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Reliability of Aircraft Fleet: Binary p-set and Lambda-set Functions

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Abstract: Definitions of binary p-set and binary λ-set functions are given. The use of these definitions allows for any unknown parameter, θ, of the model of fatigue failure of aircraft to provide the economical effectiveness of an airline under a limitation of the fatigue failure rate (FFR) and the limitation of any fatigue failure probability of (FFP) in a fleet of aircraft . The solution of the problem using theory of semi-Markov process with rewords is based on a result of an acceptance full-scale fatigue test of an aircraft structure. For this case the maximum of FFP and the maximum of FFR can be limited for any unknown parameter, θ.

Keywords: *inspection program, Markov chain, Monte-Carlo, reliability, p-set function.*

1. Introduction

This paper is a development and some correction of an inspection program planning solution provided in [1] and [2]. Some additional details and examples are discussed. Definitions of a binary p-set and a binary λ-set function are introduced. The use of these definitions allows for any unknown parameter, θ, of the model of fatigue failure of aircraft to provide the economical effectiveness of an airline under a limitation of the fatigue failure rate (FFR) and the limitation of any fatigue failure probability (FFP) in a fleet of aircraft. The solution of the problem using the theory of semi-Markov process with rewords is based on a result of an acceptance full-scale fatigue test of an aircraft structure.

There is a long history of the solution of the fatigue problem of an aircraft (AC). The survey of the problem was made in [1]. Now there are two main approaches to the solution of fatigue problem of aircraft: safe-life and damage tolerance (DT) approaches. In first case the reliability of AC is provided by the limitation of its service time by a specified life (SL) (it is a time of retirement). In the second approach the reliability of AC is provided by a program of inspections [3].

A great deal of research has been devoted to this problem. Some state-of-the-art of solutions of some specific versions of it was discussed during 8th IMA International Conference on Modelling in Industrial Maintenance and Reliability (MIMAR), Oxford, Institute of Mathematics and its Applications, 2014. Here we mention some papers which are the most close to the considered in this paper problem. The paper [4] focus on a critical single-component system which is classified in three distinct states: normal, defective and failed. A system failure may lead to human injury. Such consequences of system failures can be taken into account by converting human injuries to monetary value.

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But this might raise ethical concerns. Therefore, scientific literature considers safety constraints [5] such as a constraint for system reliability. Some research dealt with the optimal decision making problem for stochastically deteriorating systems and typically formulated it as a Markov decision process [6]. In [7] a cooperative game-based decision method for aircraft fleet condition based maintenance is considered.

After analysis of these and similar publications it should be noted:

1) In most publications the reliability problem is considered as a problem of the theory of probability when the cumulative distribution function (cdf) of corresponding random variables (a fatigue life, fatigue crack model parameters …) is known already. But in this paper the main attention will be devoted to the statistical problem when this cdf is not known.
2) For such specific system as aircraft fleet (large price of an aircraft, great "cost" of a failure, very limited number of test for estimation of reliability...) it is necessary to study at least two stages: a) beginning of service of fleet of new aircraft fleet, b) regular service of airline. Usually they consider only the last stage.

In offered paper we take into account both these notes.

Let us remind that if SL is already nominated but it is necessary to check the required reliability then the final decision is defined by the fatigue life binary p-bound based on a processing the result of acceptance full-scale fatigue test [1]. If some specific requirements are not met the use of a new type of AC in regular service is not allowed. Preliminary, the redesign of AC should be made. We suppose to know the model of the fatigue crack growth but some parameter of the model, θ, is not known. The estimate of it, $\hat{\theta}$, can be obtained by processing the result of full-scale fatigue test. If some specific requirements to the acceptance test are met, we say: $\hat{\theta} \in \Theta_0$, where Θ_0 is some subset of the parameter space, Θ. If these requirements are not met, $\hat{\theta} \notin \Theta_0$, the use of a new type of AC in regular service is not allowed. The corresponding probability of failure will be equal to zero. But if $\hat{\theta} \in \Theta_0$ we chose a vector of time moments of inspections $t_{1:n} = (t_1,...,t_n)$. And let T_d be a random time moment when fatigue crack become detectable, T_c - a time moment when it reaches critical size and the failure takes place. We suppose that the fatigue crack is detectable with probability equal to 1 if the inspection is made within the interval (T_d, T_c) and it is equal to zero if inspection time moment is lower than T_d. The vector $t_{1:n}$ defines the set of intervals $\{(t_{i-1}, t_i],\ i = 1,...,n\}$ and corresponding probability that the fatigue crack grows up to critical time before it will be detected. A definitions of a binary p-set function of the estimate $\hat{\theta}$ for the vector $Z = (T_d, T_c)$ as a generalization of the right-hand binary p-bound [1] will be introduced in the section 4.

