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# Wavelet Neural Networks for Volatility Forecasting. Comparative Analysis with Stochastic Models

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**Abstract.** This paper provides volatility forecasting models by using wavelet analysis, variance analysis and NARX architecture neural networks. Implementation of volatility forecasting is applicable for financial market players: institutional investors (banks, insurance companies, private and state pension funds) as well as private investors for volatility forecasting in financial time series. Developed models are compared with worldwide used stochastic models like GARCH(P,Q), EGARCH(P,Q) and GJR(P,Q). According to research results developed models provide better forecasting results in terms than conditional variance models.

**Keywords:** nonlinear autoregressive with exogenous inputs (NARX) neural networks, wavelet analysis, wavelet neural networks, direct discrete wavelet transform, volatility, historical volatility, implied volatility, volatility index (VIX), variance, the north-east volatility wind' effect, stochastic models, conditional variance models, GARCH(P,Q), EGARCH(P,Q) , GJR(P,Q), time series, forecasting.

## 1. Introduction

Current paper might be applicable for financial market players: institutional investors (banks, insurance companies, private and state pension funds) as well as private investors for volatility forecasting in financial time series.

Volatility is applied for option pricing. Option price is directly related to implied volatility, which is a measure of financial market stability. There are financial instruments, which gain market profit from market instability (price fluctuations). Volatility is also important in asset management strategies. Investors are interested to leave market (or reduce open positions) before the market is ceased by instability. In this case volatility is used as a measure of risk.

There are two types of volatility: **Historical volatility - HV** and **Implied volatility - IV**. Both serve as the measure of scattering, but historical volatility is a function of (asset) past market prices, e.g. German DAX30 stock index, Hang-Seng stock index or stock prices BMW, Volkswagen, Alianz, Adidas, IBM stock prices). The term historical volatility in financial mathematics and econometrics implies measures of scattering - variance and standard deviation.

But implied volatility is a function of option prices. Options are derivative contracts. Historical volatility is not tradable and is used for risk management unlike implied volatility which is tradable; therefore, implied volatility dynamics forecasting has a high practical intended use and volatility is traded, for example, example by using **VIX implied volatility index** trades.

This paper continues the scope of publications about volatility forecasting: [12] and publications about so called north-east volatility wind effect [7-12]. So called north-east volatility wind effect, discovered by author, is keeping following idea: small changes in low frequency signal component volatility leads to significant volatility growth in high-frequency components and consequently to overall volatility growth. [10] Further this idea was implemented in volatility forecasting with wavelet based neural networks in paper [12] (which can be conditionally -

called wavelet neural network models). For this purpose NARX (nonlinear autoregressive with exogenous inputs) [3]. Research is realized with Signal decomposition (wavelet decomposition, described in [4, 15]) with subsequent variance analysis on signal components. For modeling wavelet coefficient variance indicator and absolute changes in trend (low-frequency component) in past are used to forecast historical volatility (it is better to say expected volatility) or implied volatility.

Developed wavelet based neural network models are comparable to stochastic models which are classical used for volatility forecasting: GARCH(P,Q), EGARCH(P,Q) and GJR(P,Q) by institutional investors [17]. According research results developed wavelet neural network models are showing better forecasting performance results from MSE (mean squared error) and R (Pearson correlation) perspectives.

## 2. Volatility Forecasting Problems With Stochastic Models

There are several models for financial time series and volatility forecasting in mathematical statistics and probability theory, e.g. Brownian motion, fractal Brownian motion, conditional variance models (CVM). In CVM family the most popular are generalized autoregressive conditional heteroscedastic - GARCH(P,Q), exponential generalized autoregressive conditional heteroscedastic – EGARCH(P,Q) and Glosten, Jagannathan, Runkle – GJR model.

The key problem of volatility forecasting by using CVM family models in it converges to unconditional variance by increasing forecast horizon [6]. This problem is illustrated in Fig. 1 on GARCH (1,1) example, where by rising conditional variance (in other words forecasted volatility value) is getting towards unconditional variance.

