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Editors:

Trond Kvamsdal
Department of Mathematical Sciences
Norwegian University of Science and Technology (NTNU),
Trondheim, Norway

Kjell Magne Mathisen
Department of Structural Engineering
Norwegian University of Science and Technology (NTNU),
Trondheim, Norway

Bjørnar Pettersen
Department of Marine Technology
Norwegian University of Science and Technology (NTNU),
Trondheim, Norway

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COMPUTATIONAL MODELLING OF ADAPTIVE COMPOSITE ELEMENTS

JANIS BRAUNS[*] AND KARLIS ROCENS[†]

[*]Department of Structural Engineering
Latvia University of Agriculture
19 Academy Street, Jelgava, Latvia
e-mail: Janis.Brauns@llu.lv

[†]Institute of Building and Reconstruction
Riga Technical University
16 Azenes Street, Riga, Latvia
e-mail: Rocensk@latnet.lv

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Summary. Numerical analysis is performed to investigate deformation behavior and strength of in-plane balanced antisymmetric laminates under tension caused by centrifugal load. It is determined that antisymmetric orientation of external layers of in-plane balanced laminate can be used to ensure the necessary adaptive warping and strength of the laminate under action of axial load.

1 INTRODUCTION

Laminates can be designed to provide the desired strength and stiffness characteristics required for specific applications. The material anisotropy can be exploited to induce coupling between deformation modes. The use of fibre reinforced composite rotor blades enables a number of possible passive aerodynamic control options[1]. By using adaptive wind rotor blades with twist coupling there is possibility to keep good, steady power-production and smooth out unwanted peaks in loading. The blades may be made wholly or partially from carbon fibre, which is a lighter, but costlier material with high strength[2]. In this study, a numerical analysis is performed to investigate deformation behavior and strength of in-plane balanced antisymmetric laminates under tension caused by centrifugal load.

2 STRESS AND STRAIN RELATIONSHIPS OF LAYERED STRUCTURE

Within a layered composite of thickness h, deformation of one layer is constrained by the other ones of different orientations. The stresses in the elementary layers are different and the stress state of the composite is inhomogeneous. By using a static equivalent system of average force stresses σ_j and moment stresses μ_j acting on a composite material, the constitutive relations for the mid-plane strains ε_i^0 and the curvatures κ_i in matrix notations are given by[3]

$$\left[\frac{\varepsilon^0}{\kappa}\right]=\left[\begin{array}{c|c}\alpha & \beta\\ \hline \beta^T & \delta\end{array}\right]\cdot\left[\frac{\sigma}{\mu}\right] \tag{1}$$

where α, β, δ are compliances of layered composite; superscript T denotes transposition operation. The force stresses and moment stresses in layered composite are calculated by averaging. The stresses in the elementary layers (Fig. 1) in the coordinates of composite $\{x_i\}$ can be determined by using the layer stiffness A'_{ij} in the local coordinate system $\{x'_i\}$ and stress transformation matrix[4]. The compliance matrices in equation (1) are represented in terms of composite stiffness characteristics determined by integration.

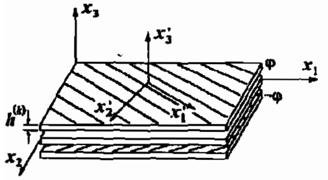

Figure 1: Multilayer model of composite structure.

3 STRENGTH ANALYSIS OF DESIGNED LAMINATE

The failure criterion for orthotropic materials under plane stress state is expressed as[5]

$$\begin{aligned} f(\sigma_{ij}) &= F_{11}\sigma_{11} + F_{22}\sigma_{22} + 2F_{12}\sigma_{12} + F_{1111}\sigma_{11}^2 + F_{2222}\sigma_{22}^2 + \\ &+ 4F_{1212}\sigma_{12}^2 + +2F_{1122}\sigma_{11}\sigma_{22} + 4F_{1112}\sigma_{11}\sigma_{12} + 4F_{2212}\sigma_{22}\sigma_{12} = 1. \end{aligned} \tag{2}$$

The coefficients in strength function (2) are components of tensors F_{ij} and F_{ijkl}. They are determined by means of the strength tensor $R_{\alpha\beta\gamma}$ determined experimentally. The mentioned criterion defines an envelope in stress space, i.e. if the stress state lies outside of this envelope then failure is predicted.