We say that it is the process of operation of an airline (AL) if after the crack detection, the fatigue failure or the retirement of AC we make the acquisition of new AC and begin its operation just from the zero time moment. The description of operation of

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AL can be given by the use of the theory of semi-Markov process with reword. In this case, to provide the reliability of AL it is necessary to limit the intensity of the failure (a rate of the failure), λ (the inverse value of a mean time between failures). The choice of the vector $t_{1:n}$ as a result of processing the data set of full-scale fatigue test of AC structure defines the binary λ-set function of the estimate $\hat{\theta}$ for the vector $Z=(T_d,T_c)$. It is some specific variation of the binary p-set function. A formal definition of it will be given in section 4 also.

The numerical examples are given.

2. Solution of the problem for the known θ

For the known θ, there are two decisions: 1) the aircraft is good enough and the operation of this aircraft type can be allowed, 2) the operation of the new type of AC is not allowed. A redesign of AC should be made. In the case of the first decision, the vector $t_{1:n}=(t_1,\ldots,t_n)$, where t_i, $i=1,...,n$, is the time moment of *i*-th inspection, should be defined also. If θ is known the different rules can be offered for the choice of structure of the vector $t_{1:n}$: 1) every interval between the inspections is equal to the constant $t_{SL}/(n+1)$, where t_{SL} is the aircraft specified life (SL) (the retirement time), 2) the probabilities of a failure in every interval are equal to the same value… In this paper we consider the first approach, but really, our considerations can be applied and in a more general case when the vector $t_{1:n}$ is defined by two parameters, the fixed t_{SL} and the the number of inspections, n, in such a way that probability of failure tends to zero when n tends to infinity.

For the choice of inspection number we should know FFP of an AC and FFR and the gain of an AL as a functions of vector $t_{1:n}$. For this purpose the service process of an AC can be considered as an absorbing Markov chain (MCh) with $(n+4)$ states. The states $E_1,E_2,\ldots,E_{n+1}$ correspond to an AC operation in the time intervals $[t_0,t_1),[t_1,t_2),\ldots,[t_n,t_{n+1})$, $t_0=0$, $t_{n+1}=t_{SL}$. States E_{n+2}, E_{n+3} and E_{n+4} are the absorbing states: AC is discarded from a service when the SL is reached or fatigue failure (FF), or fatigue crack detection (CD) takes place.

In the corresponding transition probability matrix, P_{AC}, let ν_i be the probability of a crack detection during the inspection number i, let q_i be the probability of the failure in service time interval $(t_{i-1},t_i]$, and let $u_i=1-\nu_i-q_i$ be the probability of successful transition to the next state. In our model we also assume that an aircraft is discarded from a service at t_{SL} even if there are no any crack discovered by inspection at the time moment t_{SL}. This inspection at the end of $(n+1)$-th interval (in state E_{n+1}) does not change the reliability but it is made in order to know the state of an aircraft (whether there is a fatigue crack or there is no fatigue crack). It can be shown that

$$u_i = P(T_d > t_i \mid T_d > t_{i-1}),\ q_i = P(t_{i-1} < T_d < T_c < t_i \mid T_d > t_{i-1}),$$
$$v_i = 1 - u_i - q_i,\ i = 1,\ldots,n+1. \quad (1)$$

In the three last lines of the matrix P_{AC} there are three units in the matrix diagonal because the states E_{n+2}, E_{n+3} and E_{n+4} are the absorbing states. All the other entries of this matrix are equal to zero, see Fig.1.