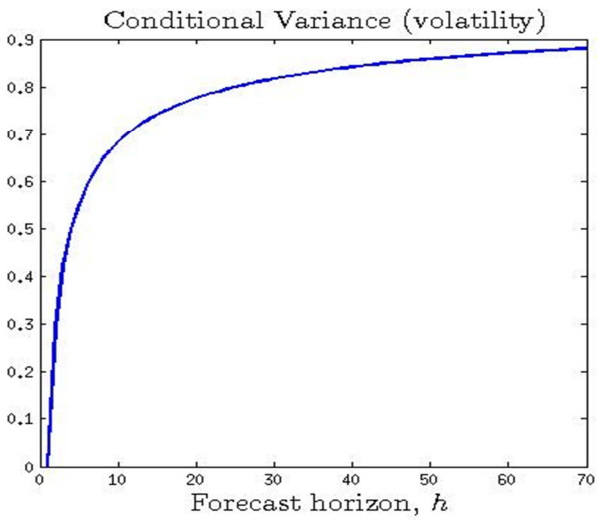


**Fig. 1.** Conditional variance by forecast horizon h for GARCH(1,1) model.

Issue described before subsequently is illustrated in volatility forecasting dynamics on German DAX30 stock index example. The forecast is done by using GARCH (1,1) model for 5 ($h$ = 5) and 12 (h = 12) trading days. For a forecast variable the mean variable of is used, which is compared to empirical values. Volatility forecasting for 5 trading days is shown in Fig. 2a) in time dynamics. Forecast error in time is shown in Fig. 2b) Relevant results shown in Fig. 2A-b) prove that the forecasted volatility is closed to empirical volatility for 5 trading days forecast horizon. Though, rising forecast horizon to dramatically increases forecast error (according the results shown in Fig. 2c-d) results). Analysis results highlight disadvantages of conditional variance model in volatility forecasting. The main reason of the weaknesses is considered in Fig. 1. It is conditional variance (fast or slow) convergence to unconditional variance. This property is typical for all conditional

cariance models. Weaknesses and improvement possibilities as well as alternative volatility forecasting models are reviewed in subsequent sections.

