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CONVECTIVE STABILITY OF A CHEMICALLY REACTING FLUID IN AN ANNULUS

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Abstract. Linear stability of a steady convective motion of a viscous incompressible fluid generated by internal heat sources in a tall vertical annulus is investigated. The heat sources are distributed within the fluid in accordance with the Arrhenius law. The problem for the determination of base flow in this case is nonlinear. The base flow velocity and temperature distribution is obtained numerically using Matlab. Linear stability of the base flow is investigated with respect to asymmetric perturbations. The stability boundary depends on the two parameters: the Prandtl number and the Frank-Kamenetskii parameter. It is shown that even for small Prandtl numbers (in contrast with the case of uniformly distributed heat sources) marginal stability curves consist of two separate brances. Calculations show that the base flow is destabilized as both parameters (the Prandtl number and the Frank-Kamenetskii parameter) increase.

1 INTRODUCTION

Analysis of processes of combustion and heat generation is aimed to develop a cleaner and more efficient energy production using different types of biomass. In order to enhance biomass thermo-chemical conversion different technical solutions are proposed [1]. Experimental investigation of the processes of biomass thermo chemical conversion is performed in [2]. Intensification of combustion processes can be achieved as a result of hydrodynamic instabilities. Linear stability of a steady convective motion between two parallel planes due to heat sources uniformly distributed within viscous incompressible fluid is analyzed in [3], [4]. The case of uniform heat sources distributed in an annulus is considered in [5]. It is shown in [3]–[5] that for small Prandtl numbers instability is associated with the shear mode due to two fluid streams moving in opposite directions while for large Prandtl numbers thermal perturbations are more important and instability occurs in the form of thermal running waves moving along the channel with large wave speed. In addition, calculations in [5] showed that asymmetric mode is the most unstable for large gaps while only axisymmetric perturbations lead to instability for small gaps.

Optimization of the processes of biomass thermo-chemical conversion should be performed under the assumption that internal heat sources are generated in the fluid as a result of exothermic chemical reaction. In the present paper we analyze one of the aspects of the problem - linear stability of a convective motion in a tall vertical annulus. Linear stability problem is solved numerically for different values of the parameters of the problem.

2 FORMULATION OF THE PROBLEM

Consider a tall vertical annulus with the inner and outer radii R_1 and R_2, respectively, filled with a viscous incompressible fluid. The annulus is closed so that the total fluid flux through the cross-section of the annulus is equal to zero. The internal heat sources are distributed within the annulus in accordance with the Arrhenius law [6]:

$$\tilde{Q} = Q_0 k_0 \exp\left[-E/(R\tilde{T}\right], \tag{1}$$

where $\tilde{T}$ is the absolute temperature, R is the universal gas constant, Q_0 and k_0 are given constants and E is the activation energy.The system of the Navier-Stokes equations under the Boussinesq approximation has the form

$$\begin{aligned} \frac{\partial \mathbf{v}}{\partial t} + Gr(\mathbf{v}\nabla)\mathbf{v} &= \nabla p + \Delta \mathbf{v} + T\mathbf{k}, & (2)\\ \frac{\partial T}{\partial t} + Gr\mathbf{v}\nabla T &= \frac{1}{Pr}\Delta T + \frac{F}{Pr}\exp\left[T/(1+bT)\right], & (3)\\ \nabla \mathbf{v} &= 0. & (4) \end{aligned}$$

Here $\mathbf{v}$, T and p are dimensionless velocity vector, temperature and pressure. The following dimensionless quantities are chosen as measures, respectively, of length, R_2, time, R_2^2/ν,velocity, $g\beta R_2^2 RT_0^2/(\nu E)$, temperature,$RT_0^2/E$, and pressure, $\rho g\beta R_2 RT_0^2/E$. Here ρ is the density, β is the coefficient of thermal expansion, and ν is the kinematic viscosity of the fluid. There are four dimensionless parameters characterizing the problem: the Grasshof number $Gr = g\beta RT_0^2 R_2^3/(\nu^2 E)$, the Prandtl number, $Pr = \nu/\chi$, the Frank-Kamenetsky number $F = [Q_0 k_0 E R_2^2/(\kappa RT_0^2)]\exp\left[-E/(RT_0)\right]$, and the parameter $b = RT_0/E$.

