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VIBRATION-BASED APPROACH FOR STRUCTURAL DAMAGE DETECTION

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In this paper, a mode shape curvature based method for detection and localization of damage in beam-like and plate-like structures is described. The proposed method requires only the mode shape curvature data of the damaged structure. The damage index is defined as the absolute difference between the measured curvature of the damaged structure and the smoothed polynomial representing the curvature of the healthy structure. To examine the advantages and limitations of the proposed method, several sets of numerical simulations are carried out. Simulated test cases considering different levels of damage severity, measurement noise and sensor sparsity are studied to evaluate the robustness of the method under the noisy experimental data and limited sensor data. The modal frequencies and the corresponding mode shapes are obtained via finite element models for numerical simulations.

1. Introduction

Design of modern civil, transport, and aerospace engineering structures such as automotive and aerospace facilities, stadiums, dams, tunnels, skyscrapers etc. is becoming more complex and is expected to be fully functional under severe environmental conditions. Their failure can lead to tragic consequences and therefore structures have to undergo regular inspections. Rigorous inspection of structures on a regular basis is usually a time-consuming and costly procedure; therefore non-destructive structural health monitoring (SHM) methods have become an important research area in civil, mechanical and aerospace engineering communities.

In recent years, various vibration-based damage detection methods have been proposed for SHM. Many of them use various transformations of measured dynamic response of a structure. Dynamic responses, which, in many cases, can be easily obtained, offer damage information, such as the location and severity. These methods are based on the fact that dynamic characteristics, i.e., natural frequencies, mode shapes and modal damping, are directly related to the stiffness of the structure. Therefore changes in these characteristics will indicate a loss of stiffness. Extensive literature surveys on damage identification using vibration based methods are given in [1-3]. Qiao et al. in [3] classified different damage identification methods for beam-type and plate-type structures into four categories, namely, natural frequency-based, mode shape-based, curvature of mode shape-based and the ones that combine both frequencies and mode shapes. Several advantages and limitations of these methods were also pointed out. Multiple crack scenarios for beam structures were studied by Khiem et al. in [4]. Results indicated that the proposed algorithm was successful at damage identification even at sparse and noisy data conditions.

Out of these methods a special attention is paid to mode shape curvature methods [5-7], which is based on modal strain energy [8] or modal stiffness [9] comparison between healthy and damaged structure. The need for the modal data of a healthy structure (baseline modal data) is one of the most significant drawbacks of those methods, as baseline modal data is sometimes difficult or even impossible to obtain. This issue is usually solved by employing Gapped Smoothing Techniques to generate a smoothed surface of mode shape curvature obtained from the damaged structure, thus simulating the healthy state of a structure [10].

In this paper, a mode shape curvature based method for damage identification in aluminium plate containing one site of mill-cut damage and aluminium beam containing two sites of mill-cut damage is described. The basic idea of the method is that the mode shape curvature of a healthy structure has a smooth surface, and it can be approximated by a polynomial. Using a mode shape curvature data of the damaged structure and a regression analysis with a polynomial approximation, smooth mode shape curvature surfaces of the healthy plate structure are estimated. Smooth mode shape curvatures of a beam structure are estimated using interpolation of mode shape curvature of a damaged structure with Fourier series functions of different orders. The damage index is defined as the absolute difference between the measured curvature of the damage structure and the smoothed polynomial representing the healthy structure and the maximum value indicates the location and size of the damage. To examine the advantages and limitations of the proposed method, several sets of numerical simulations considering different levels of damage severity, measurement noise and sensor sparsity are carried out.

2. Damage detection algorithm

Most of the mode shape curvature damage detection methods require the baseline data of the healthy structure. In many cases, such as for structures that are already in service for some time, the baseline modal parameters are rarely available. To overcome this difficulty, a damage index that uses exclusively mode shape curvature data of a damaged structure is proposed.