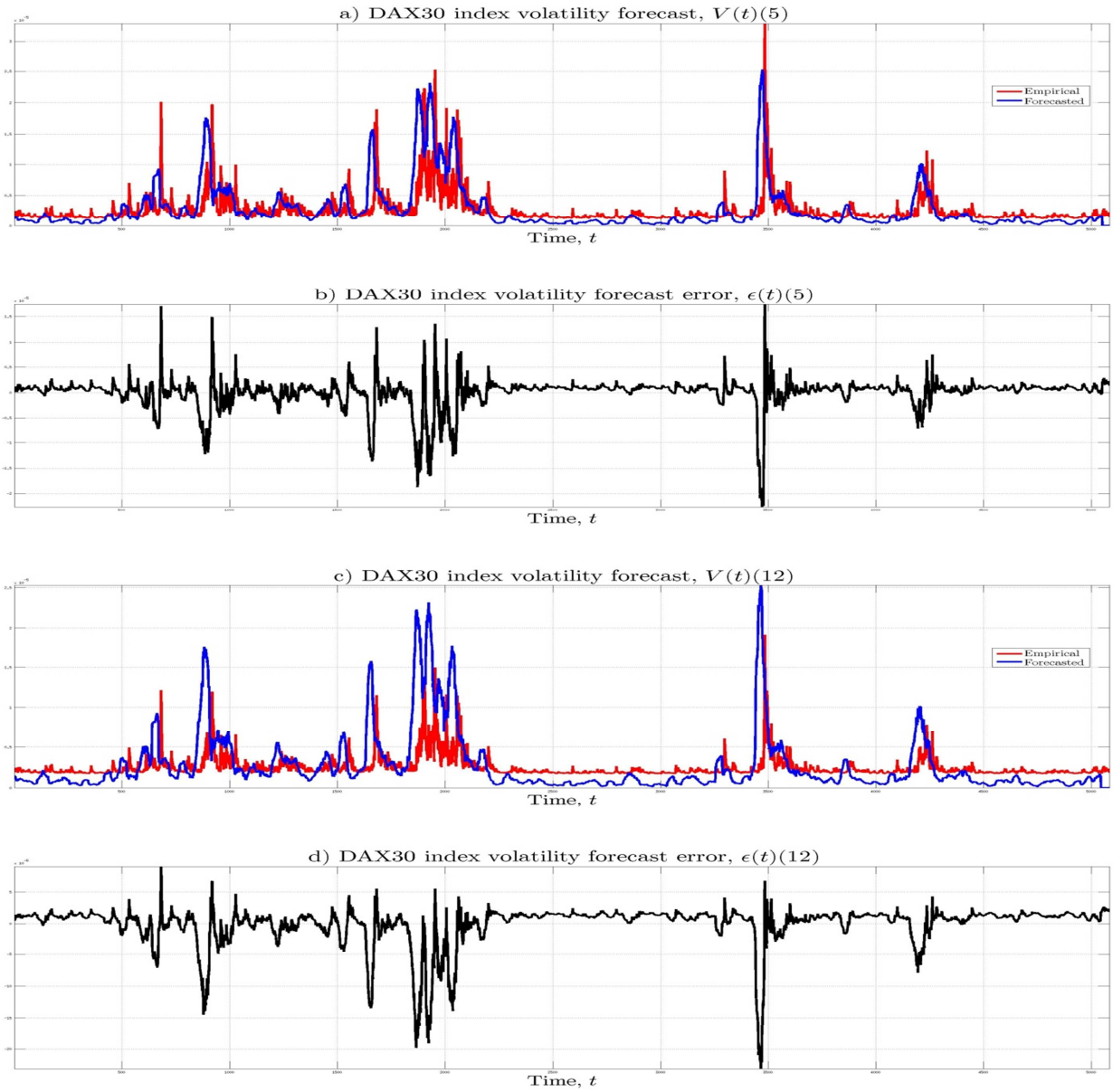


**Fig. 2.** Volatility forecasting at different forecast horizons.

# 3. Wavelet Neural Network Models

## 3.1. Principal Model Components and Functional Dependences

Current section provides analysis of volatility forecasting model development. Developed models use background of north-east volatility wind effect modeling [7, 8, 11] with some simplification of wavelet analysis in order to reduce the number of predictors (factors in models). After discussion and model consideration, the following components of developed models are selected:

wavelet analysis part, variance analysis and neural networks. Model component is shown in Fig. 3.

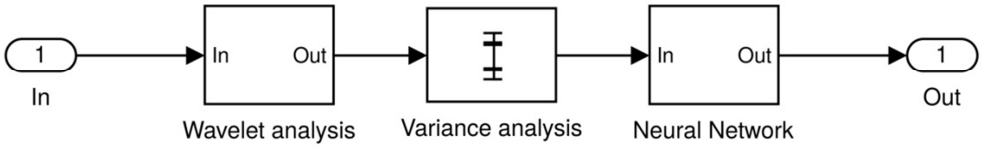


**Fig. 3.** Volatility forecasting model components.

In accordance with the author's experience, the best model for volatility forecasting is nonlinear auto regression implemented in neural network model with NARX (nonlinear autoregressive with exogenous inputs) architecture. The model equation is expressed in equation (1).

$$HV(t+h) = \sum_{d=1}^{D}\sum_{hn=1}^{H} F\big(W_{HV}(d,hn)HV(t-d)\big) + \\ + \sum_{d=1}^{D}\sum_{hn=1}^{H} F\big(W_{dSa}(d,hn)dSa(t-d)\big) + \\ + \sum_{d=1}^{D}\sum_{hn=1}^{H} F\big(W_{V}(d,hn)V(t-d)\big) + b. \quad (1)$$

Nonlinear dependencies are described by sigmoidal function

$$F(u) = \frac{1}{1-e^{-u}}.$$

Formula (1) shows that each argument $HV, dSa, V$ is related to forecasted volatility $HV(t+h)$ by weight matrices. Each weight matrix has lag parameter $D$ and hidden neuron number parameter $H$. The model work is affected with parameters $H$ and $D$ which are related to neural network architecture. Neural network architecture can be optimized by using discrete optimization in genetic algorithm way or by using enumeration approach [5]. Modeling of algorithm is considering in details in other papers of the author.

### 3.2. Model Performance Results

This subsection contains modeling results. The relevant research results, worked out via wavelet neural network models show high performance results in volatility forecasting. For comparative analysis two indicators were selected for outputs and targets data performance: Pearson correlation (R) and mean squared error (MSE). For analysis several stock index data were selected.

Comparative analysis includes German DAX30 index, American SP500 index, Japan NIKKEI225 and China Hang-Seng indexes. Forecast results are compared for developed (author) models and best stochastic model for certain index. Stochastic models are selected from range: GARCH(1-3,1-3), EGARCH(1-3,1-3), GJR(1-3,1-3). The forecast is done for more than 70 trading days horizon. The results are compared on independent set (test and validation set) and on complete set. Results are illustrated graphically in Fig. 4.

