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Algorithms of calculation open queuing networks with heterogeneous requests

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In earlier studies, the author developed a method for calculating the parameters of queuing systems while taking into account the heterogeneity of the requests. In the present paper, a method for calculating the parameters of open stochastic heterogeneous networks is developed. In addition, algorithms for the calculation of the normalization constants and the parameters of an open stochastic heterogeneous network of arbitrary configuration are designed.

Keywords: Heterogeneity, approximate methods, heterogeneous service, normalization constant

Introduction

In the works of Jackson (1963) and Gordon and Newell (1967) a theory of closed stochastic networks was developed for the case of homogeneous exponential request service. In practice, not only closed stochastic networks but also open stochastic networks may often be encountered. Local or corporate computer networks may serve as examples of such networks, where the requests are the calls for programs processing, data queries, or requests for connection of users to the Internet. Boicov (2009) has described the method for calculating the characteristics of queuing systems with heterogeneous requests for closed stochastic networks. However, it is difficult to apply existing algorithms for calculating the normalization constants, their description may be found in works of Buzen (1973) and Boicov (2010) for this method. The reason is that the calculation of these constants is time consuming and usually it comes to the complete enumeration of the variables and the components of the constants of the normalization. In open systems, this calculation is even more labor intensive, since the number of requests in open systems may increase without limit. In this paper, a way to bypass this difficulty is suggested by the selection of the simplest models of queuing systems equivalent to a queuing system with heterogeneity. In addition, the reduction of the computational labor's intensiveness is obtained by finding of the values of the normalization constants in the following calculation method in the form of a series convolution of geometric progressions.

The research of the simplest queuing system with heterogeneous requests in open queuing networks

The author (Boicov, 2009) obtained the following formula to calculate the average number of requests in a queuing system with heterogeneity:

$$N = \sum_{i=1}^{R} \lambda_i / \mu_i + \sum_{i=1}^{R} \lambda_i [\sum_{i=1}^{R} [\lambda_i / \mu_i^2] / [(1 - \sum_{i=1}^{R} \lambda_i / \mu_i)]] \quad (1).$$

Here, λ_i is the average intensity of the arrivals of the ***i*** group of requests, μ_i, are the average intensities o serving these arrivals, and *R* is the total number of types of requests. A steady state of the operation of $\sum_{i=1}^{R} \lambda_i / \mu_i$, a single-channel, single phase service network with heterogeneity is possible in the total capacity factor of is less than unity. Then, for a closed stochastic network consisting of *M* nodes, the condition of the steady state may be written in the form of *M* of the following inequalities:

$$\sum_{j=1}^{M} \sum_{i=1}^{R} \lambda_{i,j} / \mu_{i,j} < 1 \qquad j = 1, 2, \ldots, M . \qquad (2).$$

Let us reduce one of these inequalities (for some *i* node) to unity. From equality (1), we find that, in this case, the number of requests in the *i* node of the network tends to infinity as the denominator of the second term tends to zero.

On the other hand, the number of requests in other nodes of the network will not grow indefinitely due to the fact that their capacity factors are different from unity. Then, with respect to the *i* node, the closed network is transformed into an open network, since, by definition (Kleinrock, 1979), the network is considered to be open if it is subjected to a source with an infinite number of requests.

At first, let us consider an elementary open heterogeneous network consisting of two nodes. Let the first node represent a device of mass service with a single channel of heterogeneous service. Using the rule of replacement for the equivalent homogeneous network (Boicov, 2009) for the analysis of this network, let us replace the first node in the network with a node of the equivalent service intensity:

$$\mu_{1equ} = 1(/\lambda)\sum_{r=1}^{R} \lambda_r \mu_r \qquad (3).$$

Assume that the second node represents a source of requests with a heterogeneous flow and its total intensity determined by the following equation:

$$\lambda = \sum_{r=1}^{R} \lambda_r \qquad (4).$$

Then, assuming that $\lambda = \mu_{2equ}$, the closed network of two nodes may be represented as a network of one node and one source of requests.

On the other hand, considering this network as a closed network with two nodes, the method proposed below for calculating the normalization constants in the form of geometric progressions may be used.

To find the average length of the requests queue in the nodes of the considered network, it is proposed to use the following sequence of actions. At first, it is necessary to solve the system of algebraic equations with unknown x_i for each *i* network node and then to find the constant of normalization in the form of geometric progressions in relation to these unknowns. Thus, for a simple network consisting of two nodes, at first it is necessary to solve a system of two algebraic equations:

$$\mu_{1equ} x_1 = \mu_{1equ} x_1 \pi_{1,1} + \mu_{2equ} x_2 \pi_{2,1}$$

$$(5).$$

$$\mu_{2equ} x_2 = \mu_{1equ} x_1 \pi_{1,2} + \mu_{2equ} x_2 \pi_{2,2}$$

Here, the values of $\pi_{i,j}$;are defined by the matrix of relationships of the nodes. In our case, for a network consisting of two nodes that are closed relative to each other, we have $\pi_{i,j} = \pi_{i,j} = 0$, $\pi_{i,j} = \pi_{i,j} = 1$.

