ASMDA 2015

Proceedings

16th Applied Stochastic Models and Data Analysis International Conference

With 4th Demographics 2015 Workshop

Editor

Christos H Skiadas

30 June – 4 July 2015

University of Piraeus, Greece

e-ISBN: 978-618-5180-05-8

Application of the Stochastic Models in Operational Risk Modelling

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Abstract. Operational risk is one of the core risks of every insurance company under the Solvency II framework and can be defined as the financial losses occurred due to incorrectly defined systems or processes; failures in IT system, human mistakes or other external processes. The research is performed in order to assess the capital to cover possible losses due to the occurrence of the operational risk sub-risks and nature of an operational risk. We have shown that operational risks can be modelled by skew *t*-copula and estimated tail dependence in each situation for modelling distributions with heavier tail area. The model is prepared on a non-life insurance company's example and is based on the recorded data from loss database that encompasses historical information of five main operational sub-risks: legal, informational, organizational, human resources and expense risk.
Keywords: Operational risk, skew *t*-copula, *t*-copula, tail dependence, modelling, solvency capital, insurance.

1 Introduction

The fact is that the requirements of the Solvency II Directive are not just about capital of an insurance company but about risk assessment through the implementation and enhancement of risk measurement and risk management. Also, the Solvency II regime requires higher capital compared with the requirements of the Solvency I Directive that should ensure the solvency and financial stability of each insurance company. Moreover, the new requirements of the Solvency II Directive, which will come in force from 1st January 2016, set a lot of challenges to every insurance company in the European Union member states in relation to the establishment of more sensitive and sophisticated risk coverage in order to ensure solvency and the safety of policyholders. Based on the requirements of the Solvency II Directive, the

16th ASMDA Conference Proceedings, 30 June – 4 July 2015, Piraeus, Greece

insurance companies should hold the appropriate amount of capital that could ensure safety of policyholders and beneficiaries. The target of this research is to study the improvement possibilities of the operational risk measurement under the Solvency II regime. The object of this paper is measurement of operational risk. Operational risk is the change in value of capital needed caused by the fact that actual losses, incurred from inadequate or failed internal processes, people and systems, or from external events, including legal risk but excluding strategic and reputational risks Since 2001, when document about operational risks *Sound Practices for the Management and Supervision of Operational Risk* was published by Basel Committee on Banking Supervision [2] operational risk has been in the centre of interest of mathematicians. Because needed capital for different risks in banks is estimated by risk measure *VaR* (what is 99.9% in banks and 99.5% in insurance), it seems natural to use the same measure for operational risk too. But the problem is that *VaR* measure is not a coherent risk measure: $VaR_\alpha(\sum_{i=1}^{n} R_i) \leq \sum_{i=1}^{n} VaR_\alpha(R_i)$, where R_i, $i \in \{1,2,...,n\}$ are different risks. Therefore, different bounds for *VaR* of a portfolio of risks can be found in Chavez-Demoulin *et al*. [5] or improved bounds in Embrechts and Puccetti [10]. Further different copulas (Gumbel, Gaussian) were used for analysis of risk across a non-symmetric matrix of loss data in Embrechts and Puccetti [11]. Extreme value theory was used to evaluate operational risks in El-Gamal *et al*. [9], Chavez-Demoulin *et al.* [6]. Our aim in this paper is to show that skew *t*-copula can be used to estimate *VaR* of portfolio of different operational risks including confidence intervals for such as risk measure like *VaR* and finally calculate estimates of tail dependence for risks and for portfolio. We have worked out our methodology using data basis of recorded operational risks during one year in one insurance company of Latvia.

