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Accuracy analysis of vibration-based identification of elastic parameters

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Abstract. Proposed is the method for the analysis of parameter uncertainty effects on the accuracy of vibration-based elastic parameter identification. The method uses the approach of minimization of the discrepancy between physically measured and numerically calculated natural frequencies. The Monte Carlo simulation is used for obtaining correlations between frequency deviations. These correlation coefficients and deviations of physical frequency measurements are used for confidence interval calculations for identified parameters. The method gives the possibility of optimal choice of frequency sets and weighting coefficients for identification of each identifiable parameter.

Keywords: *identification of mechanical properties, vibration, metamodeling, composites.*

1. Introduction

The method of identification of elastic parameters (Young's moduli, shear moduli, Poisson's ratio) using eigenfrequency measurements of specimens is very old [4]. Currently there is an extensive amount of literature on the identification of elastic properties of layered composite materials using physical measurements and numerical calculations of natural frequencies, mostly using Finite Element Method (FEM) [5], [6], [7], [8], [9]. The traditional numerical-experimental identification procedure is based on the minimization of discrepancy between numerical and experimental results [5], [7], [8]. During the first years of using this method, the main problem was the minimization of the discrepancy functional. Today, using modern numerical experimental designs and nonparametric approximation methods, the discrepancy minimization is not a great problem. However, the estimation of the variance of obtained identified parameters is a pressing problem. The errors of identification depend on errors introduced by material production, cutting testing specimens, physical measurement errors and errors caused by disregarding significant factors in the finite element model. A large amount of literature is devoted to analysis of the accuracy of FEM, but the influence of errors of physical experiments, caused by parameter variance during material production, specimen preparation and errors of registration and measurement of natural frequencies, is significantly less studied.
Here we propose a method of variance calculation using Monte Carlo simulation of geometrical and physical uncertainty, but at the beginning the main idea of vibration-based identification [5], [7], [8] will be explained.

2. Vibration-based identification of elastic parmeters

Identifiable elastic parameters usually are Young's moduli, shear moduli, Poisson's ratio. The classical idea is to find those values of elastic parameters of the mathematical FEM model which will give minimal discrepancy between calculated and physically measured eigenfrequencies.

We will designate the vector-column of n physically measured natural frequencies as $\boldsymbol{f}^{EXP}$

$$\boldsymbol{f}^{\mathrm{EXP}} = \left[f_1^{\mathrm{EXP}}, f_2^{\mathrm{EXP}}, \ldots, f_n^{\mathrm{EXP}}\right]^{\mathrm{T}}, \tag{1}$$

vector-column of numerically calculated (mostly by FEM) eigenfrequencies as $\boldsymbol{f}^{\mathrm{FEM}}$

$$\boldsymbol{f}^{\mathrm{FEM}} = \left[f_1^{\mathrm{FEM}}, f_2^{\mathrm{FEM}}, \ldots, f_n^{\mathrm{FEM}}\right]^{\mathrm{T}}, \tag{2}$$

and vector-column of m physically identifiable parameters of elasticity as vector-column $\boldsymbol{E}$

$$\boldsymbol{E} = [E_1, E_2, \ldots, E_m]^{\mathrm{T}}. \tag{3}$$

A superscript T denotes the matrix transpose operation. The number m of identified parameters can be different, including elastic modulus and Poisson's ratio for different composite layers. The discrepancy Φ between measured and calculated natural frequencies is measured as weighted sum of n differences

$$\Phi = \sum_{i=1}^{n} w_i \left| f_i^{\mathrm{EXP}} - f_i^{\mathrm{FEM}} \right|^l, \tag{4}$$

where w_i – nonnegative weighting coefficient for i-th frequency, l – positive exponent.
The frequently used weighting method for discrepancy measure is squared relative error

$$\Phi = \sum_{i=1}^{n} \left(\frac{f_i^{\mathrm{EXP}} - f_i^{\mathrm{FEM}}}{f_i^{\mathrm{EXP}}} \right)^2. \tag{5}$$