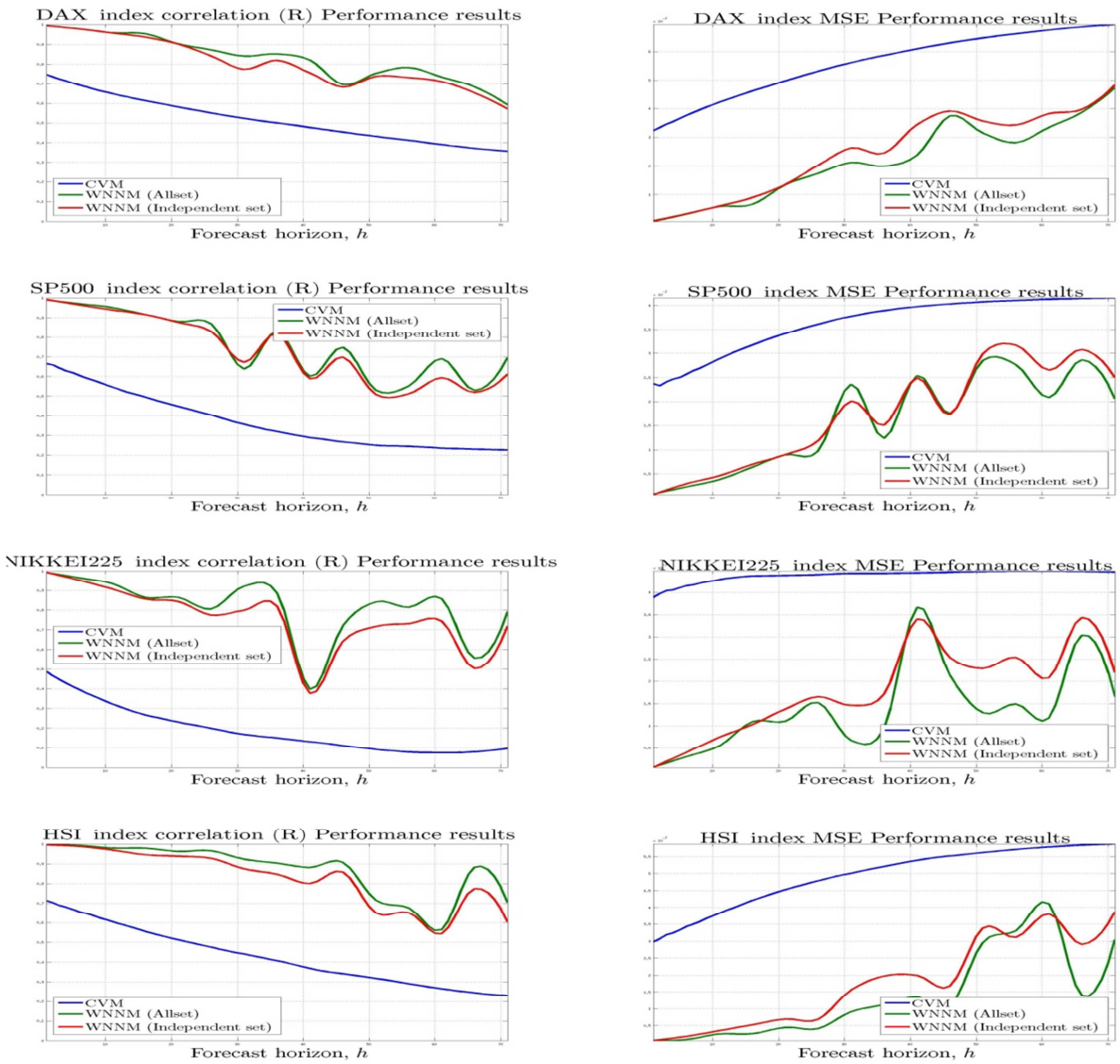


**Fig. 4.** Forecast results comparative analysis.

## 4. Conclusion

Investigating volatility forecasting results for DAX30 stock index, the author concludes the dynamics of worked out models in terms of closely approximate volatility, forecast error. Comparing acquired results to stochastic model results author concludes that developed models approximate volatility dynamics more closely than stochastic models. Hereinafter, model comparative analysis is done.

Comparative analysis includes German DAX30 index, American SP500 index, Japan NIKKEI225 and China Hang-Seng indexes. Forecast results are compared for developed (author) models and best stochastic model for certain index. Stochastic models are selected from range: GARCH(1-3,1-3), EGARCH(1-3,1-3), GJR(1-3,1-3). The forecast is done for more than 70 trading days horizon. The results are compared on independent set (test and validation set) and on complete set.

Comparing the results, the author concludes that developed models show higher performance results than stochastic models (GARCH, EGARCH, GJR) for all forecast horizons. Conclusions are binding for all analyzed stock indexes - Hang-Seng, NIKKEI225, SP500, DAX30. Developed

models show better results in all sets (including independent set). From both - correlation R and MSE (mean squared error) perspectives.

Consequently, based upon the relevant research results and additional tests done from neural network performance perspective, the author concludes that there is small difference in independent set and full set results for short-term forecasts (up to 21-26 trading days, depending on index). This gives grounds to believe that the observed horizon results are stable and safe operability in volatility forecasting.

Longer term horizon use of the developed models is possible, though it is recommended to be more selective in choice of the model which, in its turn, increases overall uptime indicator. Notwithstanding, author models provide better results than stochastic models. The above statement is illustrated in Fig. 4 model results.

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