2 Construction of skew *t*-copula

We are going to model the joint distribution of different risks via skew *t*-copula to show advantage of the last one. Usually operational risk data have univariate marginals with skewed distributions of different types. To construct a multivariate model with certain dependence structure and different marginals copula theory has been the only tool at hand so far. But most of the suggested copulas are symmetric. To join skewed marginals into a multivariate distribution it seems more natural to use a skewed multivariate distribution. There exist many different modifications and extensions of the standard multivariate *t*-distribution. An overview of these distributions is given in Kotz and Nadarajah [15], Ch. 5. We have constructed skew *t*-copula based on the multivariate *t*-distribution and skew *t*-distribution introduced in Azzalini and Capitanio [1] and corresponding copulas constructed using these distributions. Notation $t_{p,\nu}$ is used when we talk about density of the p-variate t-distribution with ν degrees of freedom and notation $g_{p,\nu}$ is used for the density of the p-variate skew t-distribution with ν degrees of freedom. Similar notations are used for the distribution functions.

DEFINITION 1. A p-dimensional random vector $\mathbf{X}=(X_1,...,X_p)^T$ *is said to have p-variate t-distribution with* ν *degrees of freedom, mean vector* $\boldsymbol{\mu}$ *and positive definite matrix* $\boldsymbol{\Sigma}$, *if its density function is given by (*Azzalini and Capitanio [1]*):*

$$t_{p,\nu}(\mathbf{x},\boldsymbol{\mu},\boldsymbol{\Sigma})=\frac{\Gamma\left(\frac{\nu+p}{2}\right)}{(\pi\nu)^{\frac{p}{2}}\Gamma\left(\frac{\nu}{2}\right)|\boldsymbol{\Sigma}|^{\frac{1}{2}}}\left[1+\frac{(\mathbf{x}-\boldsymbol{\mu})^T\boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})}{\nu}\right]^{-\frac{\nu+p}{2}}. \quad (1)$$

Next we give the definition of the p-dimensional skew $t_{p,\nu}$-distribution (Azzalini and Capitanio [1]).

DEFINITION 2. A random p-vector $\mathbf{X}=(X_1,...,X_p)^T$ *has p-variate skew* t-*distribution with parameters* $\boldsymbol{\mu}$, $\boldsymbol{\alpha}$ *and* $\boldsymbol{\Sigma}$ *, if its density function is of the form*

$$g_{p,\nu}(\mathbf{x};\boldsymbol{\mu},\boldsymbol{\Sigma},\boldsymbol{\alpha})=2\cdot t_{p,\nu}(\mathbf{x};\boldsymbol{\mu},\boldsymbol{\Sigma})\cdot T_{1,\nu+p}\left[\boldsymbol{\alpha}^T\mathbf{W}^{-1}(\mathbf{x}-\boldsymbol{\mu})\left(\frac{\nu+p}{Q+\nu}\right)^{\frac{1}{2}}\right], \quad (2)$$

where Q *denotes the quadratic form*

$$Q=(\mathbf{x}-\boldsymbol{\mu})^T\boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})$$

and $\mathbf{W}$ *is the* $p\times p$ *diagonal matrix* $\mathbf{W}=(\delta_{ij}\sqrt{\sigma_{ij}})$, $i,j=1,...,p$, *where* δ_{ij} *is the Kronecker delta.* $T_{1,\nu+p}(\cdot)$ *denotes the distribution function of the central univariate* t *- distribution with* $\nu+p$ *degrees of freedom.*

The skew t-copula is introduced in Kollo and Pettere [13]. As marginal distributions of the business lines are skewed, a skewed copula will be a natural model to give a good fit with the data.

DEFINITION 3. A copula $C_{p,\nu}$ *is called skew* $t_{p,\nu}$-*copula with parameters* $\boldsymbol{\mu}$, $\boldsymbol{\Sigma}$, $\boldsymbol{\alpha}$, *if*

$$C_{p,\nu}(u_1,...,u_p:\boldsymbol{\mu},\boldsymbol{\Sigma},\boldsymbol{\alpha})=G_{p,\nu}(G_{p,\nu}^{-1}(u_1;\mu_1,\sigma_{11},\alpha_1),...,G_{p,\nu}^{-1}(u_p;\mu_p,\sigma_{pp},\alpha_p),\boldsymbol{\mu},\boldsymbol{\Sigma},\boldsymbol{\alpha})$$

where $G_{p,\nu}^{-1}(u_i;\mu_i,\sigma_{ii},\alpha_i)$, $i\in\{1,2,...,p\}$ denotes the inverse of the univariate skew $t_{p,\nu}$*-distribution function and* $G_{p,\nu}$ *is the distribution function of p-variate skew* $t_{p,\nu}$ *-distribution with density (2).*