The discrepancy minimization approach means that input parameters which give minimal value of functional Φ will be considered as identified values for unknown parameters $\boldsymbol{E}$:

$$\boldsymbol{E}^* = \arg\min_{E} \Phi(E). \tag{6}$$

When using FEM software, the minimization needs physical experiments as well as numerical experiments; therefore this approach is called Mixed Numerical-Experimental Technique (MNET). The flowchart of this method is shown in Fig. 2.
The traditional MNET steps are:
Step 1. Preparation of specimen samples, providing frequency measurements by resonance measurements or Fourier analysis of free oscillations registered after initial excitation.
Step 2. Design of numerical experiments for FEM software. The variable input factors for eigenfrequency calculations are identifiable elastic parameters. Mostly the Latin Hypercube (LH) type designs are used. Here we use LHs optimized according to Mean Square Error space-filling criterion, introduced in [1]. The values for other input parameters (geometrical, mass, density, layer configuration and others must correspond with specimens used in physical experiments.
The number of runs for numerical experiments depends on the number m of the identifiable parameters. For relatively simple plate-type specimens the calculations are fast enough to execute $100 - 300$ trial runs in 15 minutes of computing time.
Step 3. Executing the numerical experiments, registering and grouping the eigenfrequencies according to vibration modes.
Step 4. Building the metamodel (surrogate model) for the dependency of calculated eigenfrequencies $\boldsymbol{f}^{FEM}$ on the input parameters $\boldsymbol{E}$.

$$\boldsymbol{f}^{\mathrm{FEM}} = \hat{f}(\boldsymbol{E}), \tag{7}$$

where the "hat" above a function symbol means approximation. The software **EDAOpt** [1], created at Institute of Mechanics of Riga Technical University, was used for design of computer experiments, metamodel building and minimization of discrepancy functional.
Practice shows that almost in all cases third order polynomials give the approximation of frequencies with error less than 0.01%. For some more complicated specimens the best accuracy can be obtained using nonparametric approximation methods: kriging, locally weighted polynomials.
Step 5. Finding the values of identifiable parameters by minimization of approximated discrepancy functional

$$\boldsymbol{E}^* = \arg\min_{\boldsymbol{E}} \sum_{i=1}^{n} w_i \left| \hat{f}_i(\boldsymbol{E}) - f_i^{\mathrm{FEM}} \right|^l. \tag{8}$$

The software **EDAOpt** uses modified multi-start simulated annealing method [1] and always gives global minimums of the discrepancy functional. It must be noted that the metamodels are relatively simple for evaluation of the objective function (8) and even several millions of evaluations needs only a few minutes of computing time.
Step 6. Traditionally, the next step is the recalculation of the metamodel in the sub-area near to the found values of identified parameter values, and analysis of the significance of different elasticity parameters on natural frequencies [5], [8]. However, this analysis gives insufficient information for the estimation of accuracy of identified values. Therefore in the present work the method for accuracy estimation will be proposed.

3. Estimation of the identification errors

The errors of identification depend on errors introduced by material production, cutting of testing specimens, physical measurement errors and errors of the finite element model. Practice shows that the mode measurements using the $PSV-400$ vibrometer have very high accuracy. The repeated measurements for a given specimen give almost the same results. At the same time the measurements of $3-6$ different specimens from the experimental sample give the estimate standard deviations from the mean from about 0.5 % up to 2 %. This means that the variance of parameters introduced by production (elasticity, density, thickness uniformity, etc.) and errors introduced by the preparation of the sample (geometrical errors, errors of density and weight estimation, microdamages created by sample cutting) have determining influence on the variance of identified elastic parameters.
Lot Many works have been devoted to the analysis of impact of elastic parameters on eigenfrequencies using derivative matrices [3], [5], [8]. In [5] the sensitivity of Poisson's ratio determination on the aspect ratio of a rectangular plate is studied. But it hasn't been taken into account that all frequencies are highly correlated, have different measurement errors and significance for parameter estimation.
Below is described the use of Monte Carlo simulation for estimation of identification errors.
After finding numerical values of identified parameters $\boldsymbol{E}^*$ using the discrepancy minimization method, the program **EDAOpt** gives the possibility of linearization of the metamodel (obtained with polynomial or kriging approximations) in the neighborhood of point $\boldsymbol{E}^*$ in the m-dimensional parameter space, obtaining the linear metamodel:

