strength(o) and fatigue load (*).

For these calculations it was assumed that there are some LIs with some defects (W) and there are some LIs without defects (U). The c.d.f of the loacal strength is defined by equation

$$F_{X_L}(x) = (1-p_U)F_{X_{LW}}(x) + p_U F_{X_{LU}}(x),$$

where

$F_{X_{LW}}(x) = 1-\exp(-\exp((\ln(x)-\theta_{0W})/\theta_{1W}))$, $F_{X_{LU}}(x) = 1-\exp(-\exp((\ln(x)-\theta_{0U})/\theta_{1U}))$.

The following parameters was used for calculation: $n_C = 30$, $\theta_{0U} = \log R_U = 5{,}704$ (where $R_U = 300$), $\theta_{1U} = 0{,}175$, $\theta_{0W} = \theta_{0U} - \log k_F = 4{,}962$, $k_F = 2{,}1$, $\theta_{1W} = 0.175$, $k_C = 2{,}5$, $k_T = 1{,}1$, $p_U = 0.6$, $r_F = 0{,}93$, $m = 10$. $\varepsilon = 0.1$.

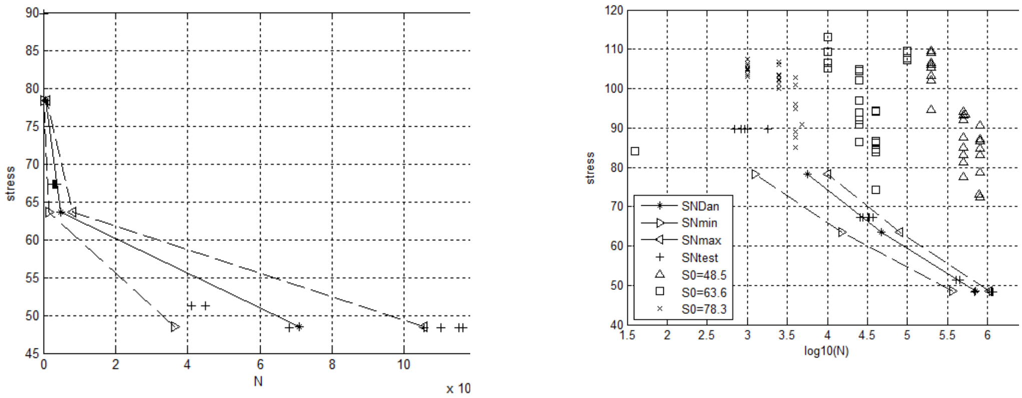


Fig.3. The test result (+), calculated mean (*) and the extreem values (►,◄) corresponding to two standard deviation from mean value. The test residual strength for preliminary load levels 48.5 MPa, 63.6 MPa and 78.3 MPa for 20%, 50% and 80% of the corresponding nominal life time.

Example 3. The modelling of tensile tests. In [14], the results of tests for the tensile strength of bundles of carbon fibers and of 14 specimens made with the use of these bundles are presented.

The data already have been studied in [4] but here for similar analysis we use the residual Daniels' function. The results of verification of the normal, lognormal and Weibull distribution laws of their strength are presented in [4] also. Usually, the verification of such hypotheses is performed visually according to the results of fitting of experimental data on the corresponding probability paper. But in [4] such a verification was carried out by using numerical ρ -criteria [12] with calculation of their efficiency. It allowes us to estimate the degree of confidence to the final conclusion objectively. As follows from results of the verification there are grounds to prefer the hypothesis about the lognormal distribution of the strength of CFRP bundles with c.d.f. $F_X(x,\theta_0,\theta_1)=\Phi((\log(x)-\theta_0)/\theta_1)$, $\theta_{0X}=6.44$, $\theta_{1X}=0.1816$.

Using this c.d.f. and Monte Carlo method for $n_C=100$ the calculation 10 realizations of the functions $r_{D,S}(s)$ and $r_{D,K}(k)$ was made. These realizations are shown in Fig.4.

