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Problems with evaluating efficiency of using satellites systems in motor vehicle

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In order to solve this issue, the authors proposed an original mathematical model of satellite receivers' use in vehicles. The model is designed on the basis of stochastic systems of queuing service. This model allows calculating the capacity of the GPS receivers used in vehicles. Efficiency of satellite management system use can be evaluated in practical work, applying this model.

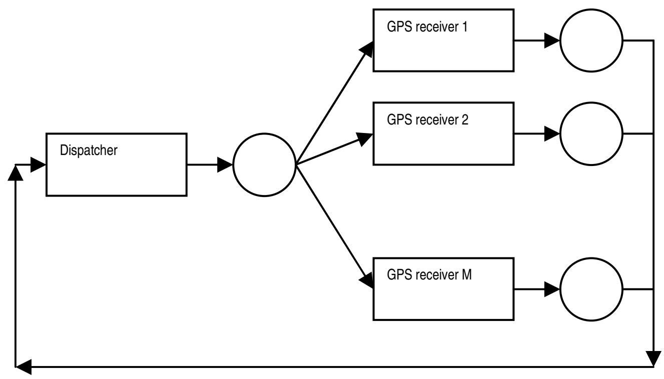
Keywords: Monitoring, request, homogeneous, normalization, queuing

Introduction

One of the main problems in improving the efficiency of GPS use in the motor vehicles is the problem related to evaluation of efficiency of such systems. Efficiency of these systems can be evaluated by mathematical calculations using queuing systems. The vehicle GPS signal receivers as such can be interpreted as separate systems of queuing which receive queries about their locations from the system dispatcher. GPS signal receivers can generate queries to the system dispatcher. Management of queries received from GPS receivers is shown in Figure 1 as a schematic image from the perspective of queuing. Each queuing system shown in Figure1 is a device that manages the GPS queries. Queries are managed on a first in, first out basis in rotation and return from GPS receiver to a dispatcher, then they are transferred back to GPS receivers from a dispatcher. One of the characteristics of this query management scheme is the availability of different rules on query management by GPS receivers. This diversity on one part can be explained by diversity of queries and the diversity of the receivers' characteristics. Unfortunately, in terms of applying queuing systems, the network device diversity is recorded extremely rarely or considered if there is a correlation between the rules on query behaviour when queuing for service and the diversity or with other queues (Boicov, 2009; Boicov and Zivitere, 2010).

As to GPS efficiency evaluation, cases should be taken into consideration when the types of queries do not depend on the status of query in the queue. Therefore this research suggests characteristics calculation method for GPS systems on the basis of the assumption that management in the network nodes is subordinated to management rules. Approbation for use of this method for corporate computer network analysis is reflected in various works of the authors (Boicov, 2011; Boicov, 2012). Along with analytical researches presented below, the authors carried out the experimental methods of GPS system assessment (Boicov and Gonzales-Ortiz, 2012) and the researches based on simulation modelling methods (Ipatovs, Bogdanov and Petersons, 2012). This research suggests selecting the mathematical tool of queuing stochastic networks as a basis for studies of the characteristics of GPS receiver network consisting of several nodes. In these networks, queries can select a network node for management randomly. It is suggested to perform the analysis of computer networks first based on the fundamental queuing system with the hyper-exponential management law and the superposition of Poisson query stream acting at the system input and then a transition to a stochastic network of queuing systems takes place. This method enables studying the networks with hierarchical organisation of the structure where subnets can be used as service units.

FIGURE 1.



Investigating the simplest queuing system with heterogeneous requests

Let us consider the single-line system with direct order of processing, in which the inbound stream of requests is divided into *M* groups. Conceding the average intensity of *i*- group as λ_i, the moments of requests' receipt of *i*- group form the Poisson stream. The uniting is performed by the way of superimpositioning the streams of groups in such a way, that the united stream is the Poisson one as well, but its average intensity is determined by the following congruence:

$$\lambda = \sum_{i=1}^{M} \lambda_i \quad (1).$$

The processing of each requests' group might be performed with an average intensity μ_i (i = 1,2, …, M) and is hypothesized to be distributed exponentially.