The basic idea of the method is that the mode shape curvature of the healthy structure has a smooth surface, and it can be approximated by a polynomial. For the one-dimensional space, an interpolation technique with a Fourier series approximation is applied on a mode shape curvature data of the damaged structure to generate smooth mode shape curvatures that are estimates of the healthy structure. The Fourier series is a sum of sine and cosine functions that describes a periodic signal. The trigonometric Fourier series form is

$$\kappa(x) = a_0 + \sum_{i=1}^{g} a_i \cdot \cos(g\omega x) + b_i \cdot \sin(g\omega x). \tag{1}$$

where a_0 models a constant (intercept) term in the data, ω is the fundamental frequency of the signal, g is the number of terms (harmonics) in the series. In this study, Fourier series functions of orders 1 to 8 ($1 \leq g \leq 8$) are used. The obtained coefficient values a_0, a_i, b_i and ω are used to reconstruct the approximation of mode shape curvature data using Eq. (1).

For the two-dimensional space a regression analysis with a polynomial approximation is used to generate smooth mode shape curvature surfaces of the healthy structure. A locally weighted scatter plot smooth using linear least-squares fitting is employed to build the smoothed surface [11]. The local regression smoothing process follows these steps for each data point:

1. Compute the *regression weights* for each data point in the span. The weights are given by the tri-cube function:

$$w_i = \left(1 - \left|\frac{x - x_i}{d(x)}\right|^3\right)^3. \tag{2}$$

where x is the predictor value associated with the response value to be smoothed, x_i are the nearest neighbours of x as defined by the span, and $d(x)$ is the distance along the abscissa from x to the most distant predictor value within the span. The weights have the following characteristics:

- The data point to be smoothed has the largest weight and the most influence on the fit;
- Data points outside the span have zero weight and no influence on the fit.

2. A weighted linear least-squares regression is performed.

3. The smoothed value is given by the weighted regression at the predictor value of interest.

The damage index is defined then as the absolute difference between the measured curvature of the damaged structure and the smoothed polynomial representing the healthy structure. The maximum value indicates the location of damage. The damage index generalized to the two-dimensional space for the n^{th} mode at grid point (i, j) is expressed in Eq. (4), while for the one-dimensional space it is given in Eq. (3):

$$DI_i^n = \left| \left(\frac{\partial^2 w^n}{\partial x^2} \right)_i^2 - \left(\kappa_x^n \right)_i^2 \right|. \tag{3}$$

$$DI_{i,j}^n = \left| \left(\frac{\partial^2 w^n}{\partial x^2} \right)_{(i,j)}^2 - \left(k_x^n \right)_{(i,j)}^2 \right| + \left| \left(\frac{\partial^2 w^n}{\partial y^2} \right)_{(i,j)}^2 - \left(k_y^n \right)_{(i,j)}^2 \right|. \tag{4}$$

where w^n is the measured transverse displacement of the structure, k_x^n, k_y^n are smoothed mode shape curvature surfaces in x and y direction, respectively, n is a mode number, i and j are numbers of grid point in x and y direction, respectively. The mode shape curvatures are calculated from the mode shapes by the central difference approximation at grid point (i, j) as:

$$\left(\frac{\partial^2 w^n}{\partial x^2} \right)_{(i,j)} = \frac{\left(w_{i+1,j}^n - 2w_{i,j}^n + w_{i-1,j}^n \right)}{h^2}, \left(\frac{\partial^2 w^n}{\partial y^2} \right)_{(i,j)} = \frac{\left(w_{i,j+1}^n - 2w_{i,j}^n + w_{i,j-1}^n \right)}{h^2}. \tag{5}$$

where h is the distance between two successive nodes.

In practice, however, experimentally measured operational deflection shapes are always corrupted by measurement noise at some extent. This may even lead to false peaks in damage index profiles misleading the data interpreter. To overcome this problem, it is proposed to calculate a summarized damage index which is defined as the average summation of damage indices for all modes N, normalized with respect to the largest value of each mode. The summarized damage index for the one-dimensional space and two-dimensional space are calculated by Eq. (6) and Eq. (7), respectively:

$$DI_i = \frac{1}{N} \cdot \sum_{n=1}^{N} \frac{DI_i^n}{DI_{\max}^n}. \tag{6}$$

$$DI_{i,j} = \frac{1}{N} \cdot \sum_{n=1}^{N} \frac{DI_{i,j}^n}{DI_{\max}^n}. \tag{7}$$