The corresponding copula density function is

$$c_{p,\nu}(\mathbf{u};\boldsymbol{\mu},\boldsymbol{\Sigma},\boldsymbol{\alpha}) = \frac{g_{p,\nu}[\{G_{1,\nu}^{-1}(u_1;0,\sigma_{11},\alpha_1),\ldots,G_{1,\nu}^{-1}(u_p;0,\sigma_{pp},\alpha_p)\};\boldsymbol{\mu},\boldsymbol{\Sigma},\boldsymbol{\alpha}]}{\prod_{i=1}^{p} g_{1,\nu}[G_{1,\nu}^{-1}(u_i;\mu_i,\sigma_{ii},\alpha_i);\mu_i,\sigma_{ii},\alpha_i]}$$

where the density function $g_{p,\nu}(\cdot;\boldsymbol{\mu},\boldsymbol{\Sigma},\boldsymbol{\alpha}): R^p \to R$ *is defined by (2) and function* $G_{1,\nu}^{-1}(u_i;\mu_i,\sigma_{ii},\alpha_i)$ *is as in Definition 3.*

We are going to apply the skew *t*-copula in a special case when the shift parameter $\boldsymbol{\mu} = \mathbf{0}$. To find a model for our data we have to estimate the parameters $\boldsymbol{\Sigma}$ and $\boldsymbol{\alpha}$. For that, we shall apply the method of moments. Parameters $\boldsymbol{\Sigma}$ and $\boldsymbol{\alpha}$ are estimated from the first two sample moments (Kollo and Pettere [13]). Let $\bar{\mathbf{X}}$ and $\mathbf{S}_{\mathbf{X}}$ denote the sample mean and the sample covariance matrix, respectively. Then the estimates are

$$\hat{\boldsymbol{\Sigma}} = \frac{\nu-2}{\nu}(\mathbf{S}_{\mathbf{X}} + \bar{\mathbf{X}}\bar{\mathbf{X}}^T) \tag{3}$$

$$\hat{\boldsymbol{\alpha}} = \frac{b(\nu)\cdot\boldsymbol{\beta}}{\sqrt{b^2(\nu) - \bar{\mathbf{X}}^T\hat{\boldsymbol{\Sigma}}^{-1}\bar{\mathbf{X}}}}\ , \tag{4}$$

where

$$\boldsymbol{\beta} = \frac{1}{b(\nu)}\hat{\mathbf{W}}\hat{\boldsymbol{\Sigma}}^{-1}\bar{\mathbf{X}}, \tag{5}$$

with $\hat{\mathbf{W}} = (\delta_{ij}\sqrt{\hat{\sigma}_{ij}})$, $i, j = 1,\ldots,p$, *where* δ_{ij} *is the Kronecker delta and*

$$b(\nu) = \left[\frac{\nu}{\pi}\right]^{\frac{1}{2}} \cdot \frac{\Gamma(\frac{\nu-1}{2})}{\Gamma(\frac{\nu}{2})}.$$

We have to assume in formula (4) that $\nu > 2$.

Variable ν is possible to estimate between every two variables using formula from Kotz and Nadarajah [15]:

$$3\nu^2 - (\nu-2)(\nu-4)(m_4(X_1) + m_4(X_2)) = 0 \tag{6}$$

where $m_4(X_i)$ denotes the sample estimate of the fourth order moments of random variable X_i , $i \in \{1,2\}$. The estimates are closest integers to the solution of equation (6) and can be found for $\nu > 4$.