	E_1	E_2	E_3	...	E_{n-1}	E_n	E_{n+1}	E_{n+2} (SL)	E_{n+3} (FF)	E_{n+4} (CD)
E_1	0	u_1	0	...	0	0	0	0	q_1	v_1
E_2	0	0	u_2	...	0	0	0	0	q_2	v_2
E_3	0	0	0	...	0	0	0	0	q_3	v_3
...	...	...	...	...	...	...	...	...	...	...
E_{n-1}	0	0	0	...	0	u_{n-1}	0	0	q_{n-1}	v_{n-1}
E_n	0	0	0	...	0	0	u_n	0	q_n	v_n
E_{n+1}	0	0	0	...	0	0	0	u_{n+1}	q_{n+1}	v_{n+1}
E_{n+2} (SL)	0	0	0	...	0	0	0	1	0	0
E_{n+3} (FF)	0	0	0	...	0	0	0	0	1	0
E_{n+4} (CD)	0	0	0	...	0	0	0	0	0	1

Figure 1: Matrix of Transition Probabilities P_{AC}

The structure of the considered matrix can be described in the following way: $P_{AC} = [Q\,R;\ 0\,I]$, where in the second line of this structure the matrix 0 is the sub matrix of zeroes, I is the sub matrix of identity corresponding to the absorbing states of the matrix P. Then the matrix of the probabilities of absorbing in the different absorbing states for the different initial transient states $B = (I - Q)^{-1} R$. The failure probability of a new AC is equal to $p_f(n,\theta) = aBb$, where the vector row $a = (1, 0, \ldots, 0)$ if all the aircraft begin an operation within the first interval (state E_1), the vector column $b = (0,1,0)'$.

We suppose that for any θ, $\theta \in \Theta_0$, there is such n that $p_f(n,\theta) = p$, where $(1-p)$ is the required reliability. For example, if all inspection intervals are equal then the required number of the inspections is defined by equation

$$n_p(p,\theta) = \min\left\{n : p_f(n,\theta) \le p, \text{ for all } n \ge n_p(p,\theta)\right\}. \tag{2}$$

We should use this equation instead of the equation $p_f(n_p,\theta) = p$ because the function $p_f(n,\theta)$ in general case is not monotone. In Fig 2 we see that some fatigue crack can be detected if $n=1$ but it is not detected if $n=2$.

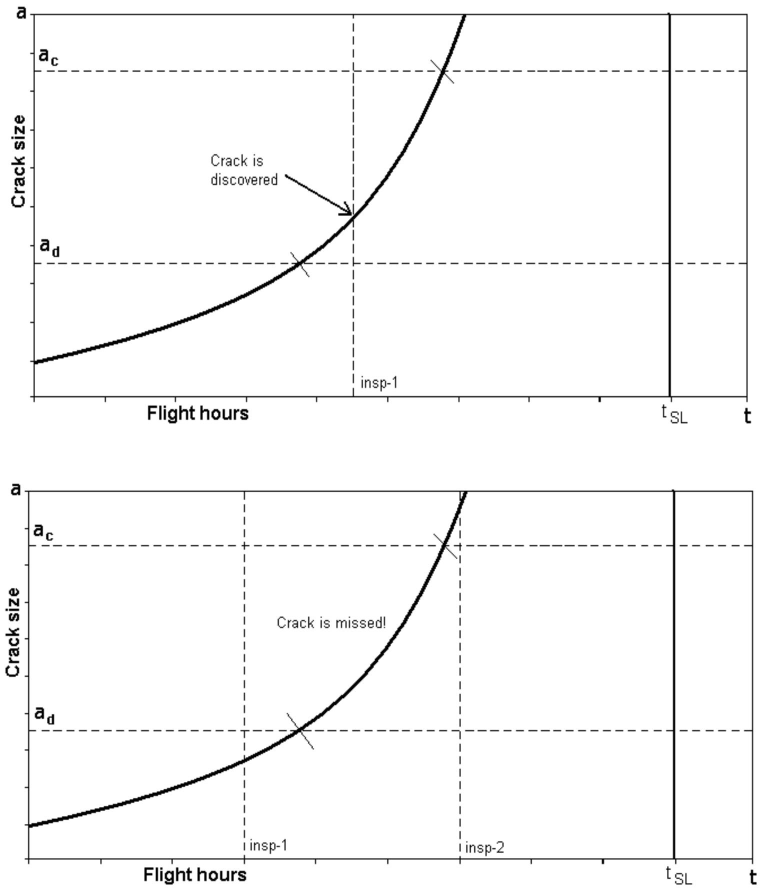

Figure 2: Demonstration of Non-Monotonous Nature of $p_f(n,\theta)$

In the corresponding matrix for the operation process of AL, P_{AL}, the states E_{n+2}, E_{n+3} and E_{n+4} are not absorbing but correspond to return of MCh to the state E_1 (in the last three lines there are the units in first column of the matrix P_{AL}; the AL operation returns to the first interval). The other lines of P_{AC} and P_{AL} are the same, see Fig. 3.