In order to evaluate the proposed damage detection algorithm the concept of the statistical hypothesis testing technique [12] is used. The damage indices determined for each node are standardised and the statistical hypothesis testing is used to classify damaged and undamaged elements. The standardised damage index (SDI) at grid point is obtained as follows

$$SDI_{i,j} = \frac{DI_{i,j} - \mu_{DI}}{\sigma_{DI}}. \tag{8}$$

where μ_{DI} and σ_{DI} are mean value and standard deviation of damage indices in Eq. (6) and Eq. (7), respectively. The decision for the localisation of damage is established based on the level of significance used in the hypothesis test which can be determined from a pre-assigned classification criterion. The typical damage threshold values for the standardised damage index widely used in literature include 1.28, 2, and 3 for 90%, 95%, and 99% confidence levels for the presence of damage.

3. Finite element model and modal analysis

3.1 Beam

The validation of proposed damage identification algorithm is performed by numerical modal analysis based on finite element method (FEM). It is conducted by using the commercial software ANSYS. An aluminium beam with two mill-cut damage sites is considered. Geometrical configuration of the beam is shown in Fig. 1. The first and second mill-cut damage sites with a depth of 2 mm and width of 50 mm are introduced at a distance of 450 mm and 750 mm from one edge of the beam, respectively. Finite element model of the beam consists of 2D beam elements. Each node has 3 degrees of freedom, namely translations along the X and Y axes and rotation along the Z axis. The beam is constructed by means of 148 equal length elements (i = 149 nodes). The elastic material properties are taken as follows: Young's modulus E = 69 GPa, Poisson's ratio ν = 0.31, and the mass density ρ = 2708 kg/m^3. The damage is modelled by reducing the flexural stiffness of the selected elements, which is achieved by decreasing the thickness of elements in the damaged region of the beam. In total, 11 modal frequencies and corresponding mode shapes are calculated.

3.2 Square plate

A square aluminium plate with dimensions of 1000 x 1000 mm and thickness of 5 mm (Fig. 2) subjected to clamped-clamped boundary conditions at all four edges is considered for this study. The following elastic material properties are taken: E = 69 GPa, ν = 0.31 and ρ = 2708 kg/m^3. The finite element model of the plate consists of eight-node shear-deformable shell elements. For the healthy plate, constant flexural stiffness is assumed for all elements, while the damaged plate is modelled by reducing the flexural stiffness of the selected elements. The damage introduced in the region with coordinates 270 mm < x < 350 mm, 640 mm < y < 700 mm is modelled by decreasing the thickness of corresponding elements. The plate is divided into 50 x 50 elements and the total of 12 modal frequencies and corresponding mode shapes are extracted from all 51 x 51 nodes of the plate.

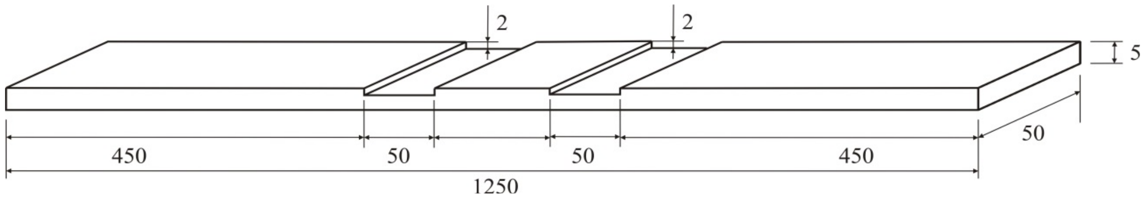

Figure 1: Geometry and dimensions of tested aluminium beam.