4 RESULTS AND ANALYSIS

Composite laminates for rotor blades incorporate a combination of unidirectional plies to support radial centrifugal loads and provide sufficient bending stiffness, and 45° plies to restrict shear and torsion. Using graphical procedure[6] and representing the needed characteristic as a function of undetermined external layer orientation angle φ the optimisation problem is solved. The procedure is suitable for in-plane balanced angle-ply laminates made up of stacks of layers with different orientation angles ±φ.

The elastic characteristics of the laminate can be determined by using lamination parameters that contain the relevant information associated with the stacking sequence and they were determined by integration according expression (3) and (4)

$$V_{1\{A,B,D\}} = \int_{-h/2}^{h/2} \cos 2\varphi \{1, x_3, x_3^2\} dx_3, \tag{3}$$

$$V_{3\{A,B,D\}} = \int_{-h/2}^{h/2} \cos 4\varphi \{1, x_3, x_3^2\} dx_3. \tag{4}$$

The lamination parameters and material invariants[7] were used for determination of stiffness characteristics A_{ij}, B_{ij} and D_{ij}[4]. By using lamination parameter diagram (Fig. 2) the region of allowable combinations of lamination parameters is determined. For a laminate of total thickness h, where the volume fraction of layers $\pm\varphi_i$ is v_i, normalized lamination parameters are given as

$$\bar{V}_1 = \frac{V_{1A}}{h} = \sum_{i=1}^{N} v_i \cos 2\varphi_i\ ; \quad \bar{V}_3 = \frac{V_{3A}}{h} = \sum_{i=1}^{N} v_i \cos 4\varphi_i\ . \tag{5}$$

The points A, B and C correspond to laminates with 0°, ±45, and 90° orientation angles, respectively. Any point inside the boundary line corresponds to laminates with two or more fiber orientation. The analysed laminates consist of 9 layers that form in-plane balanced antisymmetric system with stacking sequence [φ, 45, –45, 90, 0, 90, –45, 45, –φ]. In the lamination parameter diagram (Fig. 2) point L corresponds to the orientation of external layers φ = 0° but point N to φ = 90°. Point M belongs to the laminate configuration with φ = ±25° when laminate indicate the maximum warping under action of axial load.

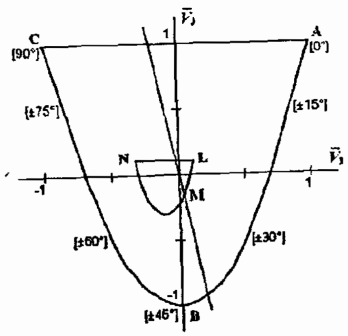

Figure 2: Lamination parameter diagrams: ABC – defined stacks of layers; LMN – analysed laminates.

In the Fig. 3 the strength function determined according to (2) and curvature of the laminate determined by using relationship (1) is shown for carbon epoxy laminate. The warping of wooden laminate is more in comparison with carbon epoxy laminate, but in both cases there are allowable intervals, where curvature changes from zero until maximum, but stress level is allowable.

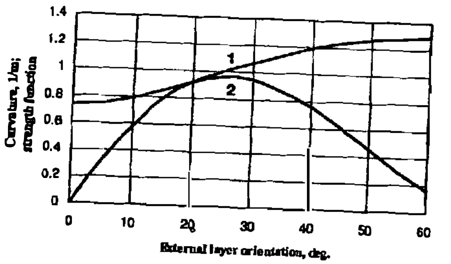

Figure 3: Strength function (1) and curvature (2) vs. external layer orientation for carbon epoxy laminate.

5 CONCLUSIONS

- Antisymmetric orientation of external layers of in-plane balanced laminate can be used to ensure the necessary adaptive warping and strength of the laminate under action of axial load.
- The use of stretching-twisting coupling can be applied to provide a control mechanism which does not have any parts moving relative to each other, and which is therefore maintenance-free.

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