Statistical Analysis of Multiple Access Interference in Chaotic Spreading Sequence Based DS-CDMA Systems

A. Litvinenko, E. Bekeris

***Abstract*— This paper presents a statistical analysis of multiple access interference (MAI) in Direct Sequence Code Division Multiple Access (DS-CDMA) communication systems based on different types of chaotic spreading sequences. The probability distribution of the interference in a system with K users causing the MAI is studied using MATLAB simulation. For chaotic spreading sequence generation six different 1-D chaotic maps are used: modified Bernoulli, modified Tent, Gauss, Sine-Circle, Cubic and Pinchers map. A brief statistical analysis of the cross-correlation properties of the chaotic sequences generated by the aforementioned maps is also presented.**

***Index Terms*—1-D chaotic maps, chaotic spreading sequences, Direct Sequence Code Division Multiple Access (DS-CDMA) communication system, multiple-access interference (MAI).**

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I. Introduction

THE possibilities of exploiting chaos phenomena for the communication area have been intensively studied for the last 30 years. One promising research direction is the use of chaotic spreading sequence for Direct Sequence Code Division Multiple Access (DS-CDMA) systems [1]-[3] and one of approaches for the performance evaluation of these DS-CDMA multi-user systems is to apply the Gaussian approximation of the multiple-access interference (MAI) [2]-[4].

This research is an extension of the work devoted to the study of MAI distribution in DS-CDMA systems based on relatively short not-return-to-zero (NRZ) chaotic spreading sequences generated by a logistic map [5]. Publication [5] shows that MAI cannot be considered as Gaussian random variable if the chaotic spreading sequence generated by Logistic map is less than 60 chips. In turn, the proposed research presents the analysis of MAI distribution in DS-CDMA systems based on chaotic spreading sequences generated by other 1-D maps: modified Bernoulli map, modified Tent map, Gauss map, Sine-Circle map, Cubic map, and Pinchers map.

According to [6], it is possible to generate a higher number of relatively short chaotic sequences with a low cross-correlation level compared with classical pseudo-noise (PN) sequences, which are widely used in DS-CDMA systems. This paper also analyzes periodic cross-correlation properties of chaotic spreading sequences generated by 1-D maps and classical pseudo-noise sequences.

Moreover, chaotic spreading sequences could be used to increase the resistance to fading in wireless sensor networks [7], to increase the security level of these networks [8], and for synchronization challenge [9]. Therefore, it is important to choose the correct method for the performance evaluation of chaotic spreading sequence based DS-CDMA multi-user system and this research verifies the correctness of Gaussian approximation for MAI simulation in the proposed systems.

The paper is organized in the following manner: the next section presents algorithms for generation of binary chaotic spreading sequences, whose periodic cross-correlation properties are analyzed in the third section; the fourth section describes the block diagram of the simulated DS-CDMA system and the results of simulation are shown in the fifth section; finally, an overview of the obtained results is presented in conclusions.

II. Generation of Chaotic Spreading Sequences

In this section algorithms of binary chaotic sequence generation are presented. The first step for obtaining the binary chaotic spreading sequence $c(n)$ is generation of a non-binary chaotic sequence $x(n)$, and for this purpose six different 1-D chaotic maps have been chosen: Bernoulli (1), Tent (2), Gauss (3), Sine-Circle (4), Cubic (5), and Pinchers map (6). The parameters of these 1-D maps have been selected with the goal of obtaining a chaotic behavior for the sequences.

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A. Litvinenko is with the Department of Fundamentals of Electronics, Riga Technical University, Azenes St. 12, Riga, Latvia (e-mail: anna.litvinenko@rtu.lv).

E. Bekeris is with the Department of Fundamentals of Electronics, Riga Technical University, Azenes St. 12, Riga, Latvia (e-mail: elmars.bekeris@rtu.lv).

TABLE I
SPECIFIED THRESHOLD *TH* VALUE FOR CHAOTIC MAPS

Map type	Bernoulli	Tent	Gauss	Sine-Circle	Cubic	Pinchers
Th value	0.5	0.5	0.01	3.25	0	0.38

Bernoulli map (p = 1.999 and $x \in (0, 1)$):

$$x(n+1) = \mathrm{mod}(px(n),1)\,. \quad (1)$$

Tent map (μ = 1.999 and $x \in (0, 1)$):

$$x(n+1) = \begin{cases} \mu(1-x(n), & x(n) \geq 0.5 \\ \mu x(n), & x(n) < 0.5 \end{cases}. \quad (2)$$