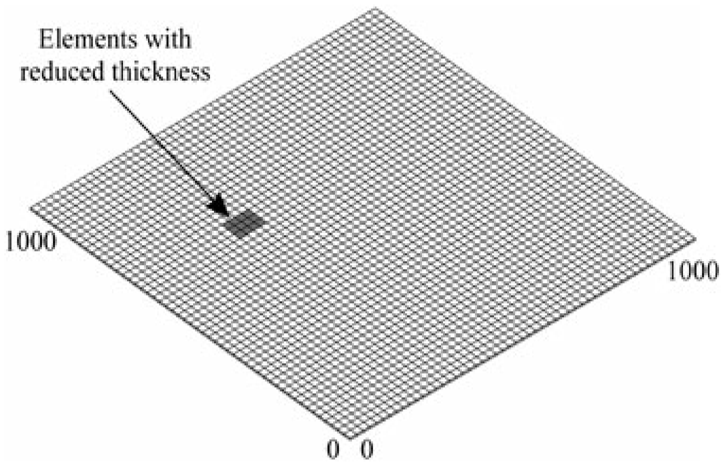

Figure 2: Finite element model for a square aluminium plate with a damage zone.

3.3 Numerical simulations

To examine the advantages and limitations of the proposed method, several sets of numerical simulations are carried out. To ascertain the sensitivity of the proposed algorithm to noisy experimental data, a series of uniformly distributed random variables is added to the numerical mode shapes for both the one-dimensional and two-dimensional spaces to generate the noise-contaminated mode shapes

$$w^n = \tilde{w}^n (1 + \delta(2r - 1)) . \quad (9)$$

where $\tilde{w}^n$ is noise free transverse displacement of the structure, r is the uniformly distributed random values in the interval $(0,1)$, δ is the noise level. In this study four different noise levels δ = 0.05%; 0.1%; 0.5% and 1% for the plate model and six different noise levels δ = 0.1%; 0.3%; 0.5%; 1%; 2% and 4% for the beam model are examined.

Although the latest measurement systems such as scanning laser vibrometer allow obtaining high-density mode shape data, in practice mode shapes often can be only measured using relatively sparse distribution of sensors and thus the robustness of the proposed method under limited data points is of interest. In order to evaluate the effect of different number of measured mode shape data points on the performance of the method, it is proposed to divide the initial vector of 149 data points for the beam model by integer values of p = 1, 2, 3, 4, 5 and 6. In these six cases, the following mode shape vectors of: 149, 75, 50, 38, 30 and 25 mode shape data points are obtained.

To evaluate the sensitivity of the proposed algorithm to severity of damage, five different levels of cut depths, namely h_1= 0.5; 1; 1.5; 2 and 2.5 mm are introduced for the plate model.

4. Results and discussion

To quantify the reliability of the proposed method for damage identification, a new parameter, called „*damage estimate reliability*" (DER) is introduced. For the one-dimensional space DER is calculated as follows:

- The whole interval along the axis of the beam (x axis) is split into 2 parts:
  - the 1st part (a) is the one which does not contain damage, namely
    0 mm < x < 450 mm, 500 mm < x < 750 mm and 800 mm < x < 1250 mm (part a);
  - the 2nd part (b), containing damage, is 450 mm < x < 500 mm and 750 mm < x < 800 mm.
- In each of these parts standardized damage indices (SDIs) from equation (8) of a respective approximation function are summed and divided by the number of data points in this particular interval, giving average amplitude of SDI ($\overline{SDI_i}$).
- DER is equal to average SDI in the area of damage (part b) divided by average SDI in all parts combined. It is expressed in percentage in Eq. (10).
- The respective DER value for the two-dimensional space is calculated by the same principle, the only difference being the calculation of SDI over an area of the structure, giving rise to the $\overline{SDI_{i,j}}$ instead of the linear portions of the structure with $\overline{SDI_i}$.

$$DER_{i,j} = \frac{\overline{SDI_{i,j}}(b)}{\overline{SDI_{i,j}}(a) + \overline{SDI_{i,j}}(b)} \times 100\% . \quad (10)$$

4.1 Beam

The reliability analysis of the proposed method for damage identification is performed for each of the Fourier fitting functions, labeled F1 through F8. The respective DER values for damage index sum over all modes at p =1 are shown in Table 1. The best fitting function turns out to be F3 with a DER = 93.78%, which is then used to examine the advantages and limitations of the proposed method for the beam model.