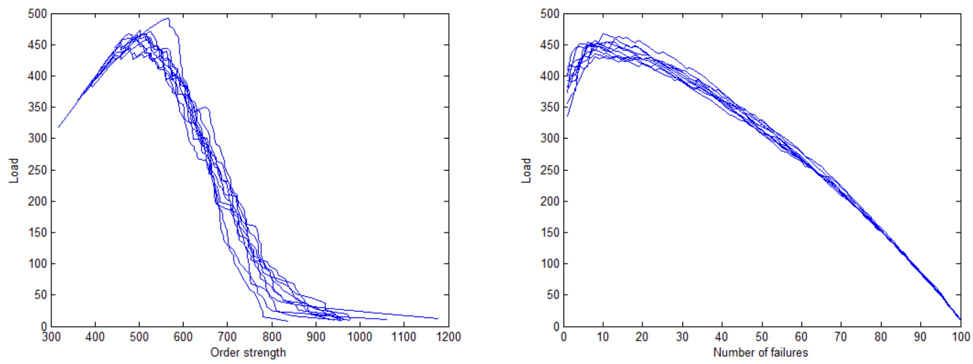


Fig. 4. Modelling of 10 realizations of the functions $R_{D,s}(s)$ and $R_{D,k}(k)$.

To get the hystogram of the value of the random variable, $K_{\max}$, corresponding to the maximum of the function $R_{D,k}(k)$, Fig. 5, the calculations of 100 realizations of this function was made.

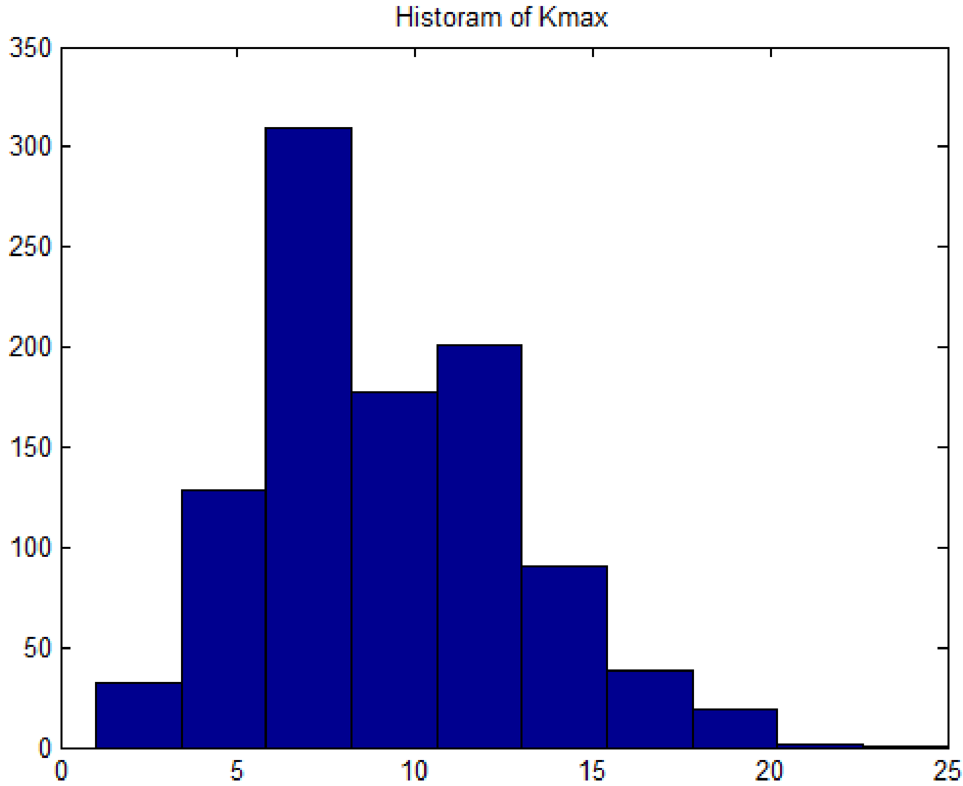


Fig.5. Hystogramm of $K_{\max}$.