	E_1	E_2	E_3	...	E_{n-1}	E_n	E_{n+1}	E_{n+2} (SL)	E_{n+3} (FF)	E_{n+4} (CD)
E_1	0	u_1	0	...	0	0	0	0	q_1	v_1
E_2	0	0	u_2	...	0	0	0	0	q_2	v_2
E_3	0	0	0	...	0	0	0	0	q_3	v_3
...	...	...	...	...	...	...	...	...	...	...
E_{n-1}	0	0	0	...	0	u_{n-1}	0	0	q_{n-1}	v_{n-1}
E_n	0	0	0	...	0	0	u_n	0	q_n	v_n
E_{n+1}	0	0	0	...	0	0	0	u_{n+1}	q_{n+1}	v_{n+1}
E_{n+2} (SL)	1	0	0	...	0	0	0	0	0	0
E_{n+3} (FF)	1	0	0	...	0	0	0	0	0	0
E_{n+4} (CD)	1	0	0	...	0	0	0	0	0	0

Figure 3: Matrix of Transition Probabilities P_{AL}

Using the theory of Markov process with rewards, the theory of the semi-Markov process with rewords and definition of P_{AL} we can get the vector of stationary probabilities, $\pi = (\pi_1, ..., \pi_{n+4})$

which is defined by the equation system:

$$\pi P_{AL} = \pi \ , \sum_{i=1}^{n+4} \pi_i = 1 \tag{3}$$

and the airline gain

$$g(n) = \sum_{i=1}^{n+4} \pi_i g_i(n) \ , \tag{4}$$

where

$$g_i(n) = \begin{cases} a_i u_i + b_i q_i + c_i v_i, i = 1, \ldots, n+1, \\ d_i, i = n+2, \ldots, n+4, \end{cases} . \tag{5}$$

a_i is the reward defined by the successful transition from one operation interval to the following one and the cost of one inspection; b_i, c_i and d_i correspond to transition to the states E_{n+3} (FF), E_{n+4} (CD) and then to the state E_1 (the "cost" of FF of AC, fatigue crack detection, acquisition of new AC) . Let us note that if $a_i = t_i - t_{i-1}$, $b = c = d = 0$ then

$$g(n) = \sum_{i=1}^{n+4} \pi_i g_i(n) = g_t(n) = \sum_{i=1}^{n+1} \pi_i (t_i - t_{i-1}) \tag{6}$$

and $L_j = g_t(n,\theta) / \pi_j$ defines the mean time to return to the same state E_j, $\lambda_F(n,\theta) = 1 / L_{n+3}$ is the FFR.

If θ is known we calculate the gain as a function of n, $g(n,\theta)$, and choose the number n_g corresponding to the maximum of the gain :

$$n_g(\theta) = \arg\max_n g(n,\theta). \tag{7}$$

Then we calculate FFR as function of n, $\lambda_F(n,\theta)$, and choose n_λ in such a way that for any $n \ge n_\lambda$ the function $\lambda_F(n,\theta)$ will be equal or less than some value λ:

$$n_\lambda(\lambda,\theta) = \min\{n : \lambda_F(n,\theta) \le \lambda, \text{ for all } n \ge n_\lambda(\lambda,\theta)\}. \tag{8}$$

And finally

$$n = n_{g\lambda}(\lambda,\theta) = \max(n_g, n_\lambda). \tag{9}$$

3. Probability of any fatigue failure in the fleet of aircraft

We consider the case when the operation of all N aircraft will be stopped if any fatigue crack will be detected. So in order to limit the probability of any fatigue failure in a fleet (FFPN) it is enough to find at least one fatigue crack before the failure of any aircraft in the fleet takes place. The corresponding probability is equal to the expected value of the random variable $P_{fNW} = (1-w)^R$, where w is a human factor (it is a probability, that the planned inspection will be made), *R* is the total random number of inspections before the first failure in the whole fleet. Let t_k^+, $t_{k-1}^+ < t_k^+$, $t_0^+ = 0$, to be "calendar" time moment when *k*-th aircraft begin the service, $T_{d_k}^+ = t_k^+ + T_{d_k}$, $T_{c_k}^+ = t_k^+ + T_{c_k}$, $\mathrm{k} = 1, 2, ..., N$ to be the random calendar time moments when fatigue crack can be discovered and fatigue failure of AC takes place correspondingly, see Fig. 4. And let $K_{SL} = \{k : T_{ck} < t_{SL},\ k = 1, 2, ..., N\}$ be a set of indexes of aircraft, the failure of which can take a place, if any inspection does not take the place,