Table 1: DER values for Fourier series fitting functions, sum of all modes at $p = 1$.

Fitting function	F1	F2	F3	F4	F5	F6	F7	F8
DER (%)	93.50	93.71	93.78	93.51	93.20	93.00	92.82	92.77

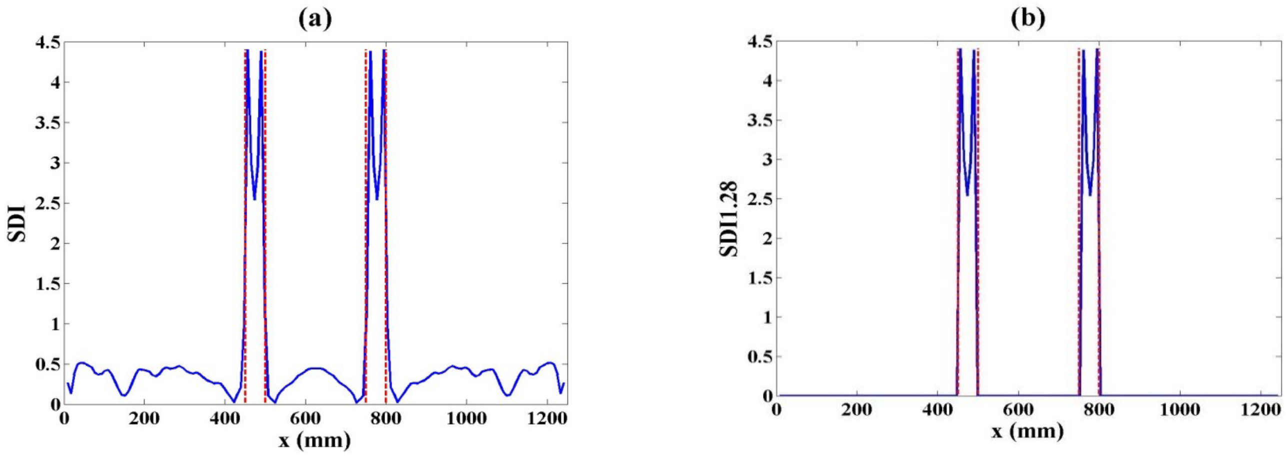

Figure 3: SDI(x) for the beam structure: *a*) SDI, *b*) $SDI_{1.28}$.

Plots of standardized damage index distributions over the coordinate of the beam SDI(x) are shown in Fig. 3. A thresholded SDI distribution is obtained from the original SDI(x) plot by setting to zero those SDI values that do not exceed the value of the threshold. This level of threshold is selected such that a 100% DER value is achieved. In all cases it turns out to be 1.28, therefore plots of $SDI_{1.28}$ are shown. It can be seen, that in both cases damage identification algorithm manages to capture the location of damage as highest peaks appear between 2 red vertical lines (depicting the predetermined area of damage).

DER values of respective SDI were calculated according to Eq. (10) for all p values at each of noise levels, forming a DER matrix of size 6x6 where columns correspond to p values and rows – to noise levels. This matrix is shown in Table 2. It can be seen that for all of the cases DER values are above 89 %, suggesting that algorithm is able to correctly identify damage. Several observations can be made. First of all, there is a tendency for noise to be more pronounced as sensor density increases. Also, noise levels from 0.1 % till 0.5 % appear to have a minor impact on DER values for largest sensor density. At higher noise levels, however decrease in DER becomes steeper.

Table 2: DER matrix for the beam model.

$p \mid \delta$ (%)	0.1	0.3	0.5	1	2	4
1	93.49	93.69	92.79	89.78	86.30	80.67
2	93.81	93.99	93.92	93.84	92.80	91.59
3	93.25	93.19	93.30	93.10	93.06	92.20
4	93.31	93.47	93.42	93.39	92.96	92.27
5	92.83	92.78	93.19	93.00	92.87	92.86
6	89.88	89.85	89.84	90.30	89.74	90.43

4.2 Plate

Standardized damage index profiles for plate model corresponding to noise free mode shape data are shown in Fig. 4. In general, standardized damage indices clearly point out the location of damage, however some low amplitude peaks are present (see Fig. 4 (a)). In order to classify the damaged elements it is proposed to truncate the SDI values smaller than 1.28 according to the standardized damage index threshold value for the 90% confidence level for the presence of damage. As can be seen in Fig. 4 (b), now the damage detection results clearly reveal the pre-determined damage location.