The resulting processing intensity, in case of superimposition of the streams, subordinate to Hyper-exponential functioning, or the general average intensity of processing in one node is determined by the expression:

$$\mu = \sum_{i=1}^{M} \mu_i$$

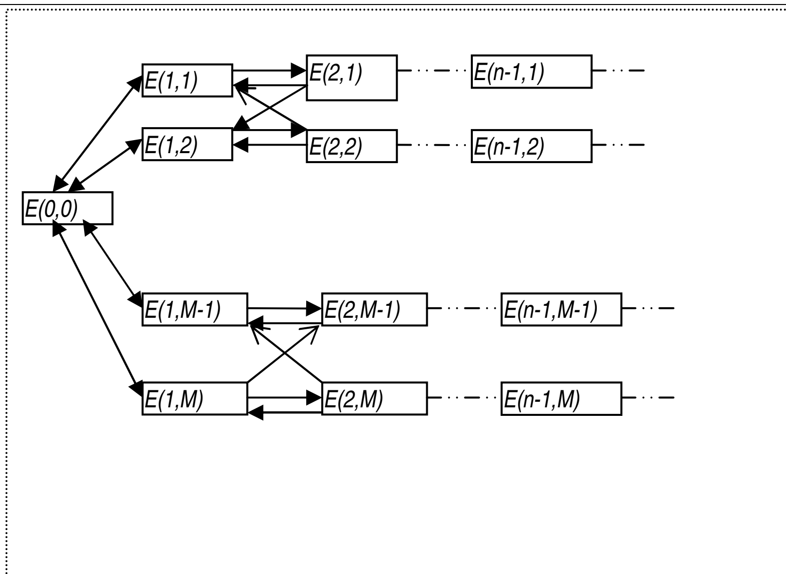
The probability of the receipt of ***i-*** type requests (q_i) for processing is determined by the relation of the average intensity of requests' incoming λ_i to resulting intensity λ, i. e. $q_i = \lambda_i / \lambda$. By reason of the performed assumption (1) we deduce the following congruence:

$$q_i = \lambda_i / \lambda, \; u \; \sum_{i=1}^{M} q_i = 1 \qquad (2).$$

The conditions of the model under consideration we will characterize with the help of the vectors $E_{n,i}(t)$, which mean, that at the moment *t*, n requests are located in the queue and the request of *i*-type (n = 0, 1, 2, …, ∞, i= 0, 1, 2, … *M*) is at processing.

The process $E_{n,i}(t)$ maybe considered to be the Markovian one (Kleinrock, 1979), as the time for any one-step transition is distributed in accordance with the exponential functioning and the probability of transitions being independent variables. The simplified graph of transition of processes $E_{n,i}(t)$ is depicted at Figure 2.

FIGURE 2.



Each vertical line of the graph's apex is in accord with the quantity of requests in a system's queue, each horizontal line is in accord with the type of a request, located for processing.

Separately considering each of the graph's vertical lines, excluding the first one, we come to the conclusion that transitions into any x-state $E_{n,i}$ (n = 2, 3, …, ∞; i=1, 2, …, M) are possible under the following circumstances: firstly, whenever the system was in the state $E_{n-1,i}$ and one of the requests (of any type) entered the system; and secondly, whenever the system was in state $E_{n+1,j}$ (j=1, 2, …, M), the processing took place, and the next request, being in the queue, is of i-type. On the other hand, whenever the system was in the state $E_{n,i}$, it would remain in it, if no processing was performed and no request arrived. As to the first vertical line, the above-mentioned inherence is also true, excluding only the fact, that transitions from state $E_{0,0}$ to state $E_{1,i}$ are possible in case of **i**-type requests' receipt, but not of any other certain type. These properties allow to write the differential equation system and to recast it in a usual way into the following infinite system of homogeneous differential equations as aspects of the steady-state probabilities:

$$P_{n,i}(\lambda+\mu_i)=\lambda P_{n-1,i}+q_i\sum_{j=1}^{M}P_{n+1,j}\mu_j \qquad (4),$$

$$P_{n,i}(\lambda+\mu_i)=\lambda_i P_{0,0}+q_i\sum_{j=1}^{M}P_{2,j}\mu_j \qquad (5),$$

where, $P_{i,j}$ is the steady-state probabilities of states $E_{i,j}$, but μ_j stands for the average intensities of j - type requests' processing .

For the present system the following normalization requirement is true:

$$P_{0,0}+\sum_{n=1}^{\infty}\sum_{j=1}^{M}P_{1,j}\mu_j=1 \qquad (6).$$

Analyzing the possible transitions across every vertical line of the graph at Image 1, on the basis of equilibrium condition for steady-state probabilities, it's possible to write down the following additional system of equations:

$$\lambda\sum_{j=1}^{M}P_{n-1,j}=\sum_{j=1}^{M}P_{n,j}\mu_j \qquad (7),$$

and for n =1, $$\lambda\sum_{j=1}^{M}P_{0,0}=\sum_{j=1}^{M}P_{1,j}\mu_j \qquad (8).$$