Gauss map (α = 9.75, β = -0.53, and $x \in (-0.5, 0.5)$):

$$x(n+1) = e^{-\alpha x(n)^2} + \beta\,. \quad (3)$$

Sine-Circle map (H = 12, Ω = 0.2, and $x \in (0, 6.5)$):

$$x(n+1) = \mathrm{mod}(x(n) + \Omega - \frac{H}{2\pi}\sin(2\pi x(n)), 2\pi)\,. \quad (4)$$

Cubic map (B = 3 and $x \in (-1.1547, 1.1547)$):

$$x(n+1) = Bx(n)(1-x(n)^2)\,. \quad (5)$$

Pinchers map (s = 2, C=0.5, and $x \in (0, 0.76)$):

$$x(n+1) = \left|\tanh s(x(n)-C)\right|\,. \quad (6)$$

The generated non-binary chaotic sequence *x(n)* is converted into the binary NRZ chaotic sequence *c(n)* using the comparison rule (7), where a defined threshold *Th* is equal to the mean value of *x* definition interval. The value of the threshold *Th* for each observed map is presented in Table I.

$$c(n) = \begin{cases} 1, & x(n) \geq Th \\ -1, & x(n) < Th \end{cases} \quad (7)$$

According to [4], by dividing the state space of 1-D maps into two parts and using different initial conditions, completely different binary sequences are generated.

III. CROSS-CORRELATION PROPERTIES OF BINARY CHAOTIC SEQUENCES

In this section the distribution of maximum absolute values of periodic cross-correlation among binary chaotic sequences is studied.

To analyze cross-correlation properties, 100 binary chaotic sequences with the same length for four different length cases (N = 15, 31, 63, 127) and for the six previously discussed maps (1) – (6) have been generated. The distribution of the maximum absolute values of the periodic cross-correlation $|R_{lk}|$ is analysed for all possible pair combinations from 100 sequences of the same map and same length. Table II presents statistical parameters – minimal, maximal, mean values and standard deviation of the maximum absolute values of the periodic cross-correlation for chaotic sequences. These statistical parameters of chaotic sequences are compared with the corresponding maximum values of the periodic cross-correlation for pseudo-noise sequences. Obviously, it is possible to generate binary chaotic sequences with the same or better cross-correlation properties than in the case of classical pseudo-noise sequences for code lengths of 15–63 chips. In addition, the number of chaotic sequences with those lengths could be increased, what is impossible in the case with classical pseudo-noise sequences.

TABLE II
STATISTICAL PARAMETERS OF CROSS-CORRELATION

N	15	31	63	127
Bernoulli map				
Min	0.2	0.23	0.21	0.16
Mean	0.52	0.41	0.32	0.25
Max	1	0.74	0.59	0.45
Standard deviation	0.12	0.08	0.05	0.03
Tent map				
Min	0.2	0.23	0.17	0.16
Mean	0.53	0.41	0.32	0.25
Max	1	0.87	0.62	0.39
Standard deviation	0.12	0.07	0.05	0.03
Gauss map				
Min	0.2	0.16	0.17	0.16
Mean	0.54	0.43	0.34	0.25
Max	1	0.87	0.59	0.45
Standard deviation	0.12	0.08	0.06	0.04
Sine-Circle map				
Min	0.07	0.23	0.2	0.16
Mean	0.56	0.45	0.36	0.28
Max	1	0.8	0.62	0.48
Standard deviation	0.14	0.09	0.06	0.04
Cubic map				
Min	0.07	0.16	0.17	0.16
Mean	0.54	0.44	0.35	0.27
Max	1	0.87	0.62	0.54
Standard deviation	0.15	0.09	0.06	0.04
Pinchers map				
Min	0.07	0.16	0.17	0.19
Mean	0.67	0.55	0.45	0.35
Max	1	0.94	0.8	0.46
Standard deviation	0.17	0.13	0.09	0.05
Maximum values of $|R_{lk}|$ for PN sequences				
m-seq.	0.6	0.35	0.36	0.32
Gold	0.6	0.29	0.27	0.13
Kasami	0.33	0.215	0.14	0.09

IV. MODEL OF CHAOTIC SPREADING SEQUENCE BASED DS-CDMA SYSTEM

In this section simulation parameters for a model of DS-CDMA communication system based on chaotic spreading sequences are discussed.