$$\hat{\boldsymbol{f}}(\boldsymbol{E}) = \boldsymbol{A}\boldsymbol{E} + \boldsymbol{B}, \tag{9}$$

where $\boldsymbol{A}$ – constant matrix $n \times m$, $\boldsymbol{B}$ – constant vector-column with n components.
Weighted discrepancy functional between calculated frequencies $\hat{f}$ and measured frequencies $\boldsymbol{f}^{EXP}$ in the matrix form (exponent $l = 2$):

$$\Phi = \left(\boldsymbol{AE} + \boldsymbol{B} - \boldsymbol{f}^{\mathrm{EXP}}\right)^{\mathrm{T}} \boldsymbol{W} \left(\boldsymbol{AE} + \boldsymbol{B} - \boldsymbol{f}^{\mathrm{EXP}}\right), \tag{10}$$

where $\boldsymbol{W}$ is the weighting matrix, for example, squared inverse of measured frequencies:

$$\boldsymbol{W} = \mathrm{diag}\left(\left(\frac{1}{f_1^{\mathrm{EXP}}}\right)^2, \left(\frac{1}{f_2^{\mathrm{EXP}}}\right)^2, \ldots, \left(\frac{1}{f_n^{\mathrm{EXP}}}\right)^2\right). \tag{11}$$

The discrepancy minimization approach gives following:

$$\boldsymbol{E}^* = \arg\min_{\boldsymbol{E}} \Phi(\boldsymbol{E}) \tag{12}$$

and after equating derivatives $\partial\Phi/\partial E_i$ to zero, we obtain the system of linear algebraic equations

$$\boldsymbol{A}^{\mathrm{T}} \boldsymbol{W} \boldsymbol{A} \boldsymbol{E}^* + \boldsymbol{A}^{\mathrm{T}} \boldsymbol{W} \left(\boldsymbol{B} - \boldsymbol{f}^{\mathrm{EXP}}\right) = 0 \tag{13}$$

The values of parameters which give minimal discrepancy are therefore vector-column $\boldsymbol{E}^*$

$$\boldsymbol{E}^* = \boldsymbol{S}\boldsymbol{f} - \boldsymbol{H}, \tag{14}$$

where

$$\boldsymbol{S} = \left(\boldsymbol{A}^{\mathrm{T}} \boldsymbol{W} \boldsymbol{A}\right)^{-1} \boldsymbol{A}^{\mathrm{T}} \boldsymbol{W} \tag{15}$$

and constant vector-column

$$\boldsymbol{H} = \left(\boldsymbol{A}^{\mathrm{T}} \boldsymbol{W} \boldsymbol{A}\right)^{-1} \boldsymbol{A}^{\mathrm{T}} \boldsymbol{W} \left(\boldsymbol{B} - \boldsymbol{f}^{\mathrm{EXP}}\right). \tag{16}$$

From the physical experiments with sample size k (number of specimens in the set of measurements), the mean $\bar{f}_i$ and standard deviation of i-th frequency can be calculated [2] as

$$\bar{f}_i = \frac{1}{k} \sum_{j=1}^{k} f_i^{\mathrm{EXP}(j)}, \tag{17}$$

$$\sigma_i = \sqrt{\frac{\sum_{j=1}^{k} \left(\bar{f}_i - f_i^{\mathrm{EXP}(j)}\right)^2}{k-1}}. \tag{18}$$