Having intercepted the values of steady-state probabilities with common coefficients from expressions 4, 5, 7 and 8, we deduce the following system of equations:

$$P_{1,i}(\lambda+\mu_i)=\lambda_i P_{0,0}+\lambda_i\sum_{j=1}^{M}P_{1,j}$$

$$P_{2,i}(\lambda+\mu_i)=\lambda\ P_{1,i}+\lambda_i\sum_{j=1}^{M}P_{2,j} \qquad (9)$$

$$\cdots$$

$$P_{n,i}(\lambda+\mu_i)=\lambda\ P_{n,i}+\lambda_i\sum_{j=1}^{M}P_{n,j}\ .$$

While summing all equations (9) over $n=\overline{1,\ \infty}$, we can deduce the following congruence:

$$(\lambda+\mu_i)\sum_{n=1}^{\infty}P_{n,i}=\lambda_i P_{0,0}+\sum_{n=1}^{\infty}P_{n,i}+\lambda_i\sum_{n=1}^{\infty}\ \sum_{j=1}^{M}P_{1,j} \qquad (10).$$

Inserting normalization requirement in (10) and performing re-summation over ***i***, we determine:

$$P_{0,0}=1-\sum_{j=1}^{M}\ \lambda_i/\mu_i \qquad (11).$$

Further on let's embed the following generating functions:

$$Q_1(s)=\lambda_1 P_{0,0}+\sum_{n=1}^{\infty}P_{n,1}s^n$$

$$Q_2(s)=\lambda_2 P_{0,0}+\sum_{n=1}^{\infty}P_{n,2}s^n \qquad (12)$$

$$\cdots$$

$$Q_M(s)=\lambda_M P_{0,0}+\sum_{n=1}^{\infty}P_{n,M}\,s^n$$

and the expanding generating function:

$$G\ (s)=Q_1(s)+Q_2(s)+\ldots+Q_M(s) \qquad (13).$$

Having multiplied the left and right parts of equation (9) by s^n and summing over n we determine :

$$(\lambda+\mu_i)\sum_{n=1}^{\infty}P_{n,i}s^n=\lambda_i P_{0,0}s+\sum_{n=1}^{\infty}P_{n,i}s^{n+1}+\lambda_i\sum_{n=1}^{\infty}\ \sum_{j=1}^{M}P_{n,j}s^n \qquad (14).$$

Substituting the values of generating functions (12) and (13) in the received formula (14), we'll deduce:

$$(\lambda+\mu_i)[Q_i(s)-\lambda_i P_{0,0}]=sQ_i(s)+\lambda_i[G(s)-P_{0,0}] \qquad (15).$$

Summing once more over *i* for all $Q_i(s)$ we determine:

$$G\ (s)=G\ (s)\sum_{i=1}^{M}\ \lambda_i/(1+\mu_i-s)+\sum_{i=1}^{M}\ [\lambda_i\mu_i/\lambda(1+\mu_i-s)]P_{0,0}$$

Here from the following expression for $G\ (s)$ can be deduced :

$$G\ (s)=P_{0,0}\sum_{i=1}^{M}\ [\lambda_i\mu_i/\lambda(1+\mu_i-s)]/[1-\sum_{i=1}^{M}\ \lambda_i/(1+\mu_i-s)] \qquad (16).$$

On the other side, using the expressions (12) and (13) we can write:

$$G\ (s)=P_{0,0}+(P_{1,1}s+P_{2,1}s^2+...+P_{n,1}s^n+....)+(P_{1,2}s+P_{2,2}s^2+...+P_{n,2}s^n+....)...$$

If we group the components at equal powers in the expression received, we can determine the following:

$$G\ (s)=P_{0,0}+s\sum_{j=1}^{M}P_{1,j}+s^2\sum_{j=1}^{M}P_{2,j}+....+s^{\infty}\sum_{j=1}^{M}P_{1,\infty} \qquad (17).$$

Each of the summands in the expression (17) defines the probability as to whether in our system the average number of requests equals 0, 1, 2,…, ∞ correspondingly. Because of that, performing the differentiation as to *s* expression (16) and after that substituting *s=1*, it's possible to deduce the following expression, defining *E* - as the average number of requests in the system:

$$E=G^{'}(s=1)=\sum_{i=1}^{M}\ \lambda_i/\mu_i+[\sum_{i=1}^{M}\ [\lambda_i/\mu_i^{\ 2}]/[\lambda(1-\sum_{i=1}^{M}\ \lambda_i/\mu_i)] \qquad (18).$$

The situation, presuming that each request, being queued in the system under consideration, will expect its processing until the queue before it finishes, makes it possible to determine the following expression for the average time of a request's processing start within the system:

$$W=\sum_{i=1}^{M}\ [\lambda_i/\mu_i^{\ 2}]/[(1-\sum_{i=1}^{M}\ \lambda_i/\mu_i)] \qquad (19).$$

FIGURE 3.