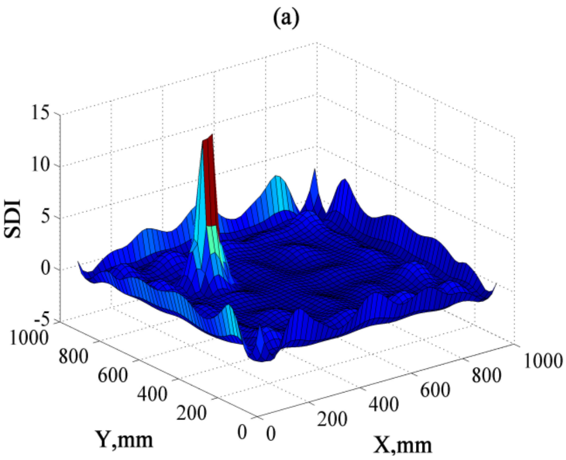

Figure 4: SDI(x, y) for the plate structure: *a*) SDI, *b*) $SDI_{1.28}$.

Combination of different noise levels and cut depths result in 20 simulated test cases for the plate model. The respective DER results for these cases according to the three confidence levels for the presence of damage are given in Table 3. It must be noted that DER values are included in the table only for those cases where standardized damage indices at the pre-determined damage location passed the SDI threshold value for the corresponding confidence level.

Table 3: DER matrix for the plate model.

	Noise level $\delta = 1\%$			Noise level $\delta = 0.5\%$			Noise level $\delta = 0.1\%$			Noise level $\delta = 0.05\%$		
Confidence level / Cut depth [mm]	90%	95%	99%	90%	95%	99%	90%	95%	99%	90%	95%	99%
0.5	-	-	-	-	-	-	98.6	99.9	100	99.3	100	100
1.0	-	-	-	92.2	98.2	99.9	98.7	99.8	100	99.8	100	100
1.5	-	-	-	92.9	98.3	100	99.0	99.9	100	100	100	100
2.0	91.7	98.4	100	93.5	98.8	100	100	100	100	100	100	100
2.5	92.1	97.8	99.8	94.3	99.1	100	100	100	100	100	100	100

It can be seen that DER values for all of the cases are higher than 91%, indicating high reliability of damage identification. The capability of the damage identification algorithm is dependent on both the introduced noise level and damage severity. As expected, the reliability of the proposed method increases as damage severity increases and decreases as the noise level increases.

5. Conclusions

In this paper a numerical study on the applicability of mode shape curvature based method for damage identification in plate-like and beam-like structures is presented. The advantage of the proposed method is that there is no need for mode shape information from the healthy state of the structure. The damage index is defined as the absolute difference between the measured curvature of the damage structure and the smoothed polynomial representing the healthy structure. Smooth mode shape curvatures of the healthy structure are estimated using interpolation of mode shape curvature of a damaged structure with Fourier series functions of different orders for the beam-like structure, while a regression analysis with a polynomial approximation is used for the plate-like structure. Several sets of numerical simulations are carried out to analyse the influence of damage severity, measurement noise and sensor spacing on the performance of the method, thus yielding a matrix where reliability of the damage identification for each of these cases is expressed in percentage through the damage estimate reliability parameter. The obtained results show that the proposed damage identification method provides reliable information about the location and size of the damage in case of the presence of damage of medium severity, relatively accurate measurement data and relatively dense distribution of sensors. The results suggest that the proposed method can be successfully used in combination with the latest scanning laser vibrometer systems which allow high-density transverse displacement measurements with a low degree of measurement noise not only for laboratory tests but also for practical structural applications.

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