It is important to notice that the value of $K_{\max}$ corresponding to the maximum of the function $R_{D,k}(k)$, is equal to small part of n_C. Mean value and standard deviation of $K_{\max}$ are equal to 9.3 and 3.8 correspondingly . After failure of this small part of LIs the „domino effect" and failure of WMV and whole composite take part.

For modelling the DS it was assumed that local strength X_L in framework of composite structure is equal to X / K_F where $\log(K_F)$ is a random variable with normal distribution with the mean value and standard deviation of which is equal to $\log(1.25) = 0.223$ and 0.07 correspondingly.

Result of modeling of the DS (-)

$s_{i+1} = k_C s_i^+ / (1 - k(s_i) / n_C), \quad i = 0,1,2,...$

for $n_C = 100$, the local stress concentration coefficient $k_T = 1.5$ is shown in Fig. 6. The residual strength (--) and process of loading (-.-) are shown in the same figure also. The sequence of growth in the nominal load is defined by

$s_i^+ = s_0^+ + (i-1)k_s E(X_L)$, $i = 1,2,...$

where $E(X_L)$ is the expected value of X_L, $k_S = 0.01$ is the conditional relative rate of external loading (the details of the choice of k_S is discussed in [4]).

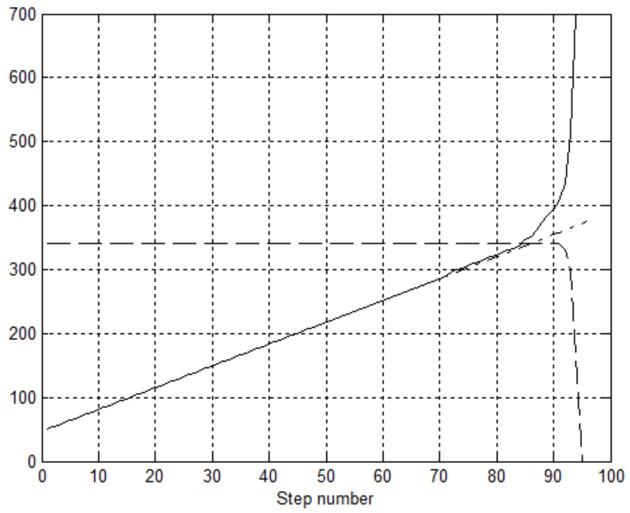


Fig. 6. DS (-) , the residual strength (--) and process of loading (-.-) for tensile strength test.

We should note that the local stress s_i is proportional to the local relative strain ε . Therefore, the relation between s_i^+ and s_i must be similar to the stress-strain curve (see Fig. 7).

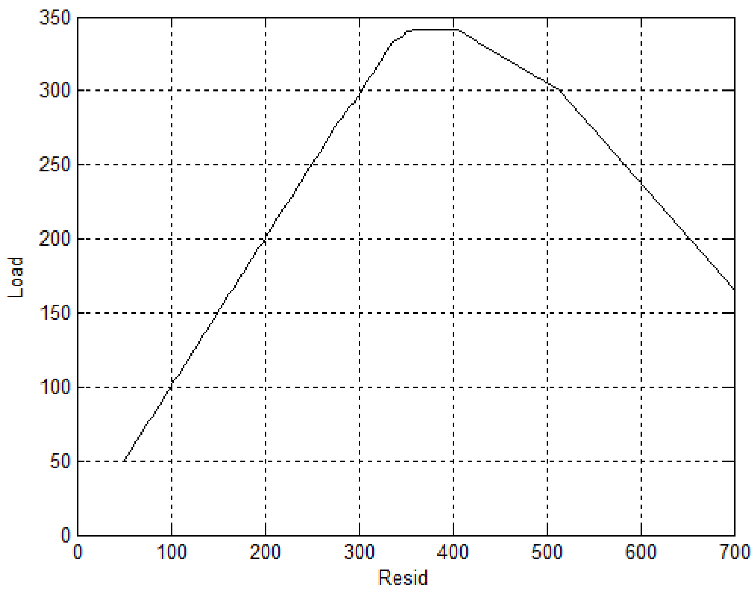


Fig. 7. The curve"the load versus residual strength" is similar to stress-strain curve in static strenth test