3 Tail dependence for skew *t*–distribution

Let us assume that (X_1, X_2) is a two-dimensional vector with univariate marginal distributions functions $F_1(x)$ and $F_2(x)$. Then the upper tail dependence coefficient is

$$\lambda_U = \lim_{u \to 1} \lambda_U(u)$$

where $\lambda_U(u) = P(F_1(x) > u / F_2(x) > u)$.

Similarly is defined the lower tail dependence coefficient

$$\lambda_L = \lim_{u \to 1} \lambda_L(u)$$

where $\lambda_L(u) = P(F_1(x) < u / F_2(x) < u)$.

For symmetric elliptical distributions $\lambda_U = \lambda_L = \lambda$, for normal distributions λ equals zero. For two-dimensional *t*-distribution with ν degrees of freedom

$$\lambda = 2T_{1,\nu}\left(-\sqrt{\frac{(\nu+1)\cdot(1-\rho)}{(\rho+1)}} \right) \tag{7}$$

where $T_{1,\nu}(.)$ is the distribution function of standard *t*-distribution with ν degrees of freedom (see Demarta and McNeil [7])

It is proved in Bortot [3] that it is sufficient to study the upper tail dependence as the lower tail dependence coefficient is determined by the upper one. To follow Bortot [3] let us denote by

$$\alpha_1^* = \frac{\alpha_1 + \alpha_2 \cdot \rho}{\sqrt{1 + \alpha_2^2 \cdot (1-\rho^2)}} \quad \text{and} \quad \alpha_2^* = \frac{\alpha_2 + \alpha_1 \cdot \rho}{\sqrt{1 + \alpha_1^2 \cdot (1-\rho^2)}} \tag{8}$$

Assume that $\alpha_1^* \le \alpha_2^*$. Then

$$\begin{aligned}
\lambda_U &= \lim_{u \to 1} \frac{P(F_1(x) > u, F_2(x) > u)}{P(F_2(x) > u)} = \lim_{x \to \infty} \frac{P(F_1(x) > F_2(x), X_2 > x)}{P(X_2 > x)} \\
&\ge \lim_{x \to \infty} \frac{P(X_1 > x, X_2 > x)}{P(X_2 > x)} \\
&= \lim_{x \to \infty} \frac{2 \cdot P(Y_1 > x, Y_2 > x) \cdot T_{1,\nu+2}\left((\alpha_1 + \alpha_2) \cdot \sqrt{\frac{(\nu+2)\cdot(\rho+1)}{2}} \right)}{2 \cdot T_{1,\nu+1}(\alpha_2^* \cdot \sqrt{\nu+1}) \cdot (1 - T_{1,\nu}(x))}
\end{aligned}.$$

or

$$\lambda_U \geq \lambda \cdot \frac{T_{1,\nu+2}\left((\alpha_1+\alpha_2)\cdot\sqrt{\frac{(\nu+2)\cdot(\rho+1)}{2}}\right)}{T_{1,\nu+1}(\alpha_2^* \cdot \sqrt{\nu+1})}. \qquad (9)$$

In the case of $\alpha_1 = \alpha_2 = \alpha$ the tail dependence coefficient can be calculated using formula:

$$\lambda_U = \lambda \cdot \frac{T_{1,\nu+2}\left(2\cdot\alpha\cdot\sqrt{\frac{(\nu+2)\cdot(\rho+1)}{2}}\right)}{T_{1,\nu+1}(\alpha^* \cdot \sqrt{\nu+1})}$$

where $\alpha^* = \frac{\alpha\cdot(1+\rho)}{\sqrt{1+\alpha^2\cdot(1-\rho^2)}}$.

The fact is that the difference of tail dependencies between *t*-distribution and skew *t*-distribution is determined by the ratio of univariate distribution functions of the *t*-distribution. It is shown in Bortot [3] that for the equal values of α the difference in tail dependence is not large.