Consider a system of cylindrical polar coordinates (r, φ, z) centered at the point O on the axes of the cylinders. The velocity components are denoted as follows: $\mathbf{v} = (v_r, v_\varphi, v_z)$. There exists a steady solution of (2)–(4) of the following form:

$$\mathbf{v} = (0, 0, W_0(r)), \quad T = T_0(r), \quad p = Cz + const. \tag{5}$$

Substituting (5) into (2)–(4) we obtain the nonlinear system of ordinary differential equations

$$\frac{d^2 W_0}{dr^2} + \frac{1}{r}\frac{dW_0}{dr} + T_0 + C = 0, \tag{6}$$

$$\frac{d^2 T_0}{dr^2} + \frac{1}{r}\frac{dT_0}{dr} + \frac{F}{Pr}\exp\left[T_0/(1+bT_0)\right] = 0. \tag{7}$$

The boundary conditions have the form

$$W_0|_{r=1} = 0, \quad W_0|_{r=R_0} = 0 \quad T_0|_{r=1} = 0, \quad T_0|_{r=R_0} = 0, \tag{8}$$

where $R_0 = R_1/R_2$. Since the annulus is closed, the total fluid flux through the cross-section of the annulus is equal to zero:

$$\int_{R_0}^{1} r W_0(r)\, dr = 0. \tag{9}$$

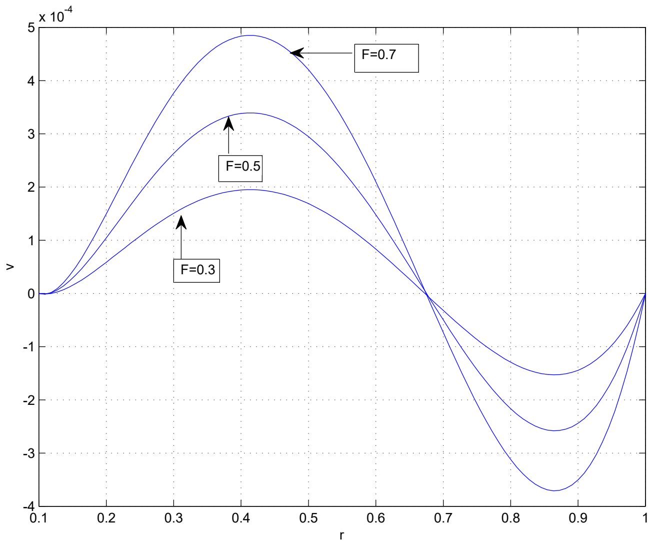

Figure 1: The base flow velocity distribution for three values of the Frank-Kamenetskii parameter F.

3 CALCULATION OF BASE FLOW

Nonlinear problem (6)–(9) is solved numerically using Matlab routune bvp4c. The value of the parameter b for all calculations below is set at 0 since it is very small in applications. First, the temperature distribution can be obtained solving (7) together with the two boundary conditiions (8) for the temperature T_0. Then equation (6) together with the boundary conditions (8) for the function W_0 is solved where the constant C is chosen to satisfy (9). It is shown in [6] that depending on the value of the paramemer b problem (6)-(9) can have several solutions. Only the solution with smallest values of the temperature T_0 is used in our calculations. The ratio of the radii of the cylinders is fixed at $R_0 = 0.1$ for all calculations (stability analysis in [5] showed that asymmetric perturbations are the most unstable for $0 < R_0 < 0.28$). The base flow velocity distribution is shown in Figure

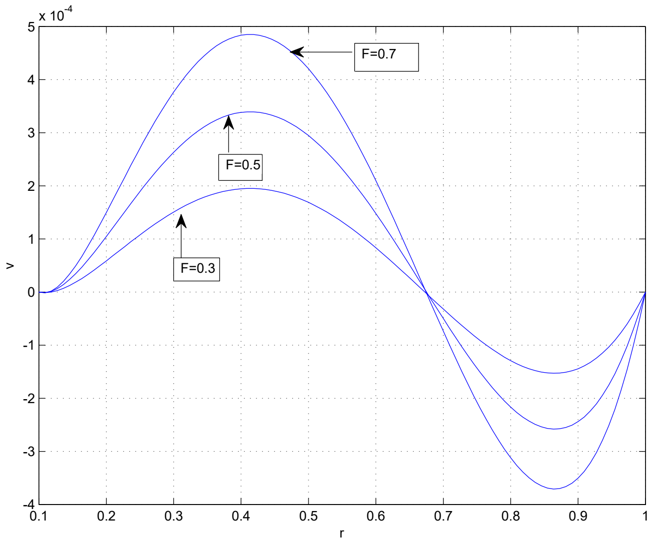

Figure 2: The base flow temperature distribution for three values of the Frank-Kamenetskii parameter F.

1 for three values of the Frank-Kamenetskii parameter F, namely, $F = 0.3$, $F = 0.5$, and $F = 0.7$. Figure 2 plots the distribution of the base temperature T_0 for the same values of the Frank-Kamenetskii parameter.