The repeated calculations by the use of the Monte Carlo method of specimen strength, S_D^+, enable us to find the average values of order statistics in a sample of the same size as in the test and to compare the predictions with real data. In [14], the test data of 14 specimens made with the use of mentioned carbon-fiber bundles are presented. The mean values (-) calculated according to the studied model and test order statistics (-.-) versus test order statistics are shown in Fig. 8. The problem on approximation of experimental data by the model calculations can be regarded as the problem of nonlinear regression. We see that the estimates of the model parameters used in corresponding calculations give a reasonably good fitting of test data.

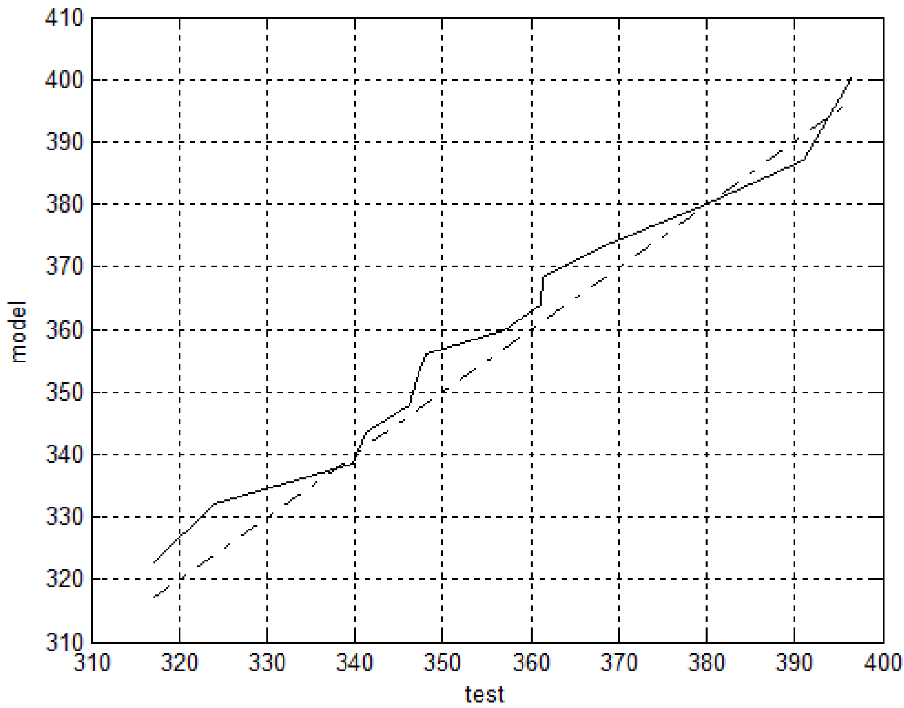


Fig. 8. The calculated mean values (-) and test order statistics (-.-) versus test order statistics.

Conclusions

The analysis of the examples of using of the developed DS approach to the description and modelling of the fatigue and the static strength tests of UFC shows that the corresponding models allows:

- to explain some specific features of these tests : 1) the existence of the fatigue strength and the possibility to have the infinite fatigue life , 2) the existence of a long period of a constancy of the residual strength;

- interpretation of the parameters of the studied models as parameters of local static strength (this is the main difference of these models from the others models).

The difference of the parameters of the c.d.f. of the local strength of LIs in framework of UFC and ones of isolated single LIs do not allows to make prediction of the parameters of UFC using parameters of LI. But these models can be used for regression analysis of test data. The search for the parameters of nonlinear regression for the above-described models is a difficult task, but we think that, in due course, the structure of models suggested will be of interest not only for graduation theses of students, but also for engineering applications, in particular, for predicting the variations in the parameters of the strength and the fatigue life of UFCs upon changes in the parameters of their components. The mathematics of the DeS allows to make the modeling and the prediction for any loading sequence, $s^+_{0:\infty} = \{s^+_0, s^+_1, s^+_2, ...\}$, in **any program of the fatigue and the tensile tests** but an **accuracy** of this prediction needs to be studied.

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