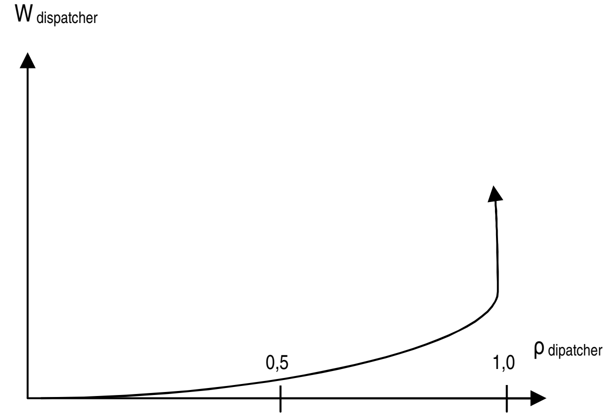
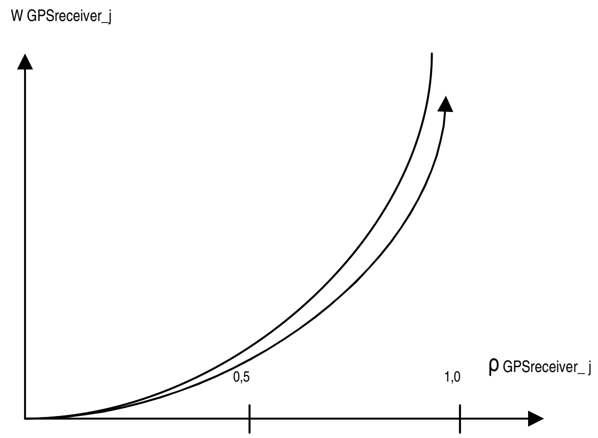


FIGURE 4.



Assessments of vehicles service delays

Expressions (19) make it possible to estimate the average delay times of services in system dispatcher - GPS receiver. The dependences of the delay requests from the load systems are an important characteristic in terms of the effectiveness of the devices. Using the above-introduced concept of higher system loads via $\varrho_{dispatcher}$ and $\varrho_{GPSreceiver\_j}$, expression (19) can be represent for dispatcher and receiver as follows:

$$W_{dispatcher} = (\varrho_{dispatcher} / \mu_{dispatcher}) / (1- \varrho_{dispatcher}) \quad (20),$$

$$W_{GPSreceiver\_j} = (\varrho_{GPSreceiver\_j} / \mu_{GPSreceiver\_j}) / (1- \varrho_{GPSreceiver\_j}) \quad (21).$$

Using the scale factor for the $\mu_{dispatcher}$ and $\mu_{GPSreceiver\_j}$, we can plot graphic dependencies for $W_{dispatcher}$ and $W_{GPSreceiver\_j}$ versus the load. These graphs are shown in Figure 3 and Figure 4.

The graphs, presents in Figure 3 and Figure 4. show that the changes in the service time delay increased tenfold compared with load of 0.1 for the GPS receiver and dispatcher system at load close to 0.8 - 0.9. For the dispatchers, it is the most pronounced.

According to queuing theory (Kleinrock, 1979), the queue's length and the time of request staying in queuing systems can be significantly reduced if you changed the services discipline. In this case, a rational change of the services discipline is the use of services priority service. In case application priority service, one can achieve lower average residence times of queries in the system (25%). The calculated value of the mean residence time for this case in Figure 3 and Figure 4 is shown by the graph with an arrow. Minimizing the value of the average residence time of relative load system and applying preferential treatment services, you can achieve optimal usage of services by the GPS receivers and dispatcher programs.

Conclusion

Monitoring of GPS receivers and dispatcher computing networks have been investigated in this paper. It was shown that GPS receives have properties of varying load nodes, that is, properties of heterogeneity. For the analysis of these properties, there is a mathematical model of the interaction of the GPS receivers and dispatcher. The mathematical model allowed us to make estimates of the average time delay for performing in GPS receivers. These calculations have shown that a GPS receives computing network can disastrously run bad with a load close to 0.9. More efficient use of GPS receives can be achieved using a priority GPS receives discipline. The performed calculations showed that the expected gain in time of use of network servers through the use of selecting the optimal load of the nodes and the application of priority of GPS receives can exceed 20%.

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