4 LINEAR STABILITY EQUATIONS

Suppose that $\hat{\mathbf{v}}$, $\hat{T}$ and $\hat{p}$ are small unsteady perturbations of the form

$$\hat{\mathbf{v}}(r,\varphi,z) = \mathbf{u}(r)\exp\left[-\lambda t + i\alpha z + in\varphi\right] \tag{10}$$

$$\hat{T}(r,\varphi,z) = \theta(r)\exp\left[-\lambda t + i\alpha z + in\varphi\right] \tag{11}$$

$$\hat{p}(r,\varphi,z) = q(r)\exp\left[-\lambda t + i\alpha z + in\varphi\right], \tag{12}$$

where $\mathbf{u} = (u(r), v(r), w(r))$, α and n are axial and azimuthal wave numbers, respectively. The perturbed flow is assumed to be of the form $\mathbf{v}_0 + \hat{\mathbf{v}}$, $T_0 + \hat{T}$, $p_0 + \hat{p}$. Substituting the perturbed quantities into (2)–(4) and linearizing the resulting equations in the neighborhood of the base flow we obtain

$$u'' + \frac{u'}{r} - \frac{un^2}{r^2} - \alpha^2 u - \frac{u}{r^2} - \frac{2inv}{r^2} = q' + iGr\alpha u W_0 - \lambda u, \quad (13)$$

$$v'' + \frac{v'}{r} - \frac{vn^2}{r^2} - \alpha^2 v - \frac{v}{r^2} + \frac{2inu}{r^2} = \frac{inq}{r} + iGr\alpha v W_0 - \lambda v, \quad (14)$$

$$w'' + \frac{w'}{r} - \frac{wn^2}{r^2} - \alpha^2 w + \theta = i\alpha q + Gr(uW_0' + i\alpha\theta W_0) - \lambda w, \quad (15)$$

$$\frac{1}{Pr}\left(\theta'' + \frac{\theta'}{r} - \frac{\theta n^2}{r^2} - \alpha^2\theta + F\exp{[T_0]}\theta\right) = Gr(uT_0' + i\alpha\theta W_0) - \lambda\theta, \quad (16)$$

$$u' + \frac{u}{r} + \frac{inv}{r} + i\alpha w = 0. \quad (17)$$

The perturbed velocity components and temperature are equal to zero at the walls of the

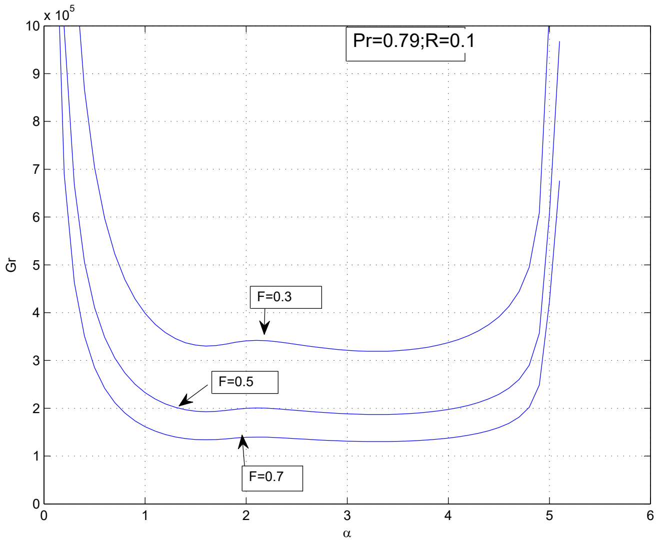

Figure 3: Marginal statbility curves for $0 < \alpha < 5$.

cylinder. In order to reduce the size of the corresponding eigenvalue problem the functions q and w are eliminated from the system. As a result, additional boundary conditions are required to solve the reduced system. These condistions are obtained from the continuity equation (17). Thus, the boundary conditions are

$$u(1) = v(1) = \theta(1) = 0, \quad u(R_0) = v(R_0) = \theta(R_0) = 0, \quad u'(1) = u'(R_0) = 0. \quad (18)$$