4 Description of the model and data

The simulation model performed during the case study is based on five risks, but it can be used for any number of risks. The model includes the following operational sub-risks:

- Legal risk (LR) means the possibility that lawsuits, adverse judgments from courts, or contracts that turn out to be unenforceable, disrupt or adversely affect the operations or condition of an insurer. The result may lead to unplanned additional payments to policyholders or that contracts are settled on an unfavorable basis, e.g. unrecoverable reinsurance.
- Organizational risk (OR) means possible losses due to unclear organizational structure (unclear processes, unclear responsibilities split between units etc.).
- Informational risk (IR) means possible losses due to failures in the IT system.
- Human Resources risk (HRR) means losses due to changes or loss of personnel, deterioration of morale, inadequate development of human resources, inappropriate working schedule, inappropriate working and safety environment, inequality or inequity in human resource management or discriminatory conduct.
- Expense risk (ER). The risk of a change in value caused by the fact that the timing and/or the amount of expenses incurred differs from those expected, e.g. assumed for pricing basis.

The historical data is based on recorded data in relation to the five risk sub-risks of operational risk from the annual loss database. The loss database introduces all incurred operational risk events with details about losses during a particular period and is important aspect of the understanding of interconnectivity of different operational sub-risks; thus is a prerequisite to controlling problems and assessing practices.

Basically, the model is based on several main steps:

1) data collection,
2) determination of a marginal distribution of each operational sub-risk,
3) simulation of 10 000 values of each risk using skew *t*-copula,
4) calculating *VaR* of each marginal,
5) finding *VaR* for total portfolio of operational risk,
6) repeating 30 times steps 3 to 5 and calculating descriptive statistics.

Descriptive statistics of the marginal distributions of the above-mentioned risks are presented in Table 1.

Table 1. Descriptive statistics of used data.

Risks	LR	OR	IR	HRR	ER
Sample size	12	12	12	12	12
Mean	7 564	45 618	5 425	1 747	2 308
Median	3 700	1 610	960	18	0
Standard deviation	11 151	143 207	9 342	4 490	6 655
Largest value	41 278	500 010	31 010	15 001	43 000
Skewness	2.92	3.45	2.21	2.82	3.24

All operational risks are skewed, but the largest maximum value has organizational risk. Risk with so large maximal value was chosen specially to check does model fit in such case too. Before fitting marginal distributions, the data were standardised and only then, the marginal distributions were approximated by exponential, gamma and normal distributions.

The testing results are shown in Table 2.

Table 2. Results of testing.

Risks	Distribution used	Parameters		Test value
LR	Exponential	λ	1.474	0.164
OR	Gamma	α	0.101	0.169
		β	3.139	
IR	Gamma	α	0.227	0.096
		β	2.098	
HRR	Gamma	α	0.152	0.338
		β	2.569	
ER	Normal	μ	3.352	0.079
		σ	1.000	

Correlation matrix between risks (order of risks from left to right and from up to down is LR, OR, IR, HRR and ER) is the following:

$$R = \begin{pmatrix} 1 & -0.143 & 0.357 & 0.183 & 0.071 \\ -0.143 & 1 & -0.118 & -0.135 & -0.085 \\ 0.357 & -0.118 & 1 & -0.086 & -0.132 \\ 0.183 & -0.135 & -0.086 & 1 & -0.063 \\ 0.071 & -0.085 & -0.132 & -0.063 & 1 \end{pmatrix}$$

One can see that smallest correlations are between LR and ER (0.071) and HRR and ER (-0.063).
Estimations of parameters were started by estimating degrees of freedom ν between each two pairs of variables by using formula (6) (see Table 3.)

Table 3. Estimated values of ν by formula (6).

i	**OR**	**IR**	**HRR**	**Ex**
LR	1.839313	1.816586	1.820098	1.833699
OR		1.831101	1.834072	1.845673
IR			1.809678	1.824871
HRR				1.828069