As usual, we can assume that measured means are random variables and have normal probability distribution. Then the standard deviations of frequency measurements (18) can be used for the calculation of the standard deviation using the linear dependence between elasticity parameters and natural frequency (14). The problem is that measured frequencies may not be considered as independent random variables but are highly correlated with each other.
The Pearson correlation coefficients can be calculated using the set of measured frequency values, but the number of measurements usually is too small to calculate the correlation matrix between the $15 - 20$ frequencies used in identification.
Therefore we propose the following Monte Carlo simulation approach for the calculation of frequency correlation coefficients. The scattering of natural frequency values depends mainly on geometrical errors of the specimen and actual variance of elastic parameters and

density of material. For the rectangular plate type specimen we used the Monte Carlo method for the simulation of the influence of following geometric parameter variance:
6 error parameters of specimen cutting (plate type specimens are not exactly rectangular) see Fig. 1a, and 4 parameters describing the variation of plate thickness, see Fig. 1b:

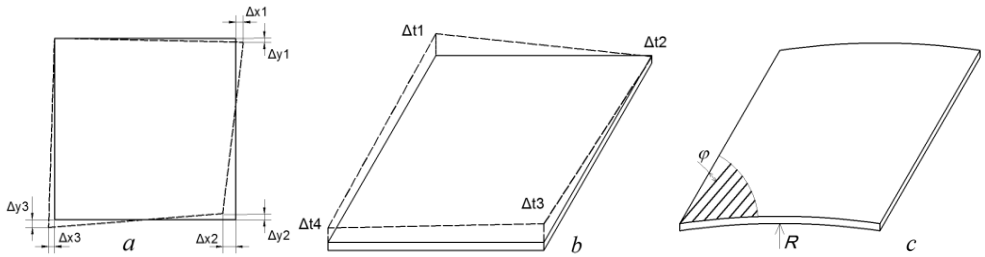

Fig. 1. The parameters of geometrical deviation.

parameter φ describing the angle between fiber orientation and edge of rectangle, parameter R - surface curvature of the plate, see Fig. 1c.

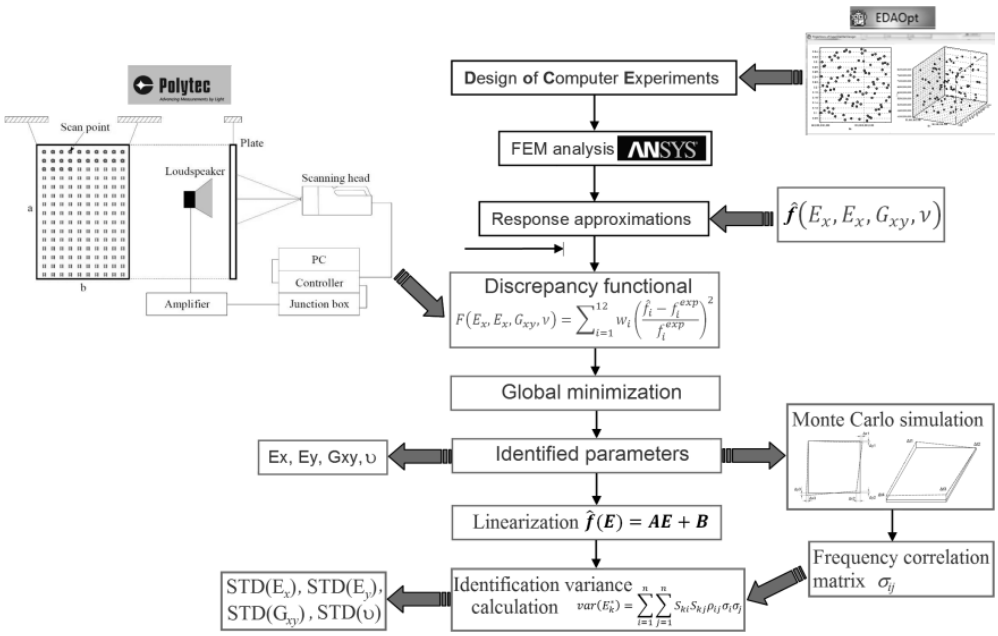

Fig. 2. Flowchart of parameter identification with variance estimation.