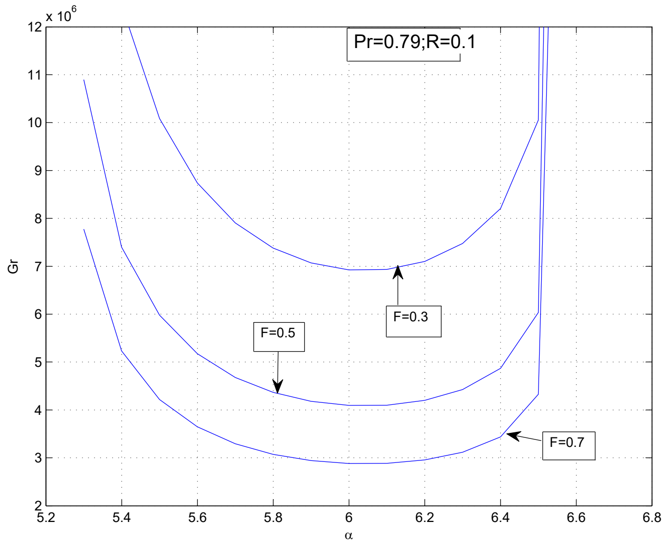

Figure 4: Marginal statbility curves for $\alpha > 5$.

5 NUMERICAL RESULTS

Collocation method based on the Chebyshel polynomials is used to solve the corresponding boundary value problem. The functions $u(r)$, $v(r)$, and $\theta(r)$ are represented in the form

$$u(x) = \sum_{m=0}^{N} a_m(1-x^2)^2 T_m(x), \quad v(x) = \sum_{m=0}^{N} b_m(1-x^2) T_m(x), \quad \theta(x) = \sum_{m=0}^{N} c_m(1-x^2) T_m(x), \tag{19}$$

where $x = 2/(1 - R_0)r - (1 + R_0)/(1 - R_0)$. Here $T_m(x) = cos(marccosx)$ is the Chebyshev polynomial of the first kind of order m and a_m, b_m, and c_m are unknown koefficients. The form of the solution (19) guarantees that all the boundary conditions (18) are automatically satisfied in terms of the transformed variable x. The collocation points are given by

$$x_j = \cos\frac{\pi j}{N}, \quad j = 0, 1, 2, \ldots, N \tag{20}$$

Using (12)–(20) we obtain generalized eigenvalue problem of the form

$$(A - \lambda B)\mathbf{a} = 0, \tag{21}$$

where $\mathbf{a} = (a_0 \ldots a_N b_0 \ldots b_N c_0 \ldots c_N)^T$. Problem (21) is solved numerically using Matlab routine eig. Marginal stability curves for the case $Pr = 0.79$ are shown in Figures 3 and 4 for the three values of the Frank-Kamenetskii parameter. It is seen from the figures that there are two separate brances of the marginal curves: the first branch corresponding to

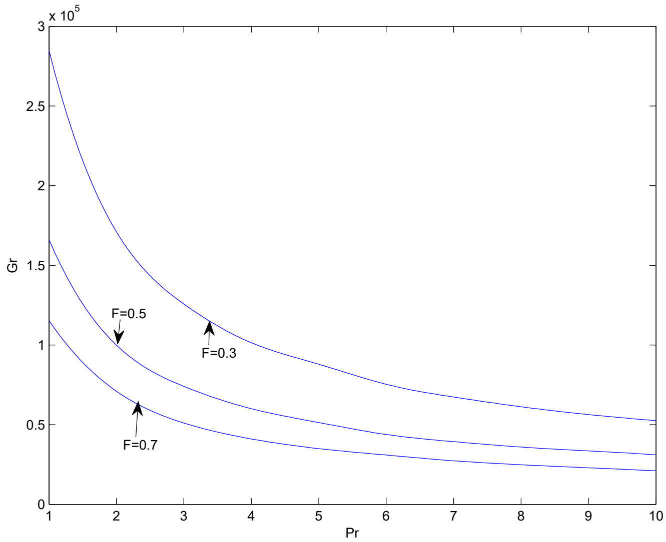

Figure 5: Critical values of the Grasshof number versus Prandtl number.

smaller values of α (and to smaller values of the Grasshof number) in the range $0 < \alpha < 5$ and the second branch corresponding to larger values of α (and to larger values of the Grasshof numbers). Note that two branches of the marginal stability curve are found to exist also in the case of a uniform heat generation, but for considerably larger values of the Prandtl numbers. The region of instability is above the curves.

The critical value of the Grasshof number is defined as the absolute minimum on the marginal curve. Figure 5 plots the critical Grasshof numbers versus the Prandtl number for the three values of the Frank-Kamenetskii parameter. It is seen from the graph that the increase in both parameters has a destabilizing influence of the flow.

6 CONCLUSIONS

Linear stability of a steady convective motion generated by heat sources due to exothermal chemical reaction is analyzed in the paper. Nonlinear problem characterizing base flow is solved numerically using Matlab. Linear stability calculations are performed for the case of asymmetric perturbations. The results show that larger values of the Prandtl number and Frank-Kamenetskii parameter destabilize the flow.

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