Nearest possible integer in all cases are 2. Formula (6) is right only if $\nu > 4$ and skew *t*-copula is possible to use from $\nu > 2$. In our case formula (4) is possible to use only with $\nu = 5$ and therefore we have chosen $\nu = 5$.
Further parameters for copula were estimated using formulas (3), (4) and (5). The obtained $\hat{\mathbf{\Sigma}}$ matrix is:

$$\hat{\mathbf{\Sigma}} = \begin{pmatrix} 0.876 & 0.044 & 0.408 & 0.268 & 0.186 \\ 0.044 & 0.661 & 0.020 & -0.007 & 0.016 \\ 0.408 & 0.020 & 0.736 & 0.060 & 0.022 \\ 0.268 & -0.007 & 0.060 & 0.691 & 0.044 \\ 0.186 & 0.016 & 0.022 & 0.044 & 0.674 \end{pmatrix}$$

Estimated values of vector $\boldsymbol{\alpha}$ are

$$\boldsymbol{\alpha}^T = (1.675\ \ 1.657\ \ 1.518\ \ 1.394\ \ 1.408)\,.$$

The simulation is based on the simulation rule for the skew $t_{p,\nu}$-distribution (Kollo and Pettere [13]):

1. Find the Cholesky decomposition $\mathbf{A}$ of $\mathbf{S}_{\mathrm{X}}$, ($\mathbf{A}\mathbf{A}^T = \mathbf{S}_{\mathbf{X}}$).
2. Simulate *p* independent values from *N(0,1)* and form *p*-vector $\mathbf{z}$.
3. Set vector $\mathbf{x} = \mathbf{A} \cdot \mathbf{z}$.
4. Simulate value w from *N(0,1)*.

5. Get realization of the skew normal vector $\mathbf{y}$ putting
$$\mathbf{y} = \begin{cases} \mathbf{x} & if \quad \boldsymbol{\alpha}^T \mathbf{x} > w \\ -\mathbf{x} & if \quad \boldsymbol{\alpha}^T \mathbf{x} \le w \end{cases} .$$

6. Simulate $h \sim \chi^2_\nu$.
7. Find vector $\mathbf{t} = \frac{\mathbf{y}}{\sqrt{h/\nu}}$.
8. Set vector $\mathbf{u}$ so that every coordinate $u_i = G_{1,\nu}(t_i; 0, \sigma_{ii}, \alpha_i)$, $i \in [1,...,p]$.
9. Set vector $\mathbf{x} = (F_1^{-1}(u_1),...,F_p^{-1}(u_p))$ where $F_i(\cdot)$ is the marginal distribution function of the initial random variable X_i.
10. Repeat previous steps 10 000 times.

5 Results

Based on the performed simulations, it is possible to conclude that the obtained portfolio *VaR* by simulations is smaller than sum of *VaR* for different risks and it means that the necessary capital to cover these risks is less by 10.3%. The main findings and results of simulation are in Table 4 and Table 5. In order to understand the information presented in Table 4, the explanation of some values are provided:

- The first line presents the 99.5% *VaR* for each sub-risk using inverse marginal distributions.
- The next lines present characteristics of 99.5% *VaR* for each sub-risk and portfolio obtained from simulations.

Table 4. 99.5% *VaR* of separate risks obtained using simulations and its characteristics.

Risks	**LR**	**OR**	**IR**	**HRR**	**ER**
99.5% VaR from distributions	40 078	947 292	55 567	28 530	19 450
Mean of *99.5% VaR*	39 980	882 287	53 803	27 247	18 936
Median	39 891	875 210	53 560	27 414	18 992
Standard deviation	908	50 990	1 700	1 395	224
Skewness	0.426	0.923	0.591	-0.023	-0.921
Coefficient of variation (%)	2.27	5.78	3.15	5.12	1.18

The same characteristics for portfolio of risks as for each risk in Table 4 are shown in Table 5.

Table 5. 99.5% *VaR* of portfolio obtained using simulations and its characteristics.