Using the principle of scaling of linear equations, it is possible to take the value of one of the unknowns for some scale factor. In our case, it means that it is possible to take $x_2 = 1$; then, from the system of equations (5), we may find $x_1 = \mu_{2equ} / \mu_{1equ}$. As noted above, the average intensity of the generated requests for the source of requests, which is the second node, is defined as $\lambda = \mu_{2equ}$. Then, the unknown x_1 of system of equations (5) is determined by the following relation:

$$x_1 = \lambda / \mu_{1equ} \qquad (6).$$

In order to find a normalization constant, let us construct a matrix on the basis of the following two provisions. Firstly, let us set the value of all the elements of the first row of the matrix equal to the scale factor. Secondly, to calculate all the matrix elements except for the elements of the first row, let us set the following rule.

Rule 1

The rule of each elements formation of the matrix, which determines the normalization constant in an open network:

1. The value of each element of the *i* column of the matrix should be calculated as the sum of the previous element of the column multiplied by the unknown x_i of the column with the corresponding previous element of the line.
2. The number of rows must tend to infinity, since the source of the open network may infinite number of requests.

Then, in the case of two nodes, the matrix will be as follows:

$$\boldsymbol{G} = \begin{vmatrix} 1, & 1 \\ x_1 & 1+x_1 \\ x_1^{\ 2} & 1+x_1+x_1^{\ 2} \\ x_1^{\ 3} & 1+x_1+x_1^{\ 2}+x_1^{\ 3} \\ \dots & \dots \\ x_1^{\ \infty} & 1+x_1+x_1^{\ 2}+x_1^{\ 3}+x_1^{\ 4}+\dots+x_1^{\ \infty} \end{vmatrix}$$

The last element of the second row of the matrix *G* will be the constant of normalization, will contain the sum of all the possible products of the unknowns x_i . However, in our case, this a geometric progression with the denominator x_1. Then, the normalization constant is determine following equation:

$$G\,(N,M) = 1/(1 - x_1) \qquad (7).$$

It is known (Gordon and Newell 1967), that the probability of the vector of stationary states of the heterogeneous may be determined by the formula:

$$\mathrm{P}(n_1, n_2, ..., n_M) = G\,(N,M) \sum_{j=1}^{M} \mu_i / \mu \;\left(\prod_{i=1}^{N} \boldsymbol{X}_i^{\,n_i} \right) \qquad (8).$$

Given that, in our example, the network consists of two nodes, the probability of the n_1 requests in the first node may be determined by the formula:

$$P(n_1) = x_1^{\,n_1} \; / (1 - x_1) \qquad (9).$$

The resulting expression x_1 is nothing but the request load intensity of the node. In queue the load of the systems is denoted by the symbol ρ. Using these arguments, expression (9) may as follows:

$$P(n_1) = \rho_1^{\,n_1} \; / (1 - \rho_1) \qquad (10).$$

Note that the value found for the probability of the number of requests n_1 in the network fully corresponds to the value of this probability in a queuing system of the M/M/1/∞ class obtained by methods (Kleinrock, 1979). Using (10) to determine the average number of requests in the system and replacing the value of x_1 with the load value ρ_1, the following expression for determining the average number of requests in an open network consisting of two nodes may be obtained:

$$N_1 = \sum_{i=1}^{R} \lambda_i / \mu_i + [\sum_{i=1}^{R} [\lambda_i / \mu_i \;] / [(1 - \sum_{i=1}^{R} \lambda_i / \mu_i)] \qquad (11).$$

A similar expression to determine the average number of requests in a heterogeneous service is obtained in Boicov (2009). It is shown (Boicov, 2010) that, under certain conditions, the expressions for N_r and N in form equivalent. Therefore, it can be argued that the analysis of an open network with request performs above may be implemented through the following three actions:

(1) The finding of the unknown x_i of the system of equation of form (5).
(2) The finding of the normalization constant according to the above mentioned Rule 1.
(3) The calculation of the network's parameters by formulas (8), (10) and (11).

The study of an open network of equivalent models with heterogeneous requests

For the analysis of more complex networks, at first, let us try to apply the calculations of the parameters of the stochastic network from the previous section to a network consisting of three nodes and then move on to a network consisting of an arbitrary number of nodes. In order to obtain an open network, let us again load one of the nodes in the network on the additional node with a load close to the limit so that it works as a source of requests.