For the evaluation of the probability distribution of the MAI in a DS-CDMA communication system based on chaotic spreading sequences, a model with K transmitters and one receiver is used and a block diagram of this mode is represented in Fig. 1. The information bit NRZ sequence *b(t)* is sent at the input of each transmitter. Then information bits are spread by chaotic NRZ sequence *c(t)*, which is unique for each transmitter. After this, the signal is transferred on carrier

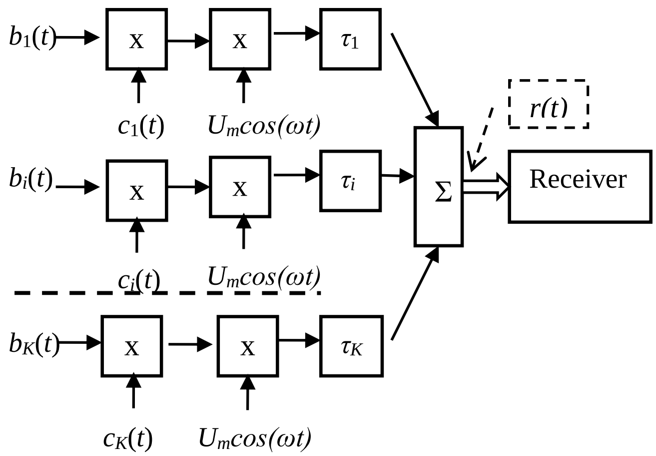

Fig. 1. Block diagram of DS-CDMA system [5].

frequency $U_m \cos(\omega_0 t)$. Each transmitter signal has a corresponding time delay τ observed by the receiver. Since the subject of this research is the probability distribution of the multiple-access interference, the channel noise during simulation is neglected.

The correlation receiver is matched to the $(K+1)$-th transmitter spreading sequence c_{K+1}. The input signal of the receiver $r(t)$ causes the multiple-access interference:

$$r(t) = \sum_{i=1}^{K} b_i(t) \cdot c_i(t) \cdot U_m \cos(\omega_0 t + \omega \tau_i). \quad (8)$$

The following parameters have been selected for simulation:

- Information bits are the random binary NRZ sequence $b_i(t)$ of duration $B_i T_b$, where T_b is the duration of one bit and B_i is the number of transmitted bit, which is randomly selected from the set (0,300) with an equal probability.
- Spreading sequence is binary NRZ chaotic sequence $c_i(t)$ generated on the basis of one of the 1-D maps (1) – (6) and comparison rule (7). The length of spreading sequence can have one of four different values: N = 15, 31, 63, 127.
- Initial values for chaotic spreading sequence generation are randomly selected from x definition interval with equal probability and correspondingly to the map type.
- Number of transmitters causing the multiple-access interference for the correlation receiver: K = 6, 18, 33, 65, 129, 259.

According to [5], at the time moments of jT_b the following values will be obtained at the output of the correlation receiver:

$$U_j = A \int_0^{jT_b} \left[\sum_{i=1}^{K} b_i(t) \cdot c_i(t) \cdot \cos(\omega \tau_i) \right] c_{K+1}(t) dt, \quad (9)$$

where A is a coefficient which depends on the parameters of correlation receiver elements.

TABLE III
RESULTS OF TESTS OF NORMALITY FOR PERFECT SYNCHRONIZATION CASE

K	Bernoulli map	Tent map	Gauss map	Sine-Circle map	Cubic map	Pinchers map	N
6	1	1	1	1	1	1	15
18	1	1	1	1	1	0	
33	1	1	1	0	1	1	
65	0	0	0	0	0	0	
129	0	0	0	1	0	0	
259	0	1	0	1	0	0	
6	1	1	1	1	1	1	31
18	1	0	1	0	1	1	
33	0	1	1	0	0	0	
65	0	1	0	0	0	0	
129	0	0	0	0	0	0	
259	0	0	0	0	0	0	
6	1	1	1	1	1	1	63
18	1	0	1	0	0	0	
33	0	1	0	1	1	1	
65	0	1	0	0	0	0	
129	1	0	1	0	0	0	
259	1	0	0	0	0	0	
6	1	1	1	1	1	1	127
18	1	0	0	0	1	0	
33	0	0	1	1	0	1	
65	0	0	0	0	0	1	
129	0	0	1	0	0	1	
259	0	0	0	0	0	0	

TABLE IV
RESULTS OF TESTS OF NORMALITY FOR ASYNCHRONOUS TRANSMISSION CASE

K	Bernoulli map	Tent map	Gauss map	Sine-Circle map	Cubic map	Pinchers map	N
6	1	1	1	1	1	1	15
18	0	0	1	0	0	1	
33	1	1	1	1	0	0	
65	0	0	0	0	0	0	
129	0	0	0	0	0	0	
259	0	0	1	0	0	0	
6	1	1	1	1	1	1	31
18	0	0	0	0	0	0	
33	1	0	0	0	0	0	
65	1	0	0	0	0	0	
129	0	0	0	0	0	0	
259	1	0	0	0	0	0	
6	1	1	1	1	1	1	63
18	0	0	0	0	0	0	
33	1	0	0	0	0	0	
65	0	0	0	0	0	0	
129	0	0	0	1	0	0	
259	0	0	0	1	0	0	
6	1	1	1	1	1	1	127
18	0	0	1	0	0	0	
33	0	0	0	0	0	0	
65	0	0	0	1	0	0	
129	0	1	0	0	0	0	
259	0	0	0	0	1	0	