	Sum of *VaR*	**Portfolio *VaR***
99.5% VaR from distributions	1 090 917	
Mean of *99.5% VaR*	1 022 333	916 576
Median	1 015 070	910 795
Standard deviation	50569	44 408
Skewness	0.938	0.937
Coefficient of variation (%)	4.95	4.84

Results in Table 4 and in Table 5 show us that simulation results from the skew *t*-copula are stable. Medians are close to means, skewness coefficients and variation coefficients are not large. Therefore, it is possible to assume approximate normal distribution of simulated mean and to calculate confidence intervals for portfolio *VaR*. Estimated confidence intervals for portfolio *VaR* are shown in Table 6.
Additionally it is possible to see from Table 5 that gain of using copula approach is EUR 105 757 or 10% decreasing in capital needed.

Table 6. Confidence interval of portfolio *VaR*.

Confidence probability	Portfolio *VaR*	Lower limit	Upper limit
99.5%	916 576	895 693	937 459

Calculated limits of confidence intervals show us that even upper limit for 99.5 % confidence is lower than portfolio *VaR* obtained simply by adding different risk *VaR*. The tail dependence coefficient calculations for given risks using formulas (7), (8) and (9) are presented in Table 7 and in Table 8. Like it is possible to see from Table 7 and Table 8, tail dependence coefficients are not large but tail dependence exists.

Table 7. Tail dependence coefficients between LR and other risks.

Risks	**LR – OR**	**LR – IR**	**LR – HRR**	**LR – ER**
λ	0.0183	0.0762	0.0486	0.0357
α_1^*	0.7489	1.2772	1.1372	1.0294
α_2^*	0.7324	1.1389	0.8822	0.7841
$T_{1,\nu+2}$	0,9996	0.9998	0.9997	0.9997
$T_{1,\nu+1}$	0.9385	0.9385	0.9385	0.9385
λ_U	**0.0196**	**0.0812**	**0.0518**	**0.0380**

Table 8. Tail dependence coefficients between other risks.

Risks	**OR – IR**	**OR – HRR**	**OR – ER**
λ_U	0.0215	0.0213	0.0241
Risks		**IR – HRR**	**IR – ER**
λ_U		0.0238	0.0206
Risks			**HRR – ER**
λ_U			0.0252

There is no possibility to calculate in direct way tail dependence for skew *t*-distribution because in formula (9) is not the equality sign. Unique what is possible to do is to estimate tail dependence coefficients between each two risks using formula (9) and to have in mind that in reality it can be slightly larger. It is shown in Kollo *et al*. [14] that the upper tail dependence coefficient for skew *t*-copula differs not much when skewness parameters have the same sign and when one of them has positive and another negative value, then skew *t*-copula can have much bigger tail dependence coefficient than the corresponding *t*-copula. Because skewness parameters, which are presented by vector, $\boldsymbol{\alpha}$ are with same sign and close to each other, we can conclude that at least in this case tail dependence does not differ much from the calculated values.

Conclusions

Risk dynamic nature in the changing market conditions sets a lot of challenges to every company. Thus, it is necessary to implement new approaches to follow the nature of risks with the aim to understand their possible impact on financial stability and further development. Under Solvency II regime insurance companies like banks will need to evaluate necessary capital to cover different risks. The largest problem can be to evaluate operational risks because of lack of data. For that reason, many different methods are created to evaluate operational risks. Many methods are based on expert evaluations (see, for example, Durfee and Tselykh [8], Jonek-Kowalska [12] and Stepcenko and Voronova [17]). However, from another side it is very natural to evaluate operational risks by using statistical methods like all other insurance and banking risks. For that is necessary to record very carefully losses in each company. If such data basis exists, we have shown that needed capital for operational risks can be evaluated by different statistical methods. Many new books have appeared in latest years about evaluation of operational risks. Latest books, for example, are Cavestany *et al*. [4] and McConnell [16]. Privilege of that paper is using skew *t*-copula modelling necessary capital to cover operational risks.

Advantages of the proposed method are:

- the skew *t*-copula has a very simple simulation rule,
- by choosing degrees of freedom is possible to find appropriate skewness of copula for simulation,

- possibility to calculate average measure of necessary characterstics,
- possibility to estimate sensitivity of calculated measure,
- possibility to calculate confidence interval of portfolio value at risk,
- tail dependence can be evaluated between risks.

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