For a network consisting of three nodes, the system of algebraic equations with the unknown parameters x_i will be as follows:

$$\mu_{1equ} x_1 = \mu_{1equ} x_1 \pi_{1,1} + \mu_{2equ} x_2 \pi_{2,1} + \mu_{3equ} x_3 \pi_{3,1}$$

$$\mu_{2equ} x_2 = \mu_{1equ} x_1 \pi_{1,2} + \mu_{2equ} x_2 \pi_{2,2} + \mu_{3equ} x_3 \pi_{3,2} \qquad (12).$$

$$\mu_{3equ} x_3 = \mu_{1equ} x_1 \pi_{1,3} + \mu_{2equ} x_2 \pi_{2,3} + \mu_{3equ} x_3 \pi_{3,3}$$

Again, assuming $x_1 = 1$ and solving system of algebraic equations (12), x_2 and x_3 may be found depending on the network configuration $\pi_{i,j}$ and μ_{iequ} for all *i* and *j* from 1 to 3. Further, in accordance with the above proposed Rule 1, the following matrix *G* may be established:

$$G = \begin{vmatrix} 1, & 1, & 1 \\ x_2, & 1+x_2, & 1+x_2+x_3, \\ x_2^2, & 1+x_2+x_2^2, & 1+x_2+x_2^2+x_3+x_2x_3+x_3^2, \\ x_2^3, & 1+x_2+x_2^2+x_2^3, & 1+x_2+x_2^2+x_2^3+x_3+x_2x_3+x_2^2x_3+x_3^2+x_2x_3^2+x_3^3 \\ \dots & \dots & \dots \end{vmatrix}$$

Let us express via x_3 with the help x_2 of some factor **α:**

$$x_3 = \alpha\, x_2 \qquad (13).$$

Now, we substitute the resulting value x_3 in the matrix *G*.

And when the members with the same powers are groped in the third column, we find:

$$G = \begin{vmatrix} 1, & 1, & 1 \\ x_2, & 1+x_2, & 1+(1+\alpha)x_2, \\ x_2^2, & 1+x_2+x_2^2, & 1+(1+\alpha)x_2+(1+\alpha+\alpha^2)x_2^2, \\ x_2^3, & 1+x_2+x_2^2+x_2^3, & 1+(1+\alpha)x_2+(1+\alpha+\alpha^2)x_2^2+(1+\alpha+\alpha^2+\alpha^3)x_2^3, \\ \dots & \dots & \dots. \end{vmatrix}$$

Note that the coefficients of x_2 the last member of the third row form a geometric progression. Thus, the last member of the third row may be represented by the following expression:

$$G\,(N,3) = \sum_{j=0}^{N} (1-\alpha^{j+1})/(1-\alpha)\; x_2^{\,j}$$

Replacing αx_2, with x_3 which is possible according to expression (13), we find:

$$G\,(N,3) = 1/[(1-x_2)\,(1-x_3)] \qquad (14).$$

Substituting this value *G(N,* 3) into (8), we may find the value of the vector of the stationary probabilities of the states for a network consisting of three nodes $\mathrm{P}(n_1, n_2, n_3)$ and thereby calculate all the numerical characteristics of this open network.

Thus, we have obtained a method for calculating the characteristics of a stochastic open network consisting of one, two, and three nodes. Using the method of mathematical induction, the characteristics of an open stochastic network consisting of n, n+1 and any arbitrary number of nodes maybe similarly calculated.

Then, for a network consisting of *M* nodes, we frame the following rule for calculating the characteristics of this network.

Rule 2

1. For any open stochastic network consisting of M nodes, it is necessary to create a matrix $\pi_{i,j}$ (i, j = 1, 2,..., M) of transition probabilities of the requests between the network nodes,
2. To solve the system of equations $\mu_{jequ}x_j = \sum_{i=1}^{M} \mu_{iequ}x_i\pi_{i,j}$ for *(j* = 1, 2 …*M),*
3. To create a matrix according to **Rule 1** to calculate the normalization constant.
4. The number of rows of the matrix must tend to infinity, and it is also necessary to calculate the matrix element *G* (∞, *M)* and
5. To calculate the values of the stationary probabilities by the formula

$$\mathrm{P}(n_1, n_2, ..., n_M) = G(\infty, M) \sum_{j=1}^{M} \mu_i / \mu \left(\prod_{i=1}^{N} X_i^{n_i} \right) \qquad (15).$$

6. Using relation (14), there is a need to find the average values for $n_1, n_2, ..., n_M$

The resulting rule (Rule 2) is actually an algorithm for calculating the parameters of a stochastic open heterogeneous network with an arbitrary configuration.

Conclusion

Summarizing the data given above, the following results may be mentioned. Firstly, the given rule (Rule 1) is actually an algorithm for calculating the normalization constants of stochastic open networks. Secondly, the obtained method for calculating the normalization constants is applied to stochastic open heterogeneous networks, as a result of which the average parameter estimates (the average number of requests in the system, the average waiting time for service) in a single-channel queuing heterogeneous system are found. Thirdly, Rule 2 is actually also an algorithm for calculating the parameters o stochastic open heterogeneous networks with arbitrary configuration.

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