V. THE RESULTS OF SIMULATION

This section describes the results of the MAI distribution analysis realized via MATLAB simulation of a DS-CDMA communication system based on chaotic spreading sequences

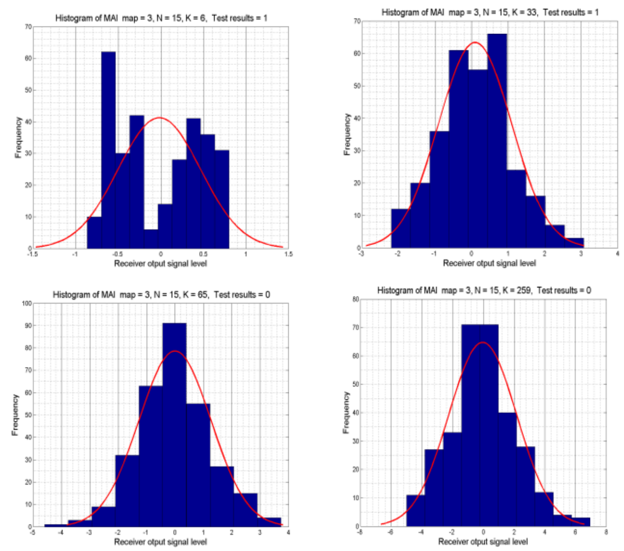

Fig. 2. Histograms of the MAI for a synchronous DS-CDMA system based on chaotic sequences with length 15 chips generated by Cubic map. The number of transmitters is respectively 6, 33, 65, 259 and the results of the test of normality for upper histograms are 1 (the number of transmitters 6, 33), and for bottom histograms are 0 (the number of transmitters 65, 259). A normal distribution curve for the corresponding mean value and variance is shown by the red line.

with K transmitters and a correlation receiver. The main purpose of the simulation is to test the hypothesis that the samples of the MAI calculated according to (9) come from a normally distributed population.

Two types of DS-CDMA system model have been examined during simulation: synchronous and asynchronous. In the case of the synchronous model, all time delays τ_i are equal. In the case of the asynchronous system, the time delays observed by the receiver are different.

Considering that the expected value and variance of the distribution are not previously specified, the Lilliefors test has been used for MAI distribution analysis. The null hypothesis of the test is as follows: MAI samples come from a normally distributed population. The results of the test for the synchronous system model are presented in Table III, and the results for the asynchronous system model are presented in Table IV. For the asynchronous system simulation the delay impact $\omega\tau_i$ is randomly selected from the set $(0, 2\pi)$.

If the result is 0, it means that the hypothesis of normality cannot be excluded. If the result is 1, it means that the hypothesis of normality can be rejected with the significance level 0.05.

Examples of histograms of the MAI and the expected normal distribution curves for 15-chips long chaotic spreading sequences generated by Cubic map and synchronous system case are presented in Fig. 2. The first upper histogram presents the MAI distribution for 6 users and the second upper histogram presents the MAI distribution for 33 users. As the Lilliefors test shows, the hypothesis of MAI normality for these cases can be rejected with the significance level 0.05. In turn, the lower histograms present the MAI distribution for 65 and 259 users. In these cases, according to the Lilliefors test, the hypothesis of MAI normality cannot be rejected. The red curves show the normal distribution for simulated MAI mean and variance values.

VI. Conclusion

The proposed research presents the multi-access interference (MAI) distribution analysis realized by MATLAB simulation of a synchronous and an asynchronous DS-CDMA communication system with K transmitters and a correlation receiver for relatively short chaotic spreading sequences. The MAI analysis has indicated overall similar results for the same sequences length, the number of transmitters and CDMA system type for the all observed 1-D maps.

Two main conclusions can be presented. For relatively short (6 - 127 chips long) binary chaotic spreading sequence, in more than half of the cases the MAI does not come from a normal distribution for the synchronous multi-user DS-CDMA communication system with the number of users less than 65. In the case of the asynchronous multi-user DS-CDMA communication system for all observed maps the MAI does not come from a normal distribution if the number of users is less than or equal to 6.

The approach for DS-CDMA multi-user system performance evaluation based on a Gaussian approximation of the MAI cannot be applied in the most of the observed cases.

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