In addition to geometrical deviations of parameters 4 values of standard deviation for elastic parameters ($E_x, E_y, G_{xy}, \upsilon$) were used for Monte Carlo simulation. All these parameters were simulated as random variables with normal density distribution and given standard deviations. The mean values for first 13 error parameters are zero, the mean values of elastic parameters and their deviations are taken from previous experiments, but can be iteratively repeated after identification.

The Monte Carlo simulation consists of $N = 2000$ calculations of eigenfrequencies using randomly generated 17 input variables. The results are N values for each of first m frequencies $f_{ij},\ i = 1, \dots, N, j = 1, \dots, m$. The Pearson product-moment correlation coefficient between i-th and j-th frequency is

$$\rho_{ij} = \frac{\sum_{k=1}^{N}(f_{ki} - \bar{f}_i)(f_{kj} - \bar{f}_j)}{\sqrt{\sum_{k=1}^{N}(f_{ki} - \bar{f}_i)^2}\sqrt{\sum_{k=1}^{N}(f_{kj} - \bar{f}_j)^2}}. \tag{19}$$

Then, according to expression (14) the standard deviation of k-th identifiable parameter can be calculated according to statistical law for correlated random variables [2].

$$STD(E_k^*) = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} S_{ki}S_{kj}\rho_{ij}\sigma_i\sigma_j}\ ,\quad k = 1, \dots, m, \tag{20}$$

where $S_{kj}\ (k = 1, \dots, m, j = 1, \dots, n)$ are elements of matrix $\boldsymbol{S}$, see (15), ρ_{ij} is the correlation coefficient between i-th and j-th natural frequency and ρ_i is the sample standard deviation of i-th measured frequency $(i, j = 1, \dots, n)$. The obtained values of standard deviations can be used for calculating confidence intervals of identified parameters.

4. Example of identification results

The proposed method was used for the identification of elastic properties of weft knitted GF/PP textile composite [10]. In our experiments with knitted GF/PP textile composite plates the $PSV - 400$ vibrometer was employed for non-contact measurement, visualization and analysis of structural vibrations. The flowchart of identification algorithm with accuracy estimation is shown in Fig. 2. The mean relative standard deviation of physical frequency measurements was about $0.5 - 2\%$. The values of correlation coefficient for first 10 frequencies vary from 0.07 (between first and second mode) up to 0.95 (between 3-th and 8-th mode).

The obtained values for E_x, E_y, G_{xy} have about $2 - 3$ % error with confidence level 95 %. At the same time the error of obtained Poisson's ratio is about $12 - 15$ %. The choice of different weighting coefficients in the discrepancy function (4) is very important for the accuracy improvement. For example, using frequencies $2, 3, 5, 6, 7, 8$ with weighting exponent $l = 2$ and $w_i = \left(f_i^{\mathrm{EXP}}\right)^{-2}$ gives 20.4 % error for Poisson's ratio (with traditional two sigma 95 % confidence level), but the same set of frequencies with weighting coefficients $w_i = \left(f_i^{\mathrm{EXP}}\right)^{-1}$ gives 15 % error for υ.

Conclusions

The presented uncertainty analysis algorithm estimates the uncertainty of the identified elastic parameters from the uncertainties of the experimentally measured quantities and uncertainties of geometrical and physical parameters of experimental specimens. This method gives the possibility of estimation of confidence intervals of determined parameters with a given confidence level. The method gives the possibility of optimal

choice of natural frequency set and weighting coefficients obtaining the best identification accuracy for each identifiable parameter separately.

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