$$T_f^+ = \min\{T_{Ck}^+ : k \in K_{SL}\},\ T_{fk}^+ = \min\{T_{Ck}^+,\ T_f^+\},\ k \in K_{SL}. \tag{10}$$

Finally

$$R = \sum\nolimits_{k \in K_{SL}} R_k \quad , \tag{11}$$

where $R_k = \max(\{[(T_{fk}^+ - t_k^+)/\Delta_t] - [(T_{dk}^+ - t_k^+)/\Delta_t]\}, 0)$, $k \in K_{SL}$, [x] is the elements of x to the nearest integers towards minus infinity. Here R_k is the random inspection number of k-th aircraft from the set K_{SL} if inspection interval $\Delta_t = t_{SL}/(n+1)$ (it is supposed a specific calendar schedule of the inspections for each aircraft: $i = 1, 2, ..., n+1$, $k \in K_{SL}$)

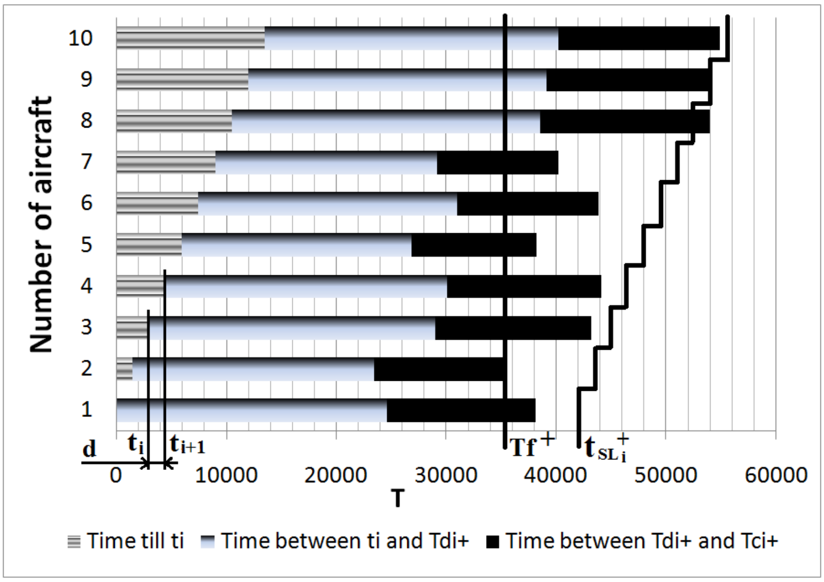

Figure 4: Inspection of N Aircraft

Random variable Q is a speed of fatigue crack growth in logarithm scale. It has the specific realization for each aircraft and $Q_1, \ldots, Q_N$ are independent random variables. So mean value of random probability of failure in the fleet

$$E\left(P_{fNW}\right) = p_{fNW}\left(n, \theta\right) = \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} (1-w)^{r(q)} \, dF_{Q_i}\left(q_1\right) \ldots dF_{Q_N}\left(q_N\right) , \tag{12}$$

where $q = (q_1, \ldots, q_N)$, $r(q)$, is realization of rv R. For large number N the Monte Carlo method is appropriate for the calculation of p_{fNW}. If this function is known then the number of the inspections, $n(p, \theta)$, required to limit the FFPN by a value p is defined by the equation

$$n(p, \theta) = \min(r : p_{fNW}(\theta, r) \leq p \ \text{ for all } r > n(p, \theta), \ r = 1, 2, ...) . \tag{13}$$

4. Solution for an unknown θ

First, we consider the problem of a limitation of FFP1 in an operation of one AC with the human factor $w=1$ and probability of the crack detection equal to unit if the inspection takes place within the interval (t_d, t_c). The limitation of FFP1 of AC is provided by the inspections in accordance with the specific p-set function, general definition of which was given in [1].

Definition 1. Let Z and X be random vectors of m and n dimensions and suppose that the class is known { P_θ , $\theta \in \Theta$ } to which the probability distribution of the random vector *W=(Z,X)* is assumed to belong. Of the parameter θ, which labels the distribution, it is presumably known only that it lies in a certain set Θ, the parameter space. Let $S_Z(x) = \bigcup_{i=1}^{r} S_{Z,i}(x)$ denotes some set of disjoint sets of *z* values as function of *x*. If

$$\sup_{\theta} \sum_{i=1}^{r} P(Z \in S_{Z,i}(X)) = p \tag{14}$$

Then the statistical decision function $S_Z(x)$ is the p-set function for the random vector *Z* on the base of a vector $x = (x_1, x_2, ..., x_n)$.

In our case $Z = (T_d, T_c)$, p-set function $S_Z(x)$ is defined by the choice of the vector t.

In [1] the connection of this definition with testing statistical hypotheses and the theory of statistical inference is given.

4.1 Binary p-set function

Now let us take into account that we consider the case when the for the estimate of unknown parameter θ, $\hat{\theta} = \hat{\theta}(x_1, ..., x_n)$, the result of acceptance test is used and the operation of a new type of aircraft will not take place if the result of the fatigue test in a laboratory is “too bad” (previously, the redesign of the new type of AC should be made). We say that in this case the event $\hat{\theta} \notin \Theta_0$, $\Theta_0 \subset \Theta$ takes place (for example, $\hat{\theta} \notin \Theta_0$ if the test fatigue life T_C is lower than some limit; or $n(p, \hat{\theta})$ is too large,…).

Let us define some binary set function

$$S(\hat{\theta}, \Theta_0, n) = \begin{cases} \bigcup_{i=1}^{n+1} S_i(n) \;\; if \;\; \hat{\theta} \in \Theta_0, \\ \varnothing, \;\; if \;\; \hat{\theta} \notin \Theta_0 \end{cases}, \tag{15}$$

where $S_i(n) = \{(t_d, t_c) : t_{i-1} < t_d, t_c \le t_i\}$, $t_i = it_{SL} / (n+1)$, $i = 1, ..., n+1$; $\varnothing$ is an empty set.

It can be shown that for very wide range of the definition of the set Θ_0 and the requirements to limit FFP1 by the value p^*, where $(1 - p^*)$ is a required reliability, there is a preliminary “designed allowed FFP1”, p_{FD}, such that corresponding set function $S(\hat{\theta},\Theta_0,n(p_{FD},\hat{\theta}))$ is *binary* p*-set function of the level* p^* *for the vector* $Z=(T_d,T_c)$ *on the base of the estimate* $\hat{\theta}$*:*

$$\sup_\theta \sum_{i=1}^{n+1} P(Z\in S_i(n(p_{FD},\hat{\theta}))\bigcap \hat{\theta}\in\Theta_0)=p^* \cdot \qquad (16)$$

This means that FFP1 will be limited by the value p^* for any unknown $\theta\in\Theta$.

4.2 Binary λ-set function

In similar way, it can be shown that for very wide range of the definition of the set Θ_0 and the requirements to limit a FFR of an AL by the value λ^*, where λ^* is a required fatigue failure intensity, there is a preliminary "designed allowed FFR", λ_{FD}, such that corresponding set function $S(\hat{\theta},\Theta_0,n_\lambda(\lambda_{FD},\hat{\theta}))$ is *a binary* λ*-set function of the level* λ^* *for the vector* $Z=(T_d,T_c)$ *on the base of the estimate* $\hat{\theta}$*:*

$$\sup_\theta E((\lambda(n_\lambda(\lambda_{FD},\hat{\theta}))\,|\,\hat{\theta}\in\Theta_0)*P(\hat{\theta}\in\Theta_0))=\lambda^* \ . \qquad (17)$$

This means that FFR will be limited by the value λ^* for any unknown $\theta\in\Theta$.

Let us note, that instead of the words a *binary* λ*-set function* we should use the words *binary* λ_g *-set function* if instead of $n_\lambda(\lambda_{FD},\hat{\theta})$ we use $n_{g\lambda}(\lambda_{FD},\hat{\theta})$.

For the requirement of a high reliability the choice of an inspection number will be defined by the limitation of FFR. For very high "cost" of FF of AC it will be defined by the maximum of the gain.

4.3 Reliability of fleet of AC

Now we consider the reliability of the fleet of N AC when there is an information exchange and the operation of all aircraft will be stopped if fatigue crack will be found during an inspection of any AC and, as it was told already, in order to prevent the failure in the fleet, it is enough to find at least one fatigue crack before the failure of any aircraft in the fleet takes place.

Let us define some multiple set function

$$S^+(\hat{\theta},\Theta_0,n)=\bigcup_{k\in K_{SL}} S_k^+(\hat{\theta},\Theta_0,n) \qquad (18)$$

where

$$S_k^+(\hat{\theta},\Theta_0,n)=\begin{cases}\bigcup_{i=1}^{n+1} S_{i,k}^+(n) \ \ if \ \ \hat{\theta}\in\Theta_0,\\ \varnothing, \ \ if \ \ \hat{\theta}\notin\Theta_0,\end{cases}$$

$S_{i,k}^+=\{(t_{d,k}^+,t_{c,k}^+):t_{(i-1),k}<t_{d,k},t_{c,k}\le t_{i,k}\}$, $t_{i,k}^+=t_k^++t_i$, $t_i=it_{SL}/(n+1)$, $i=1,...,n+1$, $k=1,2,...,N$. Example of $S^+(\hat{\theta},\Theta_0,n)$ is shown in Fig.5.

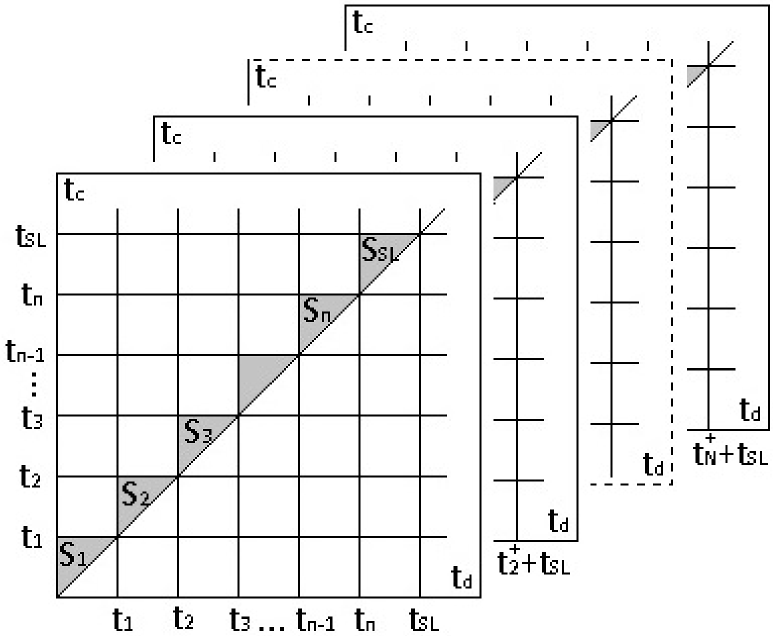

Figure 5: Example of $S^+(\hat{\theta},\Theta_0,n)$

Again, it can be shown that for very wide range of the definition of the set Θ_0 and the requirements to limit FFPN by the value p^*, there is a preliminary "designed" choice of allowed FFPN, p_{FD}, such that corresponding multiple set function $S^+(\hat{\theta},\Theta_0,n(p_{FD},\hat{\theta}))$ is p-set function of level p^* for the set of vectors $\{Z_k^+,\ k \in K_{SL}\}$, where $Z_k^+ = (T_{d,k}^+, T_{f,k}^+)$:

$$\sup_\theta E\{\sum_{k\in K_{SL}} \sum_{i=1}^{n+1} P(Z_k^+ \in S_{i,k}^+(n(p_{FD},\hat{\theta})) \bigcap \hat{\theta} \in \Theta_0)\} = p^* . \qquad (19)$$

That means that FFPN will be limited by the value p^* for any unknown θ.

5. Numerical example

The example of the solution of the reliability problem of aircraft fleet is considered in [2]. Here we consider only the problem of reliability of AL.

Despite of all the simplicity, the equation $a(t) = \alpha \exp(Qt)$ gives us rather comprehensible description of fatigue crack growth in the interval (t_d, t_c), where (we recall) t_d is the time when a fatigue crack becomes detectable ($a(t_d) = a_d$) and t_c is the time when the crack reaches its critical size ($a(t_c) = a_c$) and fatigue failure takes place. It can be assumed that corresponding random variables $T_d = (\log a_d - \log\alpha)/Q = C_d/Q$ and $T_c = (\log a_c - \log\alpha)/Q = C_c/Q$ have the lognormal distribution because, as it is assumed usually, normal distribution of $\log T_c$ can take place only if either both $\log C_c$ and $\log Q$ are normally distributed or if one of these components is normally distributed while another one is constant. We suppose also, that vector $(X,Y) = (\log(Q), \log(C_c))$ has two dimensional normal distribution with vector-parameter $\theta = (\mu_X, \mu_Y, \sigma_X, \sigma_Y, r)$. It is worth to note, that for the case when a_c and a_d are constants then cdf of C_d is completely defined by the distribution of C_c because $C_d = C_c - \delta$, where $\delta = \log(a_c / a_d)$. For numerical example it is supposed a_c =237.8 mm and a_d =20 mm.

We use the following definitions of the components of an AL income: for all $i = 1,...,n+1$ $a_i = a_0(n) + d_{isp} t_{SL}$ where $a1 = a_0(n) + d_{isp} t_{SL}$ $a_0(n) = a_{01} t_{SL} / (n+1)$, - is the reward related to successful transition from one operation interval to the following one, a_{01} defines the reward of operation in one time unit (one hour or one flight); $d_{insp} t_{SL}$ is the cost of one inspection (negative value) which is supposed to be proportional to t_{SL}; $b_i = b_{01} t_{SL}$ is related to FF (negative value), $c_i = c_{01} a_0(n)$ is the reward related to transitions from any state $E_1,...,E_{n+1}$ to the state E_{n+4} (it is supposed to be proportional to a_0 because it is a part of a_0); $d_i = d_{01} t_{SL}$ is negative reward, the absolute value of which is the cost of new aircraft acquisition after events SL, FF or CD and transition to E_1 takes place. In numerical example we have used the following values: $t_{SL} = 40000$, b_{01}=-0.3; $d_{insp} = -0.05$; a_{01}=1; c_{01}=0.1; d_{01}=- 0.3. Suppose we have the following estimate of parameter $\theta = (\mu_X, \mu_Y, \sigma_X, \sigma_Y, r)$: $\hat{\theta}$=(-8.58688044, 1.9424608, 0.155, 0.0778895, 0.796437) (see Fig 2.2 and Table 2.1 in [1]). It was assumed that the set Θ_0 corresponds to the decision to make redesign if the estimate of critical time to failure $t_C = \exp(\hat{\mu}_Y - \hat{\mu}_X)$ is too small: $t_C < 0.3 t_{SL}$.

Calculation of $w(\theta, \lambda_{FD}, \Theta_0) = E\{\lambda_F(\hat{\theta}, \lambda_{FD}, \Theta_0)\}$ was made for $(7.2029 \le \mu_X \le 9.9709)$, $(1.3972 \le \mu_Y \le 2.4877)$ assuming that the vector (σ_X, σ_Y, r) is the same for all different vectors (μ_X, μ_Y). It was found that for λ_{FD} =0.1*10-6 the maximum value of $w(\theta, \lambda_{FD}, \Theta_0)$ is equal to 0.9041*10-6 . Suppose that the value 0.9041*10-6 is required reliability. Then for the known estimate of the parameter the calculation of $n_\lambda(\lambda_{FD}, \hat{\theta})$ for λ_{FD} =0.1*10-6 gives us the required number of inspection. It is equal to 6. For the considered estimate of θ t_C is equal to 37.4574e+003 so the redesign is not needed. After the necessary calculation of $g(n,\theta)$ it is found $n_g = 4$. So finally, the required number of inspections $n = \max(n_g, n_\lambda)$ is equal to 6.

6. Conclusions

The problem of inspection planning on the bases of the result of acceptance full-scale fatigue test of an AC structure is the choice of the sequence $\{t_1, t_2, ..., t_n, t_{SL}\}$ corresponding to limitation of the FFP of an AC or the FFR of an AL if some requirements to the result of acceptance full-scale fatigue test are met. If these requirements are not met the redesign of the new type of an aircraft should be made. The definitions of binary p-set and λ-set functions are introduced for description of corresponding mathematical procedures, based on observation of some fatigue crack during acceptance full-scale fatigue test of airframe. In general case the desire to increase the gain of airline service can be taken into account but under condition that required reliability is already provided. The method of necessary